

Appendix A

The explanation for the Convex Hull Pricing (ConvHP) model about its formulation and relationship with the Locational Marginal Pricing (LMP) method is shown as follows.

(1) The explanation for the ConvHP model about its formulation

In the manuscript, the bi-level pricing problem in (4)-(5) is a more general formulation of the ConvHP problem in (2). The transformation from the ConvHP problem (2) to the general bi-level programming problem (4)-(5) is described in detail as follows.

In the manuscript, the ConvHP problem is expressed as

$$\max_{\sigma_i \geq 0, \forall t} \left(\min_{c, x, z} \sum_t \sum_i c_{i,t} - \sum_t (\sigma_t)^T \left(\sum_i \mathbf{M}_{i,t} x_{i,t} - \mathbf{b}_t \right) \right) \quad (\text{A1})$$

$$(c_i, x_i, z_i) \in V_i, \forall i \in I$$

where, i and t are respectively the indices of market participants and time intervals; c_i , x_i and z_i are respectively the operation cost variable, power output variable, binary operation state variable of participant i ; $\mathbf{M}_{i,t}$ is the coefficient vector for power output $x_{i,t}$ in the system-wide constraint; \mathbf{b}_t is the resource limit vectors of the system-wide constraint; V_i is the non-convex feasible region of participant i ; σ_t is the lagrangian dual multiplier for system-wide constraints at time t (regarded as the price of system resources).

It can be reformulated as

$$\max_{\sigma_i \geq 0, \forall t} \left(\min_{c, x, z} \left(\sum_t \sum_i c_{i,t} - \sum_t \sum_i (\sigma_t)^T \mathbf{M}_{i,t} x_{i,t} + \left(\sum_t \sum_i (\sigma_t)^T \mathbf{M}_{i,t} x_{i,t}^D - \sum_t \sum_i c_{i,t}^D \right) \right) \right. \\ \left. + \sum_t (\sigma_t)^T \mathbf{b}_t - \left(\sum_t \sum_i (\sigma_t)^T \mathbf{M}_{i,t} x_{i,t}^D - \sum_t \sum_i c_{i,t}^D \right) \right) \quad (\text{A2})$$

$$(c_i, x_i, z_i) \in V_i, \forall i \in I$$

where, $x_{i,t}^D$ and $c_{i,t}^D$ are the dispatch instructions decided by the market operator, and they are constants in the above problem.

The electricity price under the ConvHP is derived as

$$\lambda_{i,t} = (\sigma_t)^T \mathbf{M}_{i,t}, \forall i \in I, \forall t \in T \quad (\text{A3})$$

By substituting equation (A3) into (A2), we can obtain

$$\max_{\sigma_i \geq 0, \forall t} \left(\min_{c, x, z} \left(\sum_t \sum_i c_{i,t} - \sum_t \sum_i \lambda_{i,t} x_{i,t} + \left(\sum_t \sum_i \lambda_{i,t} x_{i,t}^D - \sum_t \sum_i c_{i,t}^D \right) \right) \right. \\ \left. + \sum_t (\sigma_t)^T \mathbf{b}_t - \left(\sum_t \sum_i (\sigma_t)^T \mathbf{M}_{i,t} x_{i,t}^D - \sum_t \sum_i c_{i,t}^D \right) \right) \quad (\text{A4})$$

$$(c_i, x_i, z_i) \in V_i, \forall i \in I$$

By rearranging the above equation, we can obtain

$$\max_{\sigma_t \geq 0, \forall t} \left(\min_{c, x, z} \left(\begin{aligned} &\sum_t \sum_i (\lambda_{i,t} x_{i,t} - c_{i,t}) - \sum_t \sum_i (\lambda_{i,t} x_{i,t}^D - c_{i,t}^D) \\ &(\mathbf{c}_i, \mathbf{x}_i, \mathbf{z}_i) \in \mathbf{V}_i, \forall i \in I \\ &+ \sum_t (\boldsymbol{\sigma}_t)^T \mathbf{b}_t - \sum_t \sum_i (\boldsymbol{\sigma}_t)^T \mathbf{M}_{i,t} x_{i,t}^D \end{aligned} \right) \right) \quad (\text{A5})$$

