An Efficient Localization Method Using Bistatic Range and AOA Measurements in Multistatic Radar

Zhaotao Qin, Jun Wang and Shaoming Wei
School of Electronics and Information Engineering
Beihang University
Beijing, China
qzt2012@buaa.edu.cn

***Abstract*—This paper addresses the target localization problem in the three-dimensional (3D) space using the hybrid bistatic range (BR) and angle of arrival (AOA) measurements in multistatic radar. Based on the two-stage processing, an efficient closed-form solution to this nonlinear estimation problem is developed. The covariance matrix of the proposed solution is derived under small error condition. Theoretical analysis shows that the performance of the proposed method can reach the Cramer-Rao lower bound (CRLB) for Gaussian noise over small error region. Simulations are included to validate the performance of the proposed estimator.**

Keywords—Cramer-Rao lower bound (CRLB); bistatic range; angle of arrival (AOA); source localization; multistatic radar

I. INTRODUCTION

The problem of target localization in multistatic radar systems has received considerable attentions in recent years [1]. Target localization is a fundamental problem for many applications such as airbone early warning, surveillance and tracking. Multistatic radar consists of multiple transmitter-receiver pairs, each of which is placed at different location. Generally speaking, different type of measurements can be used to locate a single target, such as bistatic range (BR) [1]–[5], Doppler shift [6], and angle of arrival (AOA) [3], [7]. The most common is BR measurement, which defines an ellipsoidal locus where the target lies on, with the associated transmitter and receiver as its foci. The intersection of multi-ellipsoids produces the target position estimate.

The localization problem is potentially challenging due to the highly nonlinear relationship between the measurements and the unknowns. A lot of elliptic localization methods have been developed. An iterative solution for the localization problem based on the Taylor-series expansion was proposed in [8]. Although it can approximate the maximum likelihood (ML) estimator, an initial position guess close to the actual target position is required to avoid divergence. Recently, various two-stage weighted least squares (WLS) estimators have been evolved [4], [5], which can reach the Cramer–Rao lower bound (CRLB) performance under small measurement noise. In [6], the moving target localization problem was considered to identify jointly the target position and velocity, using the hybrid BR and Doppler shift measurements. In [3], Rui and Ho developed a closed-form algebraic localization algorithm through parameter transformation and multistage processing, in which the time and bearing measurements were used. It was shown to be able to attain the CRLB accuracy under mild measurement noise conditions. Nevertheless, the method only applied to two-dimensional (2D) case using one-dimensional AOA, viz., bearing measurement. For the issue of three-dimensional (3D) space target positioning, 2D AOA represented by the pair of azimuth and elevation should be explored. However, to the best of our knowledge, we have not come across any closed-form method which is appropriate for 3D localization using the hybrid BR and AOA measurements.

This paper develops an efficient closed-form method for the 3D space target localization using the hybrid BR and AOA measurements in multistatic radar. The proposed method is based on two-stage processing. In the first stage, a set of pseudo-linear BR, azimuth and elevation angle equations are established by introducing appropriate nuisance parameters. In the second stage, in order to improve the localization performance, the error term of the first stage solution is estimated through exploring the relationship between the unknowns and the nuisance parameters. The obtained estimator is computationally attractive and avoids the local convergence problem which exists in iterative solutions. The new algorithm is shown, by both theory and simulations, to be able to reach the CRLB performance for small Gaussian measurement noise.

The rest of this paper is organized as follows. Section II formulates the localization scenario and data model. The CRLB is briefly summarized in Section III. In Section IV, the closed-form algorithm is presented. Section V analyzes the performance of the proposed estimator. Section VI presents simulations to verify the theoretical developments, and Section VII concludes the paper.

II. PROBLEM FORMULATION

We consider the problem of single target localization in the 3D space with M transmitters and N receivers, whose positions are denoted by $\boldsymbol{t}_i = \left[x_i^t, y_i^t, z_i^t\right]^{\mathrm{T}}$, $i = 1, 2, \cdots, M$ and $\boldsymbol{s}_j = \left[x_j^s, y_j^s, z_j^s\right]^{\mathrm{T}}$, $j = 1, 2, \cdots, N$, respectively. The true position of the target is represented by $\boldsymbol{u} = \left[x, y, z\right]^{\mathrm{T}}$, where superscript T stands for the transpose. The localization scenario is illustrated in Fig. 1.

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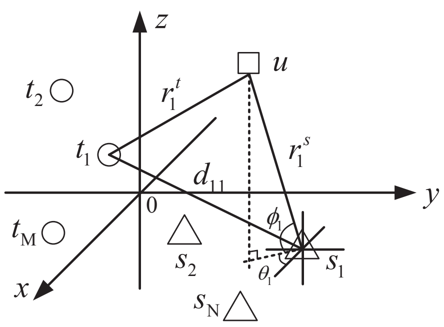

Fig. 1. Localization geometry.

