CSIE 5432/5433 HW5 PDF

YiWenLai

TOTAL POINTS

30 / 0

QUESTION 1

1 Problem 6 20 / 0

✓ **+ 20 pts Correct**

+ 10 pts (1) Correct answer with one mistake

(2) Correct answer but some descriptions not clear enough.

+ 0 pts (1) Choose "I have no idea" option.

(2) Correct answer but missing some steps.

- 10 pts Wrong answer

QUESTION 2

2 Problem 12 10 / 0

+ 20 pts Correct answer.

✓ **+ 10 pts 1 mistake.**

+ 0 pts 2 mistakes or 2 unclear places.

+ 0 pts Choose "I have no idea" option.

- 10 pts Wrong answer.

1 without explaining why is "min"

QUESTION 3

3 Problem 15 0 / 0

✓ **+ 0 pts Good submission or choose "I have no idea" option**

- 10 pts Do not select page or select wrong page

- 20 pts No submission

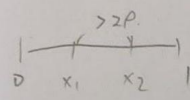
- 20 pts Only program output or terminal commands

- 20 pts submit a url of code

4.

if x_1, x_2 are both with same sign, the expected dichotomies are 2.

So we just consider the other two cases with different signs.



, $\rho \in [0, 0.5]$, $|x_2 - x_1| > 2\rho$.

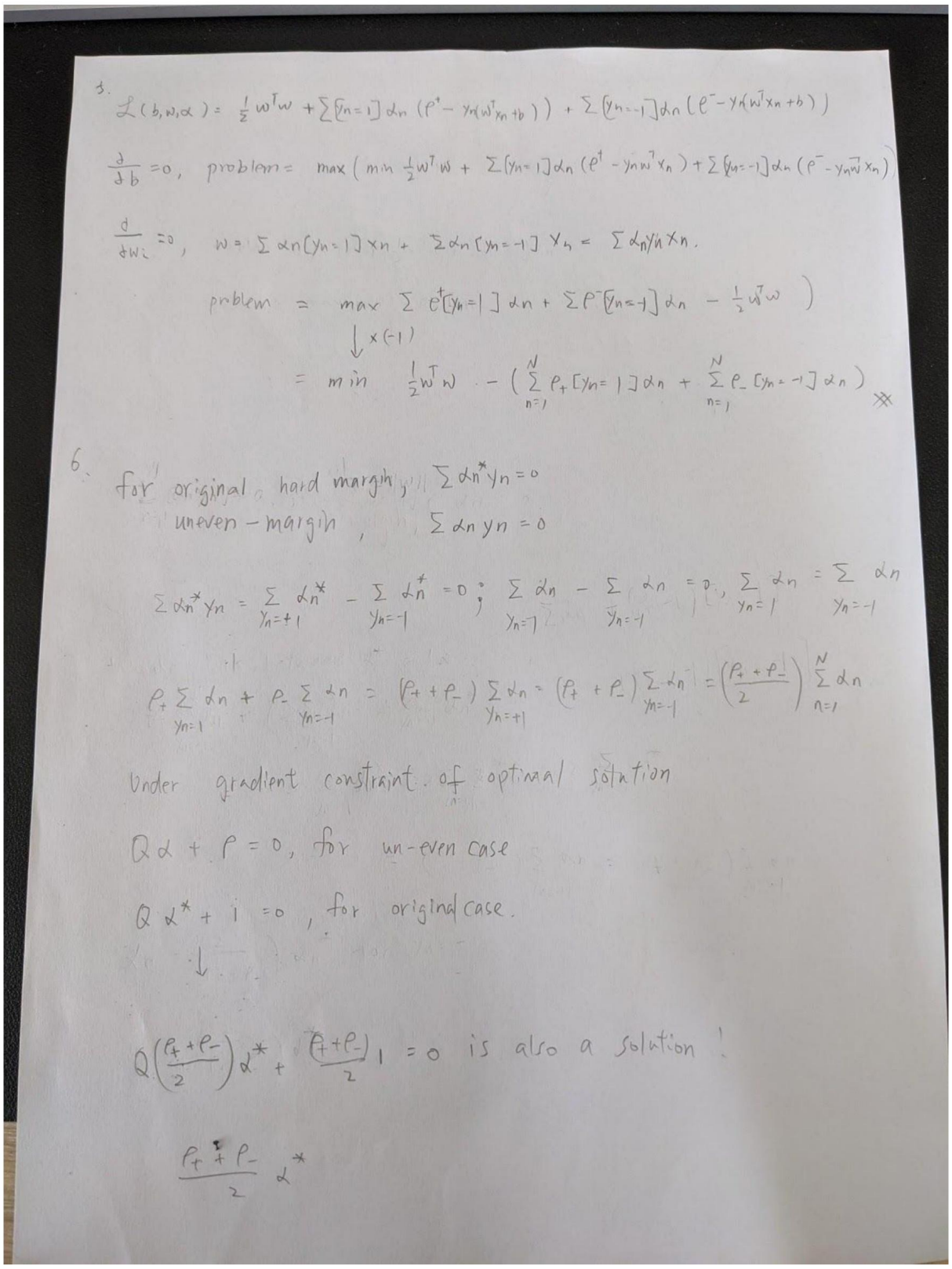
$$\int_0^{1-2\rho} \int_{x_1+2\rho}^{1} dx_2 \, dx_1 = \int_0^{1-2\rho} 1-(2\rho+x_1)\, dx_1 \text{ , if } x_1+2\rho = 1 \text{ then all } = 0.$$