By changing the sign of the objective function, the problem (A5) can be equivalently transformed as

$$\min_{\sigma_t \geq 0, \forall t} \left(\max_{c, x, z} \left(\begin{aligned} &\sum_t \sum_i (\lambda_{i,t} x_{i,t} - c_{i,t}) - \sum_t \sum_i (\lambda_{i,t} x_{i,t}^D - c_{i,t}^D) \\ &(\mathbf{c}_i, \mathbf{x}_i, \mathbf{z}_i) \in \mathbf{V}_i, \forall i \in I \\ &+ \sum_t (\boldsymbol{\sigma}_t)^T (\sum_i \mathbf{M}_{i,t} x_{i,t}^D - \mathbf{b}_t) \end{aligned} \right) \right) \quad (\text{A6})$$

Then, it can be reformulated as

① Upper-level problem

$$\min_{\lambda, \sigma} \sum_t \sum_i (\lambda_{i,t} x_{i,t}^{L*} - c_{i,t}^{L*}) - \sum_t \sum_i (\lambda_{i,t} x_{i,t}^D - c_{i,t}^D) + \sum_t (\boldsymbol{\sigma}_t)^T (\sum_i \mathbf{M}_{i,t} x_{i,t}^D - \mathbf{b}_t) \quad (\text{A7})$$

$$\begin{aligned} \text{s.t. } \lambda_{i,t} &= (\boldsymbol{\sigma}_t)^T \mathbf{M}_{i,t}, \forall i \in I, \forall t \in T \\ \sigma_t &\geq 0, \forall t \end{aligned} \quad (\text{A8})$$

where, $x_{i,t}^{L*}$ and $c_{i,t}^{L*}$ indicate the optimal solution of the lower-level problem.

The objective function of the upper-level problem means the sum of lost opportunity costs (LOCs) of the market participants $(\sum_t \sum_i (\lambda_{i,t} x_{i,t}^{L*} - c_{i,t}^{L*}) - \sum_t \sum_i (\lambda_{i,t} x_{i,t}^D - c_{i,t}^D))$ and the product revenue shortfall (PRS) of the market operator $(\sum_t (\boldsymbol{\sigma}_t)^T (\sum_i \mathbf{M}_{i,t} x_{i,t}^D - \mathbf{b}_t))$.

② Lower-level problem

$$\max_{c, x, z} \sum_t \sum_i (\lambda_{i,t}^{U*} x_{i,t} - c_{i,t}) - \sum_t \sum_i (\lambda_{i,t}^{U*} x_{i,t}^D - c_{i,t}^D) \quad (\text{A9})$$

$$\text{s.t. } (\mathbf{c}_i, \mathbf{x}_i, \mathbf{z}_i) \in \mathbf{V}_i, \forall i \in I \quad (\text{A10})$$

where, $\lambda_{i,t}^{U*}$ indicates the price signal from the upper-level problem.

As prices are fixed in the lower-level problem, $\sum_t \sum_i (\lambda_{i,t}^{U*} x_{i,t}^D - c_{i,t}^D)$ is constant in the objective function of the lower-level problem. Hence, the lower-level problem aims to maximize the self-dispatch profits of market participants under given prices.

The general form of the bi-level pricing problem is shown as follows.

① Upper-level problem

$$\min_{\lambda, \sigma} P(\boldsymbol{\lambda}, \mathbf{v}^{L*}) + F(\boldsymbol{\lambda}, \boldsymbol{\sigma}) \quad (\text{A11})$$

$$\text{s.t. } G(\boldsymbol{\lambda}, \boldsymbol{\sigma}) \geq 0 \quad (\text{A12})$$

② Lower-level problem

$$\max_{\mathbf{v}} P(\boldsymbol{\lambda}^{U*}, \mathbf{v}) \quad \text{s.t. } \mathbf{v} \in \mathbf{V} \quad (\text{A13})$$

Comparing (A7)-(A10) with (A11)-(A13), it can be observed how the general bi-

level pricing problem accommodates the ConvHP problem.