Each transmitter radiates a signal, and all receivers observe the signal from direct propagation and from indirect reflection of the target. The product of time-delay between transmitter-target-receiver which is easy to be obtained from the time difference measurement of direct and indirect propagation, and the signal propagation speed is known as BR. Moreover, the source reflecting signal can be observed by all receivers to produce multiple AOA pairs $\left(\theta_j, \varphi_j\right)$, $j = 1, 2, \cdots, N$, where θ_j and φ_j denote the azimuth and elevation respectively, and $\theta_j \in \left(-\pi, \pi\right]$, $\varphi_j \in \left(0, \pi/2\right)$. We would like to accurately estimate the target position from the observed BR and AOA measurements.

Autocorrelation of the direct and the reflected signals from transmitter i at receiver j provides the time difference of arrival τ_{ij}^o. After multiplying τ_{ij}^o by the signal propagation speed c, the range difference of arrival r_{ij}^o is given by

$$r_{ij}^o = c\tau_{ij}^o = r_i^t + r_j^s - d_{ij} \tag{1}$$

where $r_i^t = \left\|\boldsymbol{u} - \boldsymbol{t}_i\right\|$, $r_j^s = \left\|\boldsymbol{u} - \boldsymbol{s}_j\right\|$, $d_{ij} = \left\|\boldsymbol{t}_i - \boldsymbol{s}_j\right\|$, and $\left\|\cdot\right\|$ represents the l_2 norm. Obviously, the BR is obtained by the sum of r_{ij}^o and the base range d_{ij}. One BR measurement corresponds to one transmitter-receiver pair; a total of MN BR measurements are gathered from all the transmitter-receiver pairs to locate the target.

The measured version of r_{ij}^o is $r_{ij} = r_{ij}^o + \Delta r_{ij}$, where Δr_{ij} is the additive noise. Collecting all the range measurements gives

$$\boldsymbol{r} = \left[\boldsymbol{r}_1^{\mathrm{T}}, \boldsymbol{r}_2^{\mathrm{T}}, \cdots, \boldsymbol{r}_N^{\mathrm{T}}\right]^{\mathrm{T}} = \boldsymbol{r}^o + \Delta\boldsymbol{r} \tag{2}$$

where $\boldsymbol{r}_j = \left[r_{1j}, r_{2j}, \cdots, r_{Mj}\right]^{\mathrm{T}} = \boldsymbol{r}_j^o + \Delta\boldsymbol{r}_j$ contains the range measurements from the j th receiver, and $\Delta\boldsymbol{r}$ is the range noise vector.

In addition to the range measurements, each receiver can determine one azimuth-elevation pair of the target as well [9]. The AOA pairs are related to the target position by

$$\theta_j^o = \arctan\frac{y - y_j^s}{x - x_j^s}, \quad \varphi_j^o = \arcsin\frac{z - z_j^s}{r_j^s} \tag{3}$$

Each AOA pair forms a straight line that passes through the target. There are one azimuth and one elevation measurements for each receiver, giving a total of $2N$ AOA measurements to locate the target.

The AOA measurements in vector form are modeled by

$$\boldsymbol{\theta} = \left[\theta_1, \theta_2, \cdots, \theta_N\right]^{\mathrm{T}} = \boldsymbol{\theta}^o + \Delta\boldsymbol{\theta} \tag{4}$$

$$\boldsymbol{\varphi} = \left[\varphi_1, \varphi_2, \cdots, \varphi_N\right]^{\mathrm{T}} = \boldsymbol{\varphi}^o + \Delta\boldsymbol{\varphi} \tag{5}$$

where $\boldsymbol{\theta}^o$ and $\boldsymbol{\varphi}^o$ are their actual counterpart; $\Delta\boldsymbol{\theta}$ and $\Delta\boldsymbol{\varphi}$ are the corresponding noise vectors.

For notation simplicity, we stack $\boldsymbol{r}$, $\boldsymbol{\theta}$, $\boldsymbol{\varphi}$ and represent them together by the $\left(MN + 2N\right) \times 1$ measurement vector $\boldsymbol{m} = \left[\boldsymbol{r}^{\mathrm{T}}, \boldsymbol{\theta}^{\mathrm{T}}, \boldsymbol{\varphi}^{\mathrm{T}}\right]^{\mathrm{T}} = \boldsymbol{m}^o + \Delta\boldsymbol{m}$. To simplify the development, we assume $\Delta\boldsymbol{r}$, $\Delta\boldsymbol{\theta}$ and $\Delta\boldsymbol{\varphi}$ are independent of each other [3], [10]. The measurement error vector $\Delta\boldsymbol{m}$ is assumed to be zero-mean Gaussian with a covariance matrix $\boldsymbol{Q_m} = \mathrm{diag}\left(\boldsymbol{Q_r}, \boldsymbol{Q_\theta}, \boldsymbol{Q_\varphi}\right)$, where $\boldsymbol{Q_r}$, $\boldsymbol{Q_\theta}$ and $\boldsymbol{Q_\varphi}$ are the covariance matrices of $\boldsymbol{r}$, $\boldsymbol{\theta}$ and $\boldsymbol{\varphi}$, respectively.