$$= \int_0^{1-2\rho} (1-2\rho)-x_1 \, dx_1 = (1-2\rho)\Big|_0^{1-2\rho} - \frac{1}{2}(1-2\rho)^2 = \frac{1}{2}$$

$2 \cdot (\frac{1}{2}(1-2\rho)^2) = (1-2\rho)^2$, for $x_2 > x_1$, same as $x_1 > x_2$.

$\therefore$ overall, $2 + 2 \cdot (1-2\rho)^2$ ✱

5. c
6. e



3. $\mathcal{L}(b, w, \alpha) = \frac{1}{2} w^T w + \sum [y_n = 1] \alpha_n (\rho^+ - y_n (w^T x_n + b)) + \sum [y_n = -1] \alpha_n (\rho^- - y_n (w^T x_n + b))$

$\frac{\partial}{\partial b} = 0$, problem = $\max \left(\min \frac{1}{2} w^T w + \sum [y_n = 1] \alpha_n (\rho^+ - y_n w^T x_n) + \sum [y_n = -1] \alpha_n (\rho^- - y_n w^T x_n) \right)$

$\frac{d}{dw_i} = 0$, $w = \sum \alpha_n [y_n = 1] x_n + \sum \alpha_n [y_n = -1] y_n = \sum \alpha_n y_n x_n$.

problem = $\max \sum \rho^+ [y_n = 1] \alpha_n + \sum \rho^- [y_n = -1] \alpha_n - \frac{1}{2} w^T w$)

$\downarrow \times (-1)$

$= \min \frac{1}{2} w^T w - \left(\sum_{n=1}^{N} \rho_+ [y_n = 1] \alpha_n + \sum_{n=1}^{N} \rho_- [y_n = -1] \alpha_n \right)$ ※

6. for original hard margin, $\sum \alpha_n^* y_n = 0$

uneven-margin, $\sum \alpha_n y_n = 0$

$\sum \alpha_n^* y_n = \sum_{y_n = +1} \alpha_n^* - \sum_{y_n = -1} \alpha_n^* = 0$; $\sum_{y_n = 1} \alpha_n - \sum_{y_n = -1} \alpha_n = 0$, $\sum_{y_n = 1} \alpha_n = \sum_{y_n = -1} \alpha_n$

$\rho_+ \sum_{y_n = 1} \alpha_n + \rho_- \sum_{y_n = -1} \alpha_n = (\rho_+ + \rho_-) \sum_{y_n = +1} \alpha_n = (\rho_+ + \rho_-) \sum_{y_n = -1} \alpha_n = \left(\frac{\rho_+ + \rho_-}{2} \right) \sum_{n=1}^{N} \alpha_n$

Under gradient constraint of optimal solution

$Q\alpha + p = 0$, for un-even case

$Q\alpha^* + 1 = 0$, for orignal case.