(2) The explanation for ConvHP model about its relationship with the LMP

The LMP problem is expressed as

$$\max_{\sigma_i \geq 0} \left(\min_{c, x, z} \sum_t \sum_i c_{i,t} - \sum_t (\sigma_t)^T (\sum_i \mathbf{M}_{i,t} x_{i,t} - \mathbf{b}_t) \right) \quad (\text{A14})$$

$$(c_i, \mathbf{x}_i) \in \mathbf{V}_i^{\text{LMP}}, \forall i \in I$$

where, $\mathbf{V}_i^{\text{LMP}}$ is the convex feasible region of participant i . $\mathbf{V}_i^{\text{LMP}}$ is a subset of \mathbf{V}_i , which is constructed by fixing \mathbf{z}_i as certain values (e.g. its optimal solutions of the unit commitment problem).

The electricity price of LMP is derived as (A3). Compared with the ConvHP method, the LMP method differs in the feasible region of the lower-level optimization problem within its corresponding bi-level programming formulation. Furthermore, when accounting for modifications to the system-wide constraints of the primal market clearing problem, the objective function of the corresponding bi-level programming formulation will also differ.

Appendix B

(1) The compact form and concrete form of the unit commitment problem

Firstly, to facilitate the correspondence between the compact model and the concrete model, we present the compact form of primal market clearing problem, known as *unit commitment* (UC) as

$$\min_{c, x, z} \sum_t \sum_i c_{i,t} \quad (\text{B1})$$

$$\text{s.t. } \sum_i \mathbf{M}_{i,t} x_{i,t} \geq \mathbf{b}_t, t \in T \quad (\text{B2})$$

$$(c_i, \mathbf{x}_i, \mathbf{z}_i) \in \mathbf{V}_i, \forall i \in I \quad (\text{B3})$$

where, i and t are respectively the indices of market participants and time intervals; I and T are respectively the sets of market participants and time intervals; c_i , \mathbf{x}_i and \mathbf{z}_i are respectively the operation cost variable, power output variable, binary operation state variable of participant i ; $\mathbf{M}_{i,t}$ is the coefficient vector for power output $x_{i,t}$ in the constraint (B2); \mathbf{b}_t is the resource limit vectors of the constraint (B2); \mathbf{V}_i is the non-convex feasible region of participant i .

Then, the concrete form of the primal market clearing problem can be expressed as

$$\min_{c^g, x^g, z^{\text{st}}, z^{\text{su}}, z^{\text{nl}}} \sum_t \sum_i c_{i,t}^g \quad (\text{B4})$$

$$\text{s.t. } \sum_i x_{i,t}^g = \sum_n x_{n,t}^d, \quad \forall t \quad (\text{B5})$$

$$\sum_i H_{b-i}^{PTDF} x_{i,t}^g - \sum_n H_{b-n}^{PTDF} x_{n,t}^d \geq P_b^{\text{Lmin}}, \quad \forall b, \forall t \quad (\text{B6})$$

$$-\sum_i H_{b-i}^{PTDF} x_{i,t}^g + \sum_n H_{b-n}^{PTDF} x_{n,t}^d \geq -P_b^{\text{Lmax}}, \quad \forall b, \forall t \quad (\text{B7})$$

$$c_{i,t}^g = C_{i,t}^g x_{i,t}^g + C_{i,t}^{\text{st}} z_{i,t}^{\text{st}} + C_{i,t}^{\text{su}} z_{i,t}^{\text{su}} + C_{i,t}^{\text{nl}} z_{i,t}^{\text{nl}}, \quad \forall i, \forall t \quad (\text{B8})$$

$$x_{i,t-1}^g - x_{i,t}^g \geq -R_i^{\text{U}} z_{i,t-1}^{\text{nl}} - S_i^{\text{U}} z_{i,t}^{\text{st}}, \quad \forall i, \forall t \quad (\text{B9})$$

$$x_{i,t}^g - x_{i,t-1}^g \geq -R_i^{\text{D}} z_{i,t-1}^{\text{nl}} - S_i^{\text{D}} z_{i,t}^{\text{su}}, \quad \forall i, \forall t \quad (\text{B10})$$

$$-x_{i,t}^g \geq -z_{i,t}^{\text{nl}} P_i^{\text{Gmax}}, \quad \forall i, \forall t \quad (\text{B11})$$

$$x_{i,t}^g \geq z_{i,t}^{\text{nl}} P_i^{\text{Gmin}}, \quad \forall i, \forall t \quad (\text{B12})$$