The objective of this paper is to estimate the target location $\boldsymbol{u}$ as accurately as possible using the hybrid BR and AOA observations, while maintaining a manageable complexity.

III. CRLB ANALYSIS

We shall establish the performance bound through CRLB analysis for the multistatic radar localization problem. The CRLB is the lower bound on the covariance matrix of any unbiased estimators of deterministic parameters [11]. The unknowns to be estimated are the target position $\boldsymbol{u}$.

The density function of the composite measurement is

$$f\left(\boldsymbol{m} \mid \boldsymbol{u}\right) = K \exp\left(-\frac{1}{2}\left(\boldsymbol{m} - \boldsymbol{m}^o\right)^{\mathrm{T}} \boldsymbol{Q_m}^{-1}\left(\boldsymbol{m} - \boldsymbol{m}^o\right)\right) \tag{6}$$

where K is a constant. Under aforementioned Gaussian measurement noise model, the CRLB of $\boldsymbol{u}$ is given by

$$\mathrm{CRLB}(\boldsymbol{u}) = \boldsymbol{J}(\boldsymbol{u})^{-1} \tag{7}$$

where $\boldsymbol{J}(\boldsymbol{u})$ is the Fisher information matrix (FIM) equal to

$$\boldsymbol{J}(\boldsymbol{u}) = \nabla_{\boldsymbol{u}}^{\boldsymbol{m}^{o}\mathrm{T}} \boldsymbol{Q}_{\boldsymbol{m}}^{-1} \nabla_{\boldsymbol{u}}^{\boldsymbol{m}^{o}} \tag{8}$$

where $\nabla_{\boldsymbol{a}}^{\boldsymbol{b}} = \partial \boldsymbol{b} / \partial \boldsymbol{a}$, i.e., $\nabla_{\boldsymbol{u}}^{\boldsymbol{m}^{o}}$ denotes the partial derivative of the noise-free joint vector $\boldsymbol{m}^{o}$ with respect to the unknown vector $\boldsymbol{u}$.

Furthermore, we have $\nabla_{\boldsymbol{u}}^{\boldsymbol{m}^{o}} = \left[\nabla_{\boldsymbol{u}}^{\boldsymbol{r}^{o}\mathrm{T}}, \nabla_{\boldsymbol{u}}^{\boldsymbol{\theta}^{o}\mathrm{T}}, \nabla_{\boldsymbol{u}}^{\boldsymbol{\varphi}^{o}\mathrm{T}}\right]^{\mathrm{T}}$, whose elements are the partial derivatives of the range and AOA functions with respect to $\boldsymbol{u}$ evaluated at its true value.

The row of $\nabla_{\boldsymbol{u}}^{\boldsymbol{r}^{o}}$, which is a $MN \times 3$ matrix, is

$$\nabla_{\boldsymbol{u}}^{r_{ij}^{o}} = \boldsymbol{\rho}_{\boldsymbol{u},\boldsymbol{t}_i}^{\mathrm{T}} + \boldsymbol{\rho}_{\boldsymbol{u},\boldsymbol{s}_j}^{\mathrm{T}} \tag{9}$$

where $\boldsymbol{\rho}_{\boldsymbol{a},\boldsymbol{b}} = (\boldsymbol{a}-\boldsymbol{b}) / \|\boldsymbol{a}-\boldsymbol{b}\|$ denotes a unit vector from $\boldsymbol{b}$ to $\boldsymbol{a}$.

The partial derivatives of AOA measurement $\nabla_{\boldsymbol{u}}^{\boldsymbol{\theta}^{o}}$ and $\nabla_{\boldsymbol{u}}^{\boldsymbol{\varphi}^{o}}$ have the same matrix structure, whose j th row are

$$\nabla_{\boldsymbol{u}}^{\theta_j^{o}} = \left(\boldsymbol{T}\,\boldsymbol{\rho}_{\boldsymbol{Fu},\boldsymbol{Fs}_j} / \left\|\boldsymbol{Fu} - \boldsymbol{Fs}_j\right\|\right)^{\mathrm{T}} \tag{10}$$

$$\nabla_{\boldsymbol{u}}^{\varphi_j^{o}} = \left(\boldsymbol{V}\,\boldsymbol{\rho}_{\boldsymbol{Fu},\boldsymbol{Fs}_j} / \left\|\boldsymbol{u} - \boldsymbol{s}_j\right\|^{2}\right)^{\mathrm{T}} \tag{11}$$

where $\boldsymbol{T} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, $\boldsymbol{F} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, $\boldsymbol{V} = \begin{bmatrix} z_j - z & 0 & 0 \\ 0 & z_j - z & 0 \\ x - x_j & y - y_j & 0 \end{bmatrix}$.