$\downarrow$

$Q \left(\frac{\rho_+ + \rho_-}{2} \right) \alpha^* + \frac{(\rho_+ + \rho_-)}{2} 1 = 0$ is also a solution !

$\frac{\rho_+ + \rho_-}{2} \alpha^*$

7. d
8. c
9. d
10. c

1 Problem 6 **20 / 0**

✓ **+ 20 pts Correct**

+ 10 pts (1) Correct answer with one mistake

(2) Correct answer but some descriptions not clear enough.

+ 0 pts (1) Choose "I have no idea" option.

(2) Correct answer but missing some steps.

- 10 pts Wrong answer

7.

d. $\log_2 K(x,x')$ may less than 0 when $0 < K(x,x') < 1$,
which may cause the element of matrix non-positive
not positive definite matrix ZZ^T → not valid kernel ※

8.

$$\|\phi(x) - \phi(x')\|^2 = \phi(x)^T\phi(x) - 2\phi(x)^T\phi(x') + \phi(x')^T\phi(x')$$
$$= K(x,x) + K(x',x') - 2K(x,x')$$
$$= 1 + 1 - 2K(x,x') \leq 2 \text{ ※}$$

10.

$$w_{t+1} = \sum_{n=1}^{N} \alpha_{t+1}\phi(x_n) = w_t + y_{n(t)}\phi(x_{n(t)}) = \left(\sum_{n=1}^{N} \alpha_t \phi(x_n)\right) + y_{n(t)}\phi(x_n)$$

Compare coefficient of each $\phi(x)$, $\alpha_{t+1} = \alpha_t$ except $\phi(x_n)$, whose coefficient is $\alpha_{t,n(t)} + y_n(t)$ ※

9. problem can be viewed as:

$$h(x_m) = \text{sign}\left(\sum_{n=1}^{N} y_n K(x_m, x_n)\right) = y_m, \ \forall m = 1, 2 \cdots N, \ E_{in} = 0$$

$$\sum_{n=1}^{N} y_n K(x_m, x_n) = \sum_{\substack{n=1 \\ n \neq m}}^{N} y_n \exp(-r(x_m - x_n)^2) + y_m.$$