$$z_{i,t}^{\text{nl}} - z_{i,t-1}^{\text{nl}} = z_{i,t}^{\text{st}} - z_{i,t}^{\text{su}}, \quad \forall i, \forall t \quad (\text{B13})$$

$$z_{i,t}^{\text{nl}} \geq \sum_{\tau=t-T_i^{\text{up}}+1}^t z_{i,\tau}^{\text{st}}, \quad \forall i, \forall t \in [T_i^{\text{up}}, T] \quad (\text{B14})$$

$$1 - z_{i,t}^{\text{nl}} \geq \sum_{\tau=t-T_i^{\text{down}}+1}^t z_{i,\tau}^{\text{su}}, \quad \forall i, \forall t \in [T_i^{\text{down}}, T] \quad (\text{B15})$$

$$x_{n,t}^d = x_{n,t}^{\text{dO}}, \quad \forall n, \forall t \quad (\text{B16})$$

$$(z_{i,t}^{\text{st}}, z_{i,t}^{\text{su}}, z_{i,t}^{\text{nl}}) \in \{0, 1\}, \quad \forall i, \forall t \quad (\text{B17})$$

where, variables $c_{i,t}^g, x_{i,t}^g, z_{i,t}^{\text{nl}}, z_{i,t}^{\text{st}}, z_{i,t}^{\text{su}}$ are respectively the operation cost, power output, on/off state, start-up state, shut-down state of generator i at time t ; $x_{n,t}^d$ is the power demand of load n at time t ; $x_{n,t}^{\text{dO}}$ is the fixed power demand of load n at time t ; $C_{i,t}^g, C_{i,t}^{\text{nl}}, C_{i,t}^{\text{st}}, C_{i,t}^{\text{su}}$ are respectively the cost coefficient of $x_{i,t}^g, z_{i,t}^{\text{nl}}, z_{i,t}^{\text{st}}, z_{i,t}^{\text{su}}$; H_{b-i}^{PTDF} is the power transfer distribution factor, which describes the sensitivity relationship between the injected power $x_{i,t}^g$ of generator i and the power flow of branch b ; P_b^{Lmin} and P_b^{Lmax} are the minimum and maximum transmission capacity of branch b ; P_i^{Gmax} and P_i^{Gmin} are the minimum and maximum power generation limit of generator i ; $R_i^{\text{U}}, R_i^{\text{D}}, S_i^{\text{U}}, S_i^{\text{D}}$ are respectively the ramp-up limit, ramp-down limit, start-up ramp limit, shut-down ramp limit of generator i ; T^{up} and T^{down} are the minimum up and minimum down time.

The objective (B4) is to minimize the total operational costs; (B5) is the system power balance constraint; (B6) and (B7) are the transmission capacity constraints; (B9) and (B10) are the ramping constraints; (B11) and (B12) are the power

generation constraints; (B13) is relation constraint for binary operational states; (B14) and (B15) are minimum up time and minimum down time constraints; (B16) is the load demands constraint.

The correspondence between constraints and variables of the above two models is shown in the Table s-1.

Table s-1 The correspondence between two models

	Compact form model	Concrete form model
Objective function	(B1)	(B4)
Constraint	(B2)	(B5)-(B7)
	(B3)	(B8)-(B17)
Variable	\mathbf{c}_i	$c_{i,t}^g$
	\mathbf{x}_i	$x_{i,t}^g, x_{n,t}^d$
	\mathbf{z}_i	$z_{i,t}^{st}, z_{i,t}^{sh}, z_{i,t}^{nl}$

(2) The compact form and concrete form of the bi-level pricing problem

Secondly, the compact form of the ConvHP problem is expressed as

$$\max_{\substack{\mathbf{c}, \mathbf{x}, \mathbf{z} \\ \sigma_t \geq 0}} \left(\min_{\substack{\mathbf{c}_i, \mathbf{x}_i, \mathbf{z}_i}} \sum_t \sum_i c_{i,t} - \sum_t (\sigma_t)^T (\sum_i \mathbf{M}_{i,t} x_{i,t} - \mathbf{b}_t) \right) \quad (\text{B18})$$

$$(\mathbf{c}_i, \mathbf{x}_i, \mathbf{z}_i) \in \mathbf{V}_i, \forall i \in I$$

Refer to the general bi-level formulation of the pricing problem in the manuscript, it can be reformulated as