Given (8)-(11), the CRLB of unknowns can be readily obtained. We shall develop the target location method using the hybrid range and AOA measurements in the next Section.

IV. THE PROPOSED METHOD

We shall develop a computationally attractive and non-iterative estimator for the localization problem. The proposed algorithm is comprised of two stages. The first stage establishes a set of pseudo-linear equations by introducing nuisance parameters and estimates the target position using a WLS estimator [11]. The second stage estimates an error term to improve the solution of the first stage through exploring the estimates of nuisance variables [6].

The derivations ignore the second and higher order error terms, which is justified when the range and AOA measurements noise are not large.

A. First Stage

We first describe (1) as $r_{ij}^{o} - r_j^{s} + d_{ij} = r_i^{t}$. Substituting $r_{ij}^{o} = r_{ij} - \Delta r_{ij}$, rearranging and squaring both sides yield

$$\begin{aligned} 2r_i^{t}\Delta r_{ij} \simeq r_{ij}^{2} + 2r_{ij}d_{ij} + 2\left(\boldsymbol{s}_j - \boldsymbol{t}_i\right)^{\mathrm{T}} \boldsymbol{s}_j \\ -2\left(\boldsymbol{s}_j - \boldsymbol{t}_i\right)^{\mathrm{T}} \boldsymbol{u} - 2\left(r_{ij} + d_{ij}\right) r_j^{s} \end{aligned} \tag{12}$$

for $i = 1, 2, \cdots, M$ and $j = 1, 2, \cdots, N$.

The azimuth angle θ_j^{o} is related to the target position through the first equation of (3). We shall express it in terms of the noisy azimuth to create a measurement equation. The algorithm derivation requires the azimuth error to be small as $\Delta\theta_j = \theta_j - \theta_j^{o} \simeq 0, j = 1, 2, \cdots, N$ [3]. Thus, we have $\sin(\Delta\theta_j) \simeq \Delta\theta_j$ and $\cos(\Delta\theta_j) \simeq 1$. Subsequently, the two formulas $\sin(\theta_j^{o}) \simeq \sin(\theta_j) - \cos(\theta_j)\Delta\theta_j$ and $\cos(\theta_j^{o}) \simeq \cos(\theta_j) + \sin(\theta_j)\Delta\theta_j$ hold.

For notation simplicity, we define two known vectors $\boldsymbol{v}_j = [\cos(\theta_j), \sin(\theta_j), 0]^{\mathrm{T}}$, $\boldsymbol{w}_j = [-\sin(\theta_j), \cos(\theta_j), 0]^{\mathrm{T}}$. Taking tangent on both sides of left equation of (3), expressing $\tan(\theta_j)$ as $\sin(\theta_j)/\cos(\theta_j)$, replacing θ_j^{o} by $\theta_j - \Delta\theta_j$ and invoking the above proximity relation, we arrive at

$$\boldsymbol{v}_j^{\mathrm{T}}\left(\boldsymbol{u} - \boldsymbol{s}_j\right)\Delta\theta_j = \boldsymbol{w}_j^{\mathrm{T}}\boldsymbol{s}_j - \boldsymbol{w}_j^{\mathrm{T}}\boldsymbol{u} \tag{13}$$

for $j = 1, 2, \cdots, N$.

The elevation angle φ_j^{o} is related to the target position through the second equation of (3). Similarly, we assume the elevation error to be small as $\Delta\varphi_j = \varphi_j - \varphi_j^{o} \simeq 0$, $j = 1, 2, \cdots, N$. Thus, we have $\sin(\Delta\varphi_j) \simeq \Delta\varphi_j$ and $\sin(\varphi_j^{o}) \simeq \sin(\varphi_j) - \cos(\varphi_j)\Delta\varphi_j$.

Taking sine on both sides of right equation of (3), replacing φ_j^{o} by $\varphi_j - \Delta\varphi_j$ and utilizing the above proximity relation, we have

$$\cos(\varphi_j) r_j^{s} \Delta\varphi_j = \boldsymbol{k}^{\mathrm{T}}\boldsymbol{s}_j - \boldsymbol{k}^{\mathrm{T}}\boldsymbol{u} + \sin(\varphi_j) r_j^{s} \tag{14}$$

where $\boldsymbol{k} = [0, 0, 1]^{\mathrm{T}}$, $j = 1, 2, \cdots, N$.

