Computer Lab1 - 732A73

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1. Daniel Bernoulli

Let $y_1, ..., y_n|\theta \sim Bern(\theta)$, and assume a prior distribution for the parameter θ as $\theta \sim Beta(\alpha 0, \beta 0)$, with $\alpha_0 = \beta_0 = 7$. We have observed a sample of $n = 78$ trials with $f = 35$ failures. Using Bayes' theorem, the posterior distribution of θ is

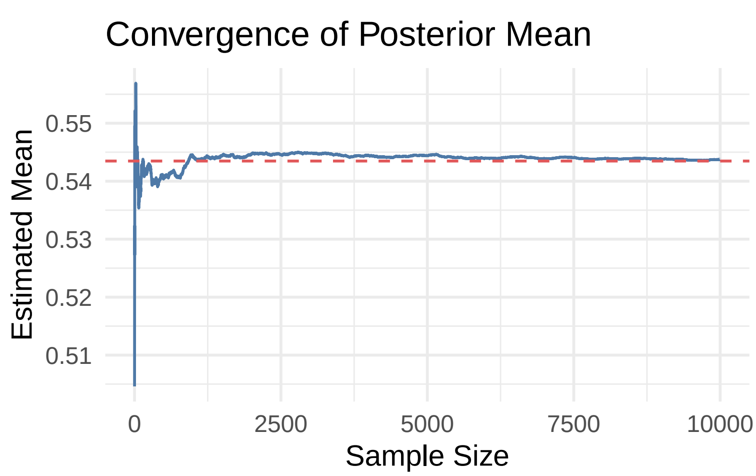
$$\theta|y \sim Beta(\alpha = \alpha_0 + s, \beta = \beta_0 + f)$$

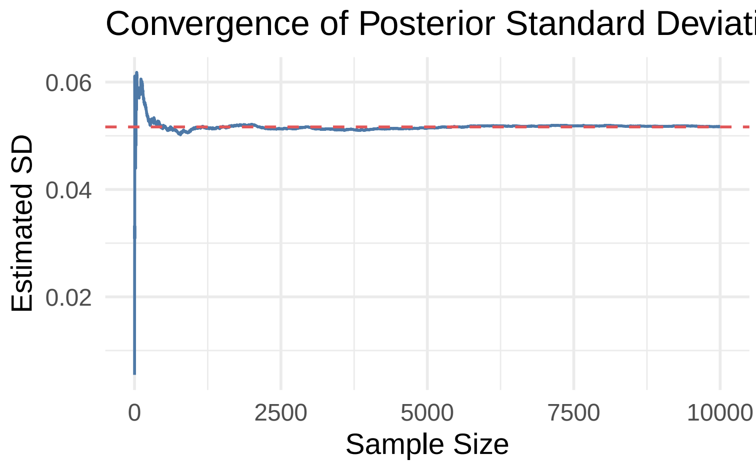
The theoretical (true) posterior mean and standard deviation are given by:

$$E[\theta|y] = \frac{\alpha}{\alpha + \beta}$$

$$SD[\theta|y] = \sqrt{\frac{\alpha\beta}{(\alpha + \beta)^2(\alpha + \beta + 1)}}$$

To examine how estimates of the posterior mean and standard deviation behave as the number of random draws increases, we simulate 10000 samples from the posterior distribution and compute running estimates for both quantities.





The plots above clearly illustrate that the estimates of the posterior mean and standard deviation converge toward their theoretical values as the number of random draws increases.

Then, we compute the posterior probability $P(\theta > 0.5|y)$. The posterior probability of $\theta > 0.5$ is 0.8055, and the exact value of $\theta > 0.5$ from the Beta posterior is 0.7990936.

Finally, we draw 10000 random values from the posterior of the odds $\phi = \frac{\theta}{1-\theta}$ and plot the posterior distribution of ϕ.