Supplementary Data and Discussion of "A Centroid Guided Cluster Transformation for Dynamic Multi-Objective Optimization Algorithm"

I. Experiments

A. Comparison Study

Meanwhile, to facilitate the reader to visualize the performance of CGCT-RM-MEDA with its peer algorithms, some sets of average IGD values under different environments are plotted in Fig. 1. As displayed in Fig. 1, the IGD values of CGCT-RM-MEDA are low and stable on most test problems, indicating its competitiveness on many problems. In other words, the algorithm shows better performance in all test problems.

B. Ablation Study

To gain insights into how the proposed strategies impact the performance of CGCT-RM-MEDA, a set of experiments are conducted.

1) Effectiveness of estimated boundaries: To verify the effectiveness of estimated boundaries, 2 variants of CGCT-RM-MEDA are introduced. Then, the 2 variants of CGCT-RM-MEDA are defined as follows. CGCT$_{b1}$, and CGCT$_{b2}$ use the cluster boundary of the second last environment and the cluster boundary of the last environment, respectively. The left part of Table I presents the comparative results measured by MIGD, and the right part presents the results measured by MHV. In addition, to examine the overall performance of the variants under different dynamic parameters, the last row of Table I also presents the results of the Friedman test for the responses. From Table I, we can see that CGCT$_{b1}$ exhibits the second performance, and CGCT$_{b2}$ performs third. Furthermore, the Friedman test results also manifest that the earlier the model information of the previous environment was less effective and that the model information of the last environment behaved closer to the new environment. Based on the results of the above statistical analyses, we conclude that there are predictable paradigms for clusters in continuous environments and that the algorithms can perform better by predicting new boundaries.

2) Effectiveness of estimated hyper-rectangle: In order to verify the effectiveness of estimated hyper-rectangle, 2 variants of CGCT-RM-MEDA are introduced. Then, the 2 variants of CGCT-RM-MEDA are defined as follows. CGCT$_{h1}$, and CGCT$_{h2}$ use the hyper-rectangle of the second last environment and the hyper-rectangle of the last environment, respectively. As in Table I, the results of the comparison between MIGD and MHV are presented in Table II. Meanwhile, the last row of Table II presents the Friedman test results for MIGD and MHV. From Table II we can see that CGCT$_{h1}$ exhibits the second performance, CGCT$_{h2}$ performs third. Furthermore, the Friedman test results also manifest that the earlier the model information of the previous environment was less effective, and that the model information of the last environment behaved closer to the new environment. Based on the results of the above statistical analyses, we conclude that there are predictable paradigms for clusters in continuous environments, and that the algorithms can be made to perform better by predicting new hyper-rectangle.

3) Effectiveness of contributing components: To verify the validity of the proposed contributing components, two variants of CGCT-RM-MEDA are introduced. CGCT$_{c1}$ and CGCT$_{c2}$ using LSTM only and clustering transform only, respectively. Meanwhile, the comparison results of MIGD and MHV measurements are shown in Table III and Table IV, respectively. From both tables, it can be seen that CGCT-RM-MEDA shows a significant performance improvement compared to the variant using one component alone. Note that in Table III, the variant CGCT$_{c1}$ using only LSTM is superior to the variant CGCT$_{c2}$ using only cluster transformation. However, the performance of the two variants is close in Table IV. Based on the above statistical analyses, we conclude that both centroid guide and cluster transformation in consecutive environments can effectively improve the quality of the initial population and that the cluster transformation strategy plays a significant role in the bi-objective problem.

4) Effectiveness of LSTM: To verify the validity of LSTM, the AR model in PPS, the baseline algorithm of this study is used instead of LSTM to obtain the variant CGCT$_{ar}$. The left part of the Table V gives the MIGD results, and the right part provides the MHV with results. The comparative results for MIGD and MHV show significant advantages, especially in the bi-objective problem. However, there is no significant difference in the results of the tri-objective problem. Using only one centroid, we assumed there was no way to represent the population movement trend in the tri-objective problem accurately. In the future, we will consider optimizing the representation of the population movement trend in the tri-objective problem. Based on the above statistical analysis results, we conclude that centroid prediction using LSTM can be better than AR.