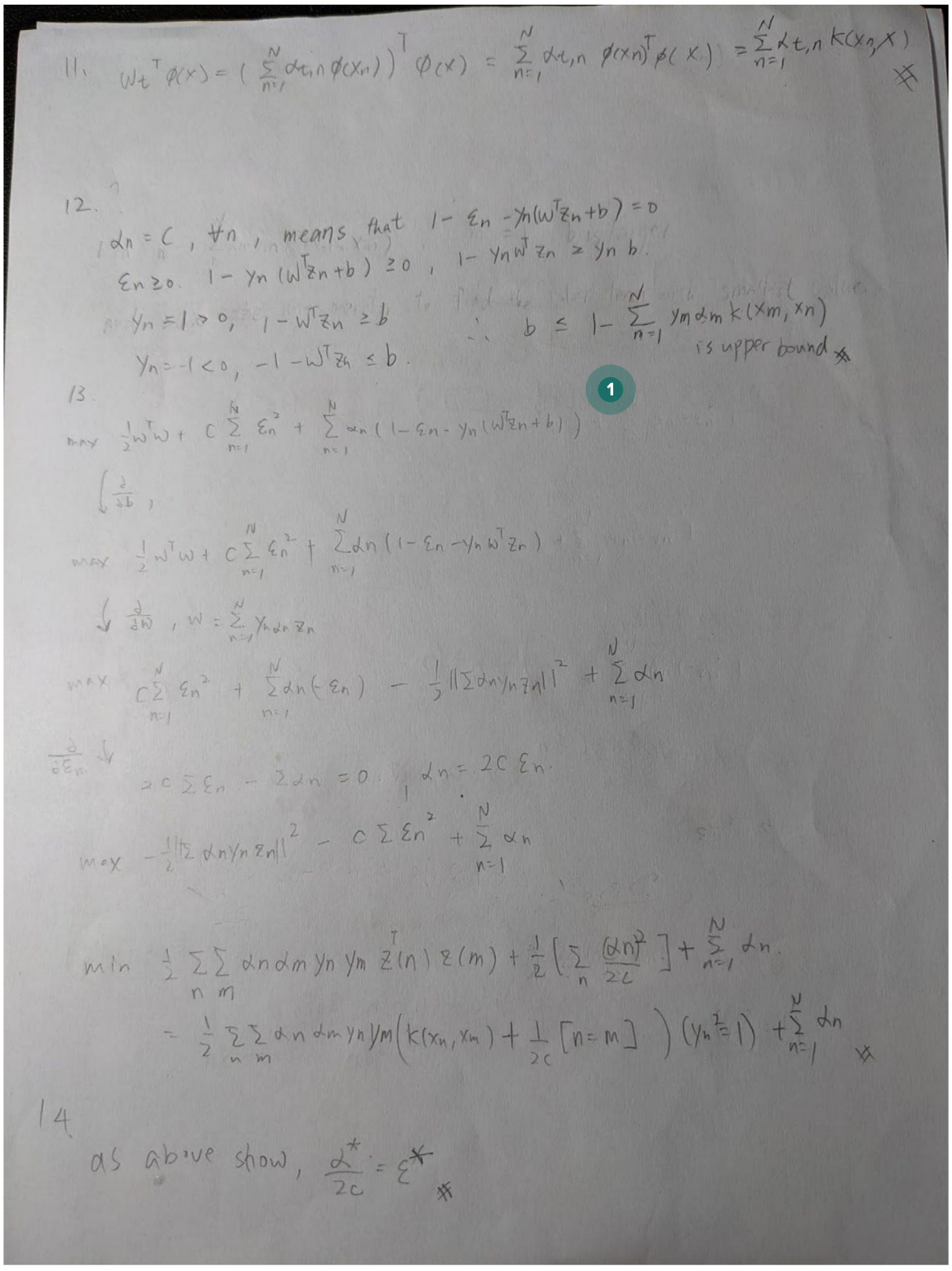
$$\left| \sum_{\substack{n=1 \\ n \neq m}}^{N} y_n \exp(-r(x_m - x_n)^2) \right| < 1, \text{ will maintain constraint.}$$

$$-1 < \sum y_n \exp(-r(x_m - x_n)^2) \leq \sum y_n \exp(-r\varepsilon^2) < 1$$

$$-1 < -(N-1)\exp(-r\varepsilon^2) < \sum y_n \exp(-r\varepsilon^2) < (N-1)\exp(-r\varepsilon^2) < 1, \text{ if } r \text{ is large enough.}$$

$\therefore \exp(-r\varepsilon^2) < \frac{1}{N-1}$ for both side $-r\varepsilon^2 < \ln\left(\frac{1}{N-1}\right)$ $r > \frac{\ln(N-1)}{\varepsilon^2}$ ※

11. a
12. b
13. e
14. e



11. $w_t^T \phi(x) = \left(\sum_{n=1}^{N} \alpha_{t,n} \phi(x_n)\right)^T \phi(x) = \sum_{n=1}^{N} \alpha_{t,n} \phi(x_n)^T \phi(x) = \sum_{n=1}^{N} \alpha_{t,n} k(x_n, x)$ #

12.