In both (12), (13) and (14), the second-order noise terms have been ignored. The N azimuth equations and N elevation equations will be combined with the MN range measurement equations to obtain a better target position estimate.

We shall assume $\left[r_1^s, r_2^s, \cdots, r_N^s\right]^{\mathrm{T}}$ and $\boldsymbol{u}$ are independent unknown variables so that (12) and (14) become pseudo-linear. Let the unknown vector be

$$\boldsymbol{\Psi}^o = \left[\boldsymbol{u}^{\mathrm{T}}, r_1^s, r_2^s, \cdots, r_N^s\right]^{\mathrm{T}} \tag{15}$$

Stacking (12), (13) and (14), and putting the $MN+2N$ equations together yield the matrix form equation

$$\boldsymbol{B}_1 \Delta \boldsymbol{m} = \boldsymbol{h}_1 - \boldsymbol{G}_1 \boldsymbol{\Psi}^o \tag{16}$$

In (16), the vector $\boldsymbol{h}_1 = \left[\boldsymbol{h}_r^{\mathrm{T}}, \boldsymbol{h}_\theta^{\mathrm{T}}, \boldsymbol{h}_\varphi^{\mathrm{T}}\right]^{\mathrm{T}}$. The entries of $\boldsymbol{h}_r$, $\boldsymbol{h}_\theta$ and $\boldsymbol{h}_\varphi$ are $r_{ij}^2 + 2r_{ij}d_{ij} + 2\left(\boldsymbol{s}_j - \boldsymbol{t}_i\right)^{\mathrm{T}} \boldsymbol{s}_j$, $\boldsymbol{w}_j^{\mathrm{T}} \boldsymbol{s}_j$, and $\boldsymbol{k}^{\mathrm{T}} \boldsymbol{s}_j$, respectively. The matrix $\boldsymbol{G}_1$ can be represented as $\boldsymbol{G}_1 = \left[\boldsymbol{G}_r^{\mathrm{T}}, \boldsymbol{G}_\theta^{\mathrm{T}}, \boldsymbol{G}_\varphi^{\mathrm{T}}\right]^{\mathrm{T}}$. The rows of $\boldsymbol{G}_r$, $\boldsymbol{G}_\theta$ and $\boldsymbol{G}_\varphi$ are $2\left[\left(\boldsymbol{s}_j - \boldsymbol{t}_i\right)^{\mathrm{T}}, \boldsymbol{0}_{j-1}^{\mathrm{T}}, \left(r_{ij} + d_{ij}\right), \boldsymbol{0}_{N-j}^{\mathrm{T}}\right]$, $\left[\boldsymbol{w}_j^{\mathrm{T}}, \boldsymbol{0}_N^{\mathrm{T}}\right]$, and $\left[\boldsymbol{k}^{\mathrm{T}}, \boldsymbol{0}_{j-1}^{\mathrm{T}}, -\sin\left(\varphi_j\right), \boldsymbol{0}_{N-j}^{\mathrm{T}}\right]$, where $\boldsymbol{0}$ denotes a corresponding dimension zero vector.

The matrix $\boldsymbol{B}_1 = blkdiag\left(\boldsymbol{B}_r, \boldsymbol{B}_\theta, \boldsymbol{B}_\varphi\right)$. $\boldsymbol{B}_r$ is equal to $\boldsymbol{B}_r = 2\mathbf{I}_N \otimes diag\left(r_1^t, r_2^t, \cdots, r_M^t\right)$, where $\mathbf{I}$ represents an identity matrix and $\otimes$ stands for the Kronecker product. The submatrix $\boldsymbol{B}_\theta$ and $\boldsymbol{B}_\varphi$ are $diag\left(\boldsymbol{v}_1^{\mathrm{T}}\left(\boldsymbol{u} - \boldsymbol{s}_1\right), \boldsymbol{v}_2^{\mathrm{T}}\left(\boldsymbol{u} - \boldsymbol{s}_2\right), \cdots, \boldsymbol{v}_N^{\mathrm{T}}\left(\boldsymbol{u} - \boldsymbol{s}_N\right)\right)$, $diag\left(\cos\left(\varphi_1\right) r_1^s, \cos\left(\varphi_2\right) r_2^s, \cdots, \cos\left(\varphi_N\right) r_N^s\right)$.