① Upper-level problem

$$\min_{\lambda, \sigma} \sum_t \sum_i (\lambda_{i,t} x_{i,t}^{L*} - c_{i,t}^{L*}) - \sum_t \sum_i (\lambda_{i,t} x_{i,t}^D - c_{i,t}^D) + \sum_t (\sigma_t)^T (\sum_i \mathbf{M}_{i,t} x_{i,t}^D - \mathbf{b}_t) \quad (\text{B19})$$

$$\text{s.t. } \lambda_{i,t} = (\sigma_t)^T \mathbf{M}_{i,t}, \forall i \in I, \forall t \in T \quad (\text{B20})$$

$$\sigma_t \geq 0, \forall t$$

where, $x_{i,t}^{L*}$ and $c_{i,t}^{L*}$ indicate the optimal solution of the lower-level problem; $x_{i,t}^D$ and $c_{i,t}^D$ are the dispatch instructions decided by the market operator, and they are constants in the above problem; σ_t is the lagrangian dual multiplier for system-wide constraints at time t (regarded as the price of system resources).

The objective function of upper-level problem means the sum of lost opportunity costs (LOCs) of the market participants ($\sum_t \sum_i (\lambda_{i,t} x_{i,t}^{L*} - c_{i,t}^{L*}) - \sum_t \sum_i (\lambda_{i,t} x_{i,t}^D - c_{i,t}^D)$) and the product revenue shortfall (PRS) of the market operator ($\sum_t (\sigma_t)^T (\sum_i \mathbf{M}_{i,t} x_{i,t}^D - \mathbf{b}_t)$).

② Lower-level problem

$$\max_{\mathbf{c}, \mathbf{x}, \mathbf{z}} \sum_t \sum_i (\lambda_{i,t}^{U*} x_{i,t} - c_{i,t}) - \sum_t \sum_i (\lambda_{i,t}^{U*} x_{i,t}^D - c_{i,t}^D) \quad (\text{B21})$$

$$\text{s.t. } (\mathbf{c}_i, \mathbf{x}_i, \mathbf{z}_i) \in \mathbf{V}_i, \forall i \in I \quad (\text{B22})$$

where, $\lambda_{i,t}^{U*}$ indicates the price signal from the upper-level problem.

As prices are fixed in the lower-level problem, $\sum_t \sum_i (\lambda_{i,t}^{U*} x_{i,t}^D - c_{i,t}^D)$ is constant in the objective function of lower-level problem. Hence, the lower-level problem aims to maximize the self-dispatch profits of market participants under given prices.

Then, the concrete form of the bi-level pricing problem (which is solved in the case study) can be expressed as

① Upper-level problem

$$\min_{\lambda_{i,t}^g, \sigma_t^{pd}, \sigma_{b,t}^{tc-}, \sigma_{b,t}^{tc+}} \left\{ \begin{aligned} & \sum_t \sum_i (\lambda_{i,t}^g x_{i,t}^{gL*} - c_{i,t}^{gL*}) - \sum_t \sum_i (\lambda_{i,t}^g x_{i,t}^{gD} - c_{i,t}^{gD}) \\ & + \sum_b \sum_t \sigma_{b,t}^{tc-} (\sum_i H_{b-i}^{PTDF} x_{i,t}^{gD} - \sum_n H_{b-n}^{PTDF} x_{n,t}^{dD} - P_b^{Lmin}) \\ & + \sum_b \sum_t \sigma_{b,t}^{tc+} (-\sum_i H_{b-i}^{PTDF} x_{i,t}^{gD} + \sum_n H_{b-n}^{PTDF} x_{n,t}^{dD} + P_b^{Lmax}) \end{aligned} \right\} \quad (B23)$$

$$\text{s.t.} \quad \lambda_{i,t}^g = \sigma_t^{pd} + \sum_b \sigma_{b,t}^{tc-} H_{b-i}^{PTDF} - \sum_b \sigma_{b,t}^{tc+} H_{b-i}^{PTDF}, \forall i, \forall t \quad (B24)$$

$$\lambda_{n,t}^d = \sigma_t^{pd} + \sum_b \sigma_{b,t}^{tc-} H_{b-n}^{PTDF} - \sum_b \sigma_{b,t}^{tc+} H_{b-n}^{PTDF}, \forall n, \forall t \quad (B25)$$