$\alpha_n = C$, $\forall n$, means that $1 - \xi_n - y_n(w^T z_n + b) = 0$

$\xi_n \geq 0$. $1 - y_n(w^T z_n + b) \geq 0$, $1 - y_n w^T z_n \geq y_n b$.

$y_n = 1 > 0$, $1 - w^T z_n \geq b$

$y_n = -1 < 0$, $-1 - w^T z_n \leq b$.

$\therefore b \leq 1 - \sum_{n=1}^{N} y_m \alpha_m k(x_m, x_n)$ is upper bound #

13.

$$\max \frac{1}{2} w^T w + C \sum_{n=1}^{N} \xi_n^2 + \sum_{n=1}^{N} \alpha_n (1 - \xi_n - y_n(w^T z_n + b))$$

$\downarrow \frac{\partial}{\partial b}$,

$$\max \frac{1}{2} w^T w + C \sum_{n=1}^{N} \xi_n^2 + \sum_{n=1}^{N} \alpha_n (1 - \xi_n - y_n w^T z_n)$$

$\downarrow \frac{\partial}{\partial w}$, $w = \sum_{n=1}^{N} y_n \alpha_n z_n$

$$\max C \sum_{n=1}^{N} \xi_n^2 + \sum_{n=1}^{N} \alpha_n (-\xi_n) - \frac{1}{2} \left\| \sum \alpha_n y_n z_n \right\|^2 + \sum_{n=1}^{N} \alpha_n$$

$\frac{\partial}{\partial \xi_n} \downarrow$

$2C \sum \xi_n - \sum \alpha_n = 0$, $\alpha_n = 2C \xi_n$.

$$\max -\frac{1}{2} \left\| \sum \alpha_n y_n z_n \right\|^2 - C \sum \xi_n^2 + \sum_{n=1}^{N} \alpha_n$$

$$\min \frac{1}{2} \sum_n \sum_m \alpha_n \alpha_m y_n y_m z(n)^T z(m) + \frac{1}{2} \left[\sum_n \frac{(\alpha_n)^2}{2C} \right] + \sum_{n=1}^{N} \alpha_n.$$

$$= \frac{1}{2} \sum_n \sum_m \alpha_n \alpha_m y_n y_m \left(k(x_n, x_m) + \frac{1}{2C} [n = m] \right) (y_n^2 = 1) + \sum_{n=1}^{N} \alpha_n$$ #

14.

as above show, $\frac{\alpha^*}{2C} = \xi^*$ #

15. d

a. result, 8.457084298367683

2 Problem 12 10 / 0

+ **20 pts** Correct answer.