The WLS solution to (16) is

$$\boldsymbol{\Psi} = \left(\boldsymbol{G}_1^T \boldsymbol{W}_1 \boldsymbol{G}_1\right)^{-1} \boldsymbol{G}_1^{\mathrm{T}} \boldsymbol{W}_1 \boldsymbol{h}_1 \tag{17}$$

where the weighting matrix $\boldsymbol{W}_1$ is $\boldsymbol{B}_1^{-\mathrm{T}} \boldsymbol{Q}_m^{-1} \boldsymbol{B}_1^{-1}$. The weighting matrix $\boldsymbol{W}_1$ depends on the true target position through $\boldsymbol{B}_1$. To solve this problem, we can first set the weighting matrix as $\boldsymbol{Q}_m^{-1}$ to produce an initial solution. The estimated target position is used to form the desired weighting matrix. Then, the final first stage solution is obtained from the new weighting matrix. According to the previous studies [3], [6], when the measurement noise is small, the first stage estimator has negligible bias, and the covariance matrix of $\boldsymbol{\Psi}$ can be approximated by $\operatorname{cov}\left(\boldsymbol{\Psi}\right) \simeq \left(\boldsymbol{G}_1^{\mathrm{T}} \boldsymbol{W}_1 \boldsymbol{G}_1\right)^{-1}$.

The first stage solution, however, cannot attain the CRLB accuracy. In the next stage, we shall explore the dependency in the elements of $\boldsymbol{\Psi}$ and estimate the error term of the first stage solution to improve the target position estimate.

B. Second Stage

The first stage solution can be represented as $\boldsymbol{\Psi} = \left[\hat{\boldsymbol{u}}^{\mathrm{T}}, \hat{r}_1^s, \hat{r}_2^s, \cdots, \hat{r}_N^s\right]^{\mathrm{T}} = \boldsymbol{\Psi}^o + \Delta\boldsymbol{\Psi}$, where $\Delta\boldsymbol{\Psi} = \left[\Delta\boldsymbol{u}^{\mathrm{T}}, \Delta r_1^s, \Delta r_2^s, \cdots, \Delta r_N^s\right]^{\mathrm{T}}$ is the associated estimation error. Then, we can replace the true value $\boldsymbol{u}$ and r_j^s with $\hat{\boldsymbol{u}} - \Delta\boldsymbol{u}$ and $\hat{r}_j^s - \Delta r_j^s$. After substituting $\boldsymbol{u}$ and r_j^s in the expression $r_j^s = \left\|\boldsymbol{u} - \boldsymbol{s}_j\right\|$, which relates the nuisance variables and the actual unknowns, we obtain

$$2\hat{r}_j^s \Delta r_j^s \simeq \hat{r}_j^{s2} - \left(\hat{\boldsymbol{u}} - \boldsymbol{s}_j\right)^{\mathrm{T}} \left(\hat{\boldsymbol{u}} - \boldsymbol{s}_j\right) + 2\left(\hat{\boldsymbol{u}} - \boldsymbol{s}_j\right)^{\mathrm{T}} \Delta\boldsymbol{u} \tag{18}$$

In (18), the second-order error terms have been ignored and $\Delta\boldsymbol{u}$ is interpreted as the unknowns. Moreover, under small measurement noise, $\boldsymbol{\Psi}$ is approximately unbiased and hence $\Delta\boldsymbol{u}$ is zero mean. So, we can introduce an additional equation

$$\Delta\boldsymbol{u} = \boldsymbol{0}_3 + \Delta\boldsymbol{u} \tag{19}$$

where the left side of equal is considered as random errors and the second term of right side is unknowns.

Equation (18) and (19) can be recast in a matrix form as

$$\boldsymbol{B}_2 \Delta\boldsymbol{\Psi} = \boldsymbol{h}_2 - \boldsymbol{G}_2 \Delta\boldsymbol{u} \tag{20}$$

In (20), $\boldsymbol{h}_2 = \left[\boldsymbol{0}_3^{\mathrm{T}}, \boldsymbol{h}_\psi^{\mathrm{T}}\right]^{\mathrm{T}}$ and $\boldsymbol{G}_2 = \left[-\boldsymbol{I}_3^{\mathrm{T}}, \boldsymbol{G}_\psi^{\mathrm{T}}\right]^{\mathrm{T}}$. The j th element of $\boldsymbol{h}_\psi$ is $\hat{r}_j^{s2} - \left(\hat{\boldsymbol{u}} - \boldsymbol{s}_j\right)^{\mathrm{T}} \left(\hat{\boldsymbol{u}} - \boldsymbol{s}_j\right)$. The j th row of $\boldsymbol{G}_\psi$ is given by $-2\left(\hat{\boldsymbol{u}} - \boldsymbol{s}_j\right)^{\mathrm{T}}$. The matrix $\boldsymbol{B}_2$ can be written as $\boldsymbol{B}_2 = blkdiag\left(\boldsymbol{I}_3, \boldsymbol{B}_\psi\right)$, where $\boldsymbol{B}_\psi = 2diag\left(r_1^s, r_2^s, \cdots, r_N^s\right)$.

