# 2.Log-normal distribution and the Gini coefficient

We consider a sample of 8 observations representing monthly incomes (in thousands of Swedish Krona). Assume the data follow a log-normal distribution:

$$y_1, ..., y_n|\mu, \sigma^2 \sim \log N(\mu, \sigma^2)$$

with known $\mu = 3.65$ and unknown variance $\sigma^2$. For the variance, we use a non-informative prior:

$$p(\sigma^2) \propto 1/\sigma^2$$

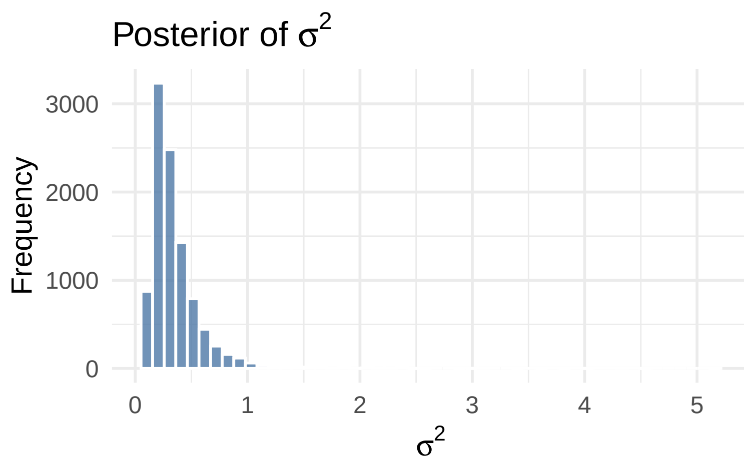
Under this model, the posterior distribution for $\sigma^2$ follows a scaled inverse-chi-squared distribution:

$$\text{Scale-inv-}\chi^2(n, \tau^2), \quad \text{where} \quad \tau = \frac{\sum_{i=1}^{n}(\log y_i - \mu)^2}{n}$$

To draw samples from this posterior, we use the fact that if:

$$\sigma \sim \text{Scale-inv-}\chi^2(n, \tau^2), \quad \text{then} \quad \sigma^2 = \frac{n\tau^2}{\chi^2_n}$$

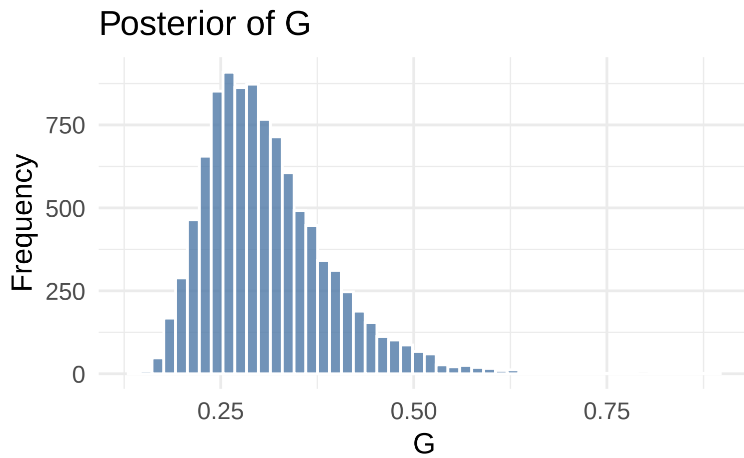
We generate 10000 posterior draws for $\sigma^2$ and visualize the resulting distribution below.




Then, the Gini coefficient is calculated using the formula:

$$G = 2\Phi(\sigma/\sqrt{2}) - 1$$

where $\Phi$ denotes the cumulative distribution function of the standard normal distribution, and $\sigma = \sqrt{\sigma^2}$. Using the posterior draws of $\sigma^2$, we obtain the posterior distribution of the Gini coefficient:




Using the posterior distribution of the Gini coefficient $G$, we compute both a 95% equal-tail credible interval and a 95% Highest Posterior Density Interval (HPDI) for $G$.

Table 1: Confidence intervals for G

| | Lower.bound | Upper.bound |
|---|---|---|
| Equal tail | 0.1930103 | 0.5080196 |
| HPDI | 0.1757738 | 0.4726963 |

The equal-tail interval removes 2.5% of the posterior mass from each tail, while the HPDI represents the narrowest interval containing 95% of the posterior mass. In this case, the HPDI is slightly narrower and starts at a lower bound compared to the equal-tail interval. This suggests that the posterior distribution of $G$ is slightly right-skewed, and the HPDI better captures the region of highest probability. Therefore, if interpretability and precision are important, the HPDI may be preferred.