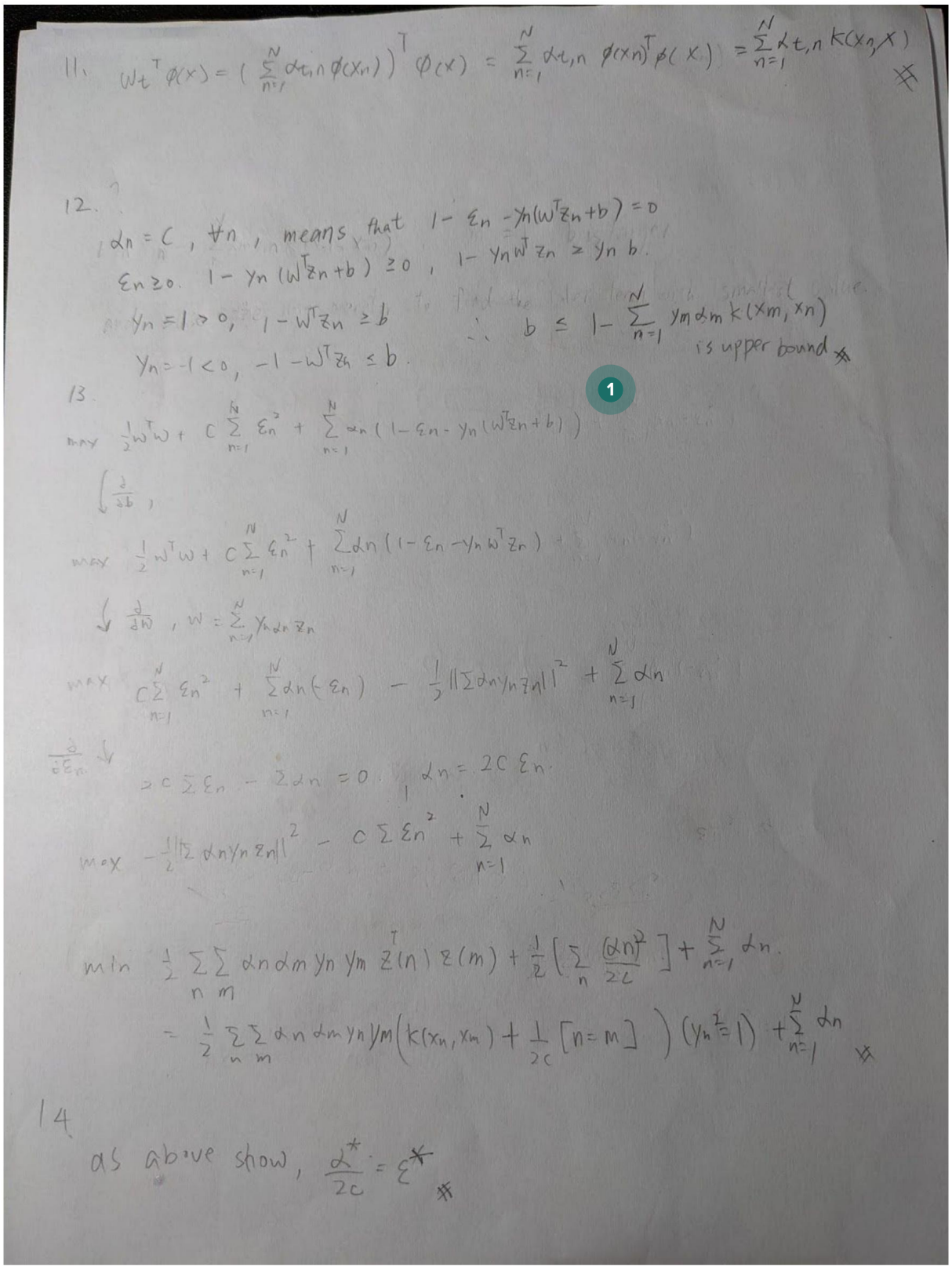
✓ **+ 10 pts** **1 mistake.**

+ **0 pts** 2 mistakes or 2 unclear places.

+ **0 pts** Choose "I have no idea" option.

- **10 pts** Wrong answer.

1 without explaining why is "min"



11. $w_t^T \phi(x) = \left(\sum_{n=1}^{N} \alpha_{t,n} \phi(x_n)\right)^T \phi(x) = \sum_{n=1}^{N} \alpha_{t,n} \phi(x_n)^T \phi(x) = \sum_{n=1}^{N} \alpha_{t,n} k(x_n, x)$ #

12.

$\alpha_n = C$, $\forall n$, means that $1 - \xi_n - y_n(w^T z_n + b) = 0$

$\xi_n \geq 0$. $1 - y_n(w^T z_n + b) \geq 0$, $1 - y_n w^T z_n \geq y_n b$.

$y_n = 1 > 0$, $1 - w^T z_n \geq b$

$y_n = -1 < 0$, $-1 - w^T z_n \leq b$.