The WLS solution to (20) is

$$\Delta\hat{\boldsymbol{u}} = \left(\boldsymbol{G}_2^{\mathrm{T}} \boldsymbol{W}_2 \boldsymbol{G}_2\right)^{-1} \boldsymbol{G}_2^{\mathrm{T}} \boldsymbol{W}_2 \boldsymbol{h}_2 \tag{21}$$

where the weighting matrix $\boldsymbol{W}_2$ is computed as $\boldsymbol{B}_2^{-\mathrm{T}} \operatorname{cov}\left(\boldsymbol{\Psi}\right)^{-1} \boldsymbol{B}_2^{-1}$. The $\boldsymbol{W}_2$ is dependent on the real target position through $\boldsymbol{B}_2$. To this end, we form $\boldsymbol{B}_2$ by replacing $\boldsymbol{u}$ with the estimated target position $\hat{\boldsymbol{u}}$.

Consequently, the final target position estimate is

$$\tilde{\boldsymbol{u}} = \hat{\boldsymbol{u}} - \Delta\hat{\boldsymbol{u}} \tag{22}$$

V. PERFORMANCE ANALYSIS

We shall examine the localization performance of the proposed method by comparing its covariance matrix with the CRLB under small noise condition. We first derive the covariance matrix of the final target position estimate.

By substituting $\hat{\boldsymbol{u}} = \boldsymbol{u} + \Delta\boldsymbol{u}$ into (22), we have $\tilde{\boldsymbol{u}} - \boldsymbol{u} = -\left(\Delta\hat{\boldsymbol{u}} - \Delta\boldsymbol{u}\right)$ and hence $\operatorname{cov}\left(\tilde{\boldsymbol{u}}\right) = \operatorname{cov}\left(\Delta\hat{\boldsymbol{u}}\right)$. Under the condition that the measurement noise is small enough, the covariance matrix of the second stage solution is given by $\operatorname{cov}\left(\Delta\hat{\boldsymbol{u}}\right) \simeq \left(\boldsymbol{G}_2^{\mathrm{T}}\boldsymbol{W}_2\boldsymbol{G}_2\right)^{-1}$. Thus, the covariance matrix of the final target estimate is obtained as $\operatorname{cov}\left(\tilde{\boldsymbol{u}}\right) = \left(\boldsymbol{G}_2^{\mathrm{T}}\boldsymbol{W}_2\boldsymbol{G}_2\right)^{-1}$. After substituting $\boldsymbol{W}_2$, $\operatorname{cov}\left(\boldsymbol{\varPsi}\right)$ and $\boldsymbol{W}_1$ in a sequence, $\operatorname{cov}\left(\tilde{\boldsymbol{u}}\right)$ can be written as

$$\operatorname{cov}\left(\tilde{\boldsymbol{u}}\right) = \left(\boldsymbol{G}_3^{\mathrm{T}}\boldsymbol{Q}_m^{-1}\boldsymbol{G}_3\right)^{-1} \tag{23}$$

where $\boldsymbol{G}_3 = \boldsymbol{B}_1^{-1}\boldsymbol{G}_1\boldsymbol{B}_2^{-1}\boldsymbol{G}_2$.

The subsequent analysis requires the following small noise conditions: (C1) $\Delta r_{ij} \ll r_{ij}^o$, $\Delta r_{ij} \ll r_i^t$, (C2) $\Delta\theta_j \simeq 0$, (C3) $\Delta\varphi_j \simeq 0$, for $i = 1,2,\cdots,M$ and $j = 1,2,\cdots,N$. After some straightforward mathematical manipulations, under the above three conditions, it can be easily verified that

$$\boldsymbol{G}_3 \simeq \nabla_{\boldsymbol{u}}^{\boldsymbol{m}^o} \tag{24}$$

where $\nabla_{\boldsymbol{u}}^{\boldsymbol{m}^o}$ denotes the partial derivative of $\boldsymbol{m}^o$ with respect to $\boldsymbol{u}$ in (8). Putting (24) into (23) and comparing to (8), we conclude that $\operatorname{cov}\left(\tilde{\boldsymbol{u}}\right) \simeq \mathrm{CRLB}\left(\boldsymbol{u}\right)$, which proves approximate efficiency of the proposed estimator under the three small noise conditions.

VI. SIMULATIONS

In this section, simulations are presented to evaluate the performance of the proposed method and validate the theoretical developments. The multistatic radar has three transmitters and four receivers, whose positions are tabulated in Table I. The target to be located is at $\boldsymbol{u} = \left[-500,500,500\right]^{\mathrm{T}}$ m.