$$\sigma_{b,t}^{tc-}, \sigma_{b,t}^{tc+} \geq 0, \forall b, \forall t \quad (B26)$$

where, $\lambda_{i,t}^g$ denotes the electricity price of generator i ; $\lambda_{n,t}^d$ denotes the electricity price of load n ; $x_{i,t}^{gL*}$ and $c_{i,t}^{gL*}$ indicate the optimal solution of the lower-level problem; $x_{i,t}^{gD}$ and $c_{i,t}^{gD}$ are the dispatch instructions decided by the market operator, and they are constants in the above problem; σ_t^{pd} is the lagrangian dual multiplier for the power balance constraint at time t (regarded as the price of the power balance resource); $\sigma_{b,t}^{tc-}$ and $\sigma_{b,t}^{tc+}$ are the lagrangian dual multipliers for the transmission capacity constraints (regarded as the prices of the transmission capacity resources).

② Lower-level problem

$$\max_{x_{i,t}^g, c_{i,t}^g, z_{i,t}^{st}, z_{i,t}^{su}, z_{i,t}^{nl}} \sum_t \sum_i (\lambda_{i,t}^{U*} x_{i,t}^g - c_{i,t}^g) - \sum_t \sum_i (\lambda_{i,t}^{U*} x_{i,t}^{gD} - c_{i,t}^{gD}) \quad (B27)$$

$$\text{s.t.} \quad c_{i,t}^g = C_{i,t}^g x_{i,t}^g + C_{i,t}^{st} z_{i,t}^{st} + C_{i,t}^{su} z_{i,t}^{su} + C_{i,t}^{nl} z_{i,t}^{nl}, \forall i, \forall t \quad (B28)$$

$$x_{i,t-1}^g - x_{i,t}^g \geq -R_i^U z_{i,t-1}^{nl} - S_i^U z_{i,t}^{st}, \forall i, \forall t \quad (B29)$$

$$x_{i,t}^g - x_{i,t-1}^g \geq -R_i^D z_{i,t-1}^{nl} - S_i^D z_{i,t}^{su}, \forall i, \forall t \quad (B30)$$

$$-x_{i,t}^g \geq -z_{i,t}^{nl} P_i^{Gmax}, \forall i, \forall t \quad (B31)$$

$$x_{i,t}^g \geq z_{i,t}^{nl} P_i^{Gmin}, \forall i, \forall t \quad (B32)$$

$$z_{i,t}^{nl} - z_{i,t-1}^{nl} = z_{i,t}^{st} - z_{i,t}^{su}, \forall i, \forall t \quad (B33)$$

$$z_{i,t}^{nl} \geq \sum_{\tau=t-T_i^{up}+1}^t z_{i,\tau}^{st}, \forall i, \forall t \in [T_i^{up}, T] \quad (B34)$$

$$1 - z_{i,t}^{nl} \geq \sum_{\tau=T_i^{down}+1}^t z_{i,\tau}^{su}, \quad \forall i, \forall t \in [T_i^{down}, T] \quad (B35)$$

$$x_{n,t}^d = x_{n,t}^{dO}, \quad \forall n, \forall t \quad (B36)$$

$$(z_{i,t}^{st}, z_{i,t}^{su}, z_{i,t}^{nl}) \in \{0,1\}, \quad \forall i, \forall t \quad (B37)$$

where, $\lambda_{i,t}^{gU*}$ indicates the price signal from the upper-level problem.

The correspondence between constraints and variables of the above two models is shown in the Table s-2.

Table s-2 The correspondence between two models

	Compact form model	Concrete form model
Objective function	(B19)	(B23)
	(B21)	(B27)
Constraint	(B20)	(B24)-(B26)
	(B22)	(B28)-(B37)
Variable	$\lambda_{i,t}$	$\lambda_{i,t}^g, \lambda_{n,t}^d$
	σ_t	$\sigma_t^{pd}, \sigma_{b,t}^{tc-}, \sigma_{b,t}^{tc+}$
	c_i	$c_{i,t}^g$
	x_i	$x_{i,t}^g, x_{n,t}^d$
	z_i	$z_{i,t}^{st}, z_{i,t}^{sh}, z_{i,t}^{nl}$