$\therefore b \leq 1 - \sum_{n=1}^{N} y_m \alpha_m k(x_m, x_n)$ is upper bound #

13.

$$\max \frac{1}{2} w^T w + C \sum_{n=1}^{N} \xi_n^2 + \sum_{n=1}^{N} \alpha_n (1 - \xi_n - y_n(w^T z_n + b))$$

$\downarrow \frac{\partial}{\partial b}$,

$$\max \frac{1}{2} w^T w + C \sum_{n=1}^{N} \xi_n^2 + \sum_{n=1}^{N} \alpha_n (1 - \xi_n - y_n w^T z_n)$$

$\downarrow \frac{\partial}{\partial w}$, $w = \sum_{n=1}^{N} y_n \alpha_n z_n$

$$\max C \sum_{n=1}^{N} \xi_n^2 + \sum_{n=1}^{N} \alpha_n (-\xi_n) - \frac{1}{2} \left\| \sum \alpha_n y_n z_n \right\|^2 + \sum_{n=1}^{N} \alpha_n$$

$\frac{\partial}{\partial \xi_n} \downarrow$

$2C \sum \xi_n - \sum \alpha_n = 0$, $\alpha_n = 2C \xi_n$.

$$\max -\frac{1}{2} \left\| \sum \alpha_n y_n z_n \right\|^2 - C \sum \xi_n^2 + \sum_{n=1}^{N} \alpha_n$$

$$\min \frac{1}{2} \sum_n \sum_m \alpha_n \alpha_m y_n y_m z(n)^T z(m) + \frac{1}{2} \left[\sum_n \frac{(\alpha_n)^2}{2C} \right] + \sum_{n=1}^{N} \alpha_n.$$

$$= \frac{1}{2} \sum_n \sum_m \alpha_n \alpha_m y_n y_m \left(k(x_n, x_m) + \frac{1}{2C} [n = m] \right) (y_n^2 = 1) + \sum_{n=1}^{N} \alpha_n$$ #

14.

as above show, $\frac{\alpha^*}{2C} = \xi^*$ #

15. d

a. result, 8.457084298367683

```
yiwenlai@YiWens-MBP ~/Desktop/html_hw5/libsvm-3.24/python INSERT python3 train.py
.....*..*.....*
optimization finished, #iter = 22874
nu = 0.108695
obj = -4784.881503, rho = 3.623003
nSV = 500, nBSV = 466
Total nSV = 500
Accuracy = 95.8061% (4249/4435) (classification)
8.457084298367683
```

b. Code

```
y, x = svm_read_problem('hw5_train')
for i in range(len(y)):
    if y[i] == 3.0:
        y[i] = 1
    else:
        y[i] = -1
m = svm_train(y, x, '-c 10 -s 0 -t 0')
p_label, p_acc, p_val = svm_predict(y, x, m)
support_vectors = m.get_SV()
support_vector_coefficients = m.get_sv_coef()
w = []
for i in range(36):
    wi = 0
    for j in range(len(support_vector_coefficients)):
        if i+1 in support_vectors[j]:
            wi +=
support_vector_coefficients[j][0]*support_vectors[j]
[i+1]
    w.append(wi)
w_2 = [x*x for x in w]
print(math.sqrt(sum(w_2)))
```

16. b

a. result is presented in Q17

17. c

a. result for Q16,17

```
yiwenlai@YiWens-MBP ~/Desktop/html_hw5/libsvm-3.24/python INSERT python3 train.py
Class:  1  Number of support vectors:  145  Accuracy:  99.93235625704622
Class:  2  Number of support vectors:  87  Accuracy:  100.0
Class:  3  Number of support vectors:  433  Accuracy:  97.76775648252537
Class:  4  Number of support vectors:  712  Accuracy:  95.98647125140924
Class:  5  Number of support vectors:  259  Accuracy:  99.32356257046223
```

b. Code for Q16,17

3 Problem 15 0 / 0

✓ **+ 0 pts** **Good submission or choose "I have no idea" option**

- **10 pts** Do not select page or select wrong page
- **20 pts** No submission
- **20 pts** Only program output or terminal commands
- **20 pts** submit a url of code