TABLE I. TRANSMITTERS AND RECEIVERS POSITIONS (KM)

Tx no. *i*	x_i^t	y_i^t	z_i^t	**Rx no. *j***	x_j^s	y_j^s	z_j^s
1	5	11	10	**1**	7	9.5	2.5
2	8.5	8	5	**2**	10	6.5	4.5
3	11	4	1	**3**	12	2	5
-	-	-	-	**4**	14	1	7.5

The signal propagation speed is $c = 3\times10^8$ m/s. The covariance matrices of the bistatic range and AOA measurements are set as $\boldsymbol{Q}_r = c^2\sigma_t^2\mathbf{I}_{MN}$ and $\boldsymbol{Q}_{AOA} = \sigma_{AOA}^2\mathbf{I}_{2N}$ separately, where σ_t and σ_{AOA} are the standard deviation of time-delay and AOA measurements. Simulation results are displayed in terms of the mean square error (MSE). Besides, we use CRLB as the benchmark for performance evaluation, which has been derived in Section III. The number of ensemble runs is 5000.

Fig. 2 illustrates the performance of the proposed estimator as the bistatic range measurement noise increases when AOA measurement error σ_{AOA} is fixed at 0.2 degree. It is obvious that the introduction of AOA measurements can provide better localization accuracy than using BR measurement alone. The improvement by adding the AOA measurements becomes apparent as σ_t increases. The proposed method works well and can attain the CRLB accuracy at the visible range of σ_t. The location method only using BR measurement begins to deviate from the CRLB when $\sigma_t = 15$ ns, while the proposed method remains to provide CRLB performance. Therefore, the proposed estimator has a higher noise tolerance than location method without AOA measurements.

The results of target position estimate is reported in Fig. 3 as the AOA measurement noise increases, when range measurement noise σ_t is equal to 40 ns. The localization accuracy of the BR-based method is independent of σ_{AOA}. So the associated localization performance curve fluctuates slightly around 25.5 dB, and the related CRLB keeps constant about 24 dB. The CRLB using hybrid measurements approaches the CRLB using BR measurement when σ_{AOA} is large, but it improves considerably at small σ_{AOA}. The proposed estimator achieves the CRLB when σ_{AOA} is small and deviates slightly from the bound when $\sigma_{AOA} > 0.5$ degree. The proposed method performs always better than the location method without AOA measurement and the improvement of localization performance becomes more and more obvious as σ_{AOA} decreases.

VII. CONCLUSION

This paper considered the problem of 3D target localization using the hybrid BR and AOA measurements in multistatic radar. An efficient closed-form method to estimate the target position was derived through two-stage processing. It requires less computation and does not rely on initial position guess. The new estimator using hybrid measurements can provide better localization accuracy than using BR measurement alone. Analysis shows that the performance of the proposed method can reach the CRLB performance for Gaussian measurement noise over the small error region. Simulations corroborate the theoretical performance of the proposed estimator.

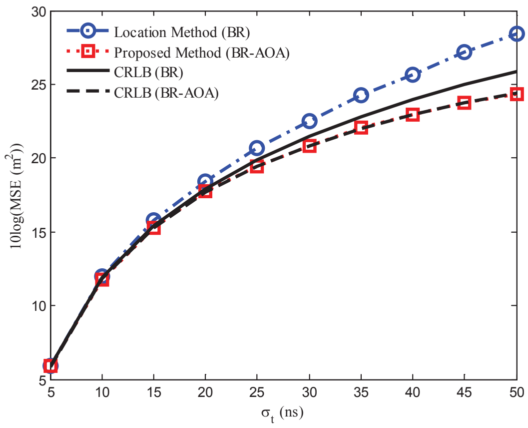

Fig. 2. Localization performance versus range measurement noise.

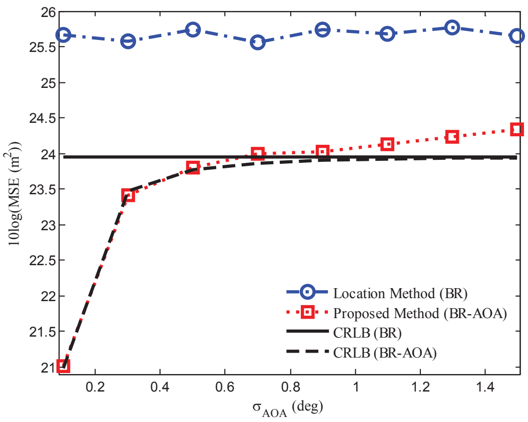

Fig. 3. Localization performance versus angle of arrival measurement noise.

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