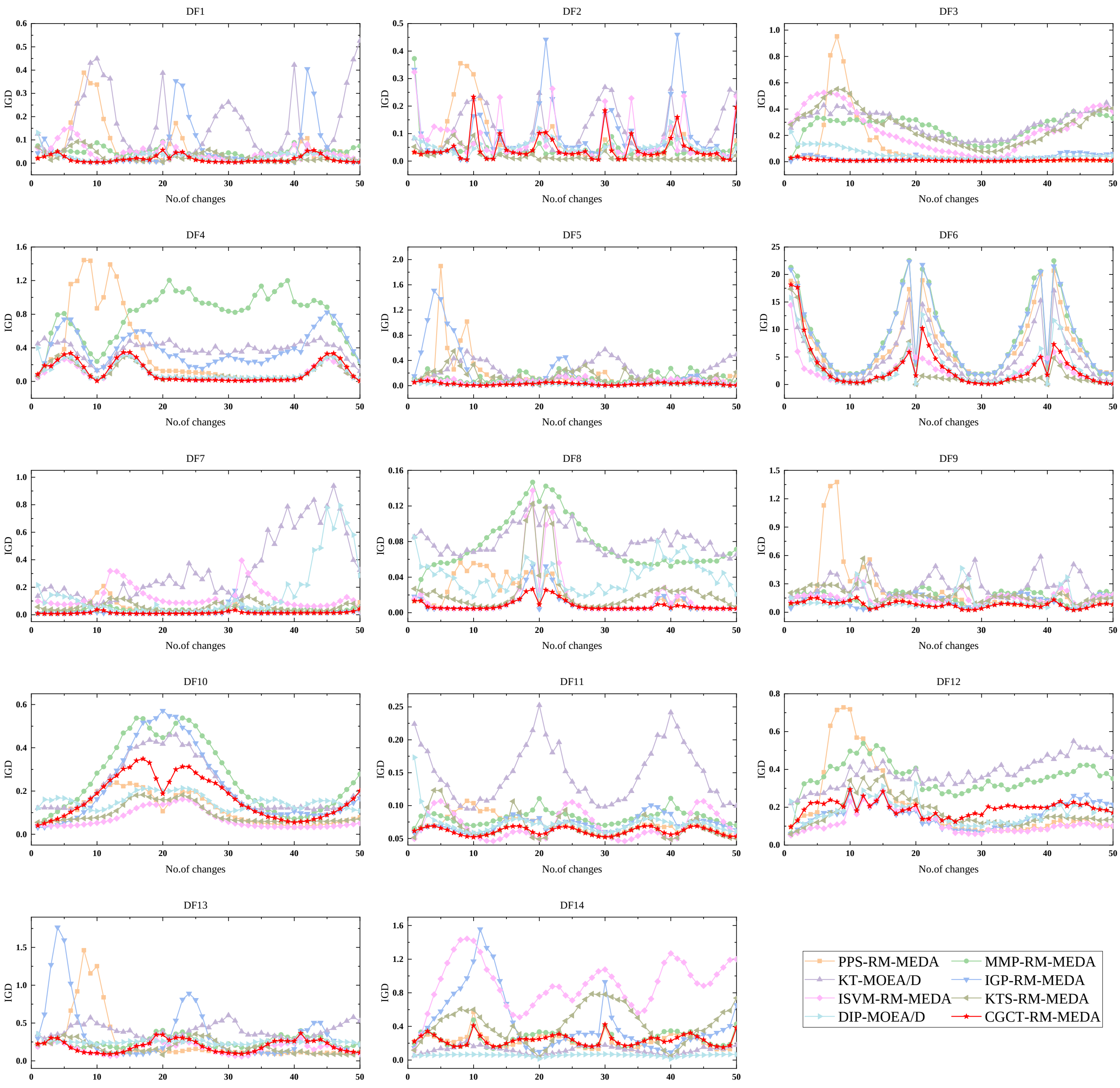

Fig. 1. Average IGD values (over 30 runs) obtained by algorithms on DMOPs with $n_t = 10, \tau_t = 10$.

TABLE I

COMPARISON RESULTS BETWEEN CGCT$_{b1}$, CGCT$_{b2}$ AND CGCT-RM-MEDA, MEASURED BY THE MIGD AND MHV VALUES

Problem	n_t, τ_t	CGCT$_{b1}$	CGCT$_{b2}$	CGCT-RM-MEDA	n_t, τ_t	CGCT$_{b1}$	CGCT$_{b2}$	CGCT-RM-MEDA
DF1	5,10	0.0667±2.27e-02+	0.0801±4.01e-02+	**0.0342±8.72e-03**	5,10	0.4665±2.08e-02+	0.4560±2.96e-02+	**0.4981±1.03e-02**
	10,10	0.0386±9.15e-03+	0.0390±1.45e-02+	**0.0183±3.78e-03**	10,10	0.4931±9.32e-03+	0.4932±1.54e-02+	**0.5180±4.88e-03**
DF2	5,10	0.0470±8.39e-03+	0.0483±1.02e-02+	**0.0337±2.59e-03**	5,10	0.6534±1.08e-02+	0.6524±1.39e-02+	**0.6714±3.90e-03**
	10,10	0.0533±6.23e-03+	0.0519±6.15e-03+	**0.0470±4.49e-03**	10,10	0.6459±7.81e-03+	0.6473±7.77e-03+	**0.6547±4.75e-03**
DF3	5,10	0.0168±1.53e-03+	0.0166±1.68e-03≈	**0.0160±8.28e-04**	5,10	0.4902±2.19e-03+	0.4905±2.27e-03≈	**0.4914±1.14e-03**
	10,10	0.0119±7.93e-04+	0.0116±6.35e-04+	**0.0106±4.06e-04**	10,10	0.4969±9.22e-04+	0.4974±7.43e-04+	**0.4984±6.51e-04**
DF4	5,10	0.1238±9.00e-03+	0.1222±1.02e-02≈	**0.1185±3.49e-03**	5,10	0.6993±5.09e-03+	0.7012±4.69e-03≈	**0.7026±2.06e-03**
	10,10	0.1156±2.80e-03≈	0.1147±2.57e-03≈	**0.1144±1.66e-03**	10,10	0.7131±2.12e-03≈	0.7146±1.38e-03≈	**0.7153±1.04e-03**
DF5	5,10	0.1593±8.85e-02+	0.1189±4.90e-02+	**0.0665±1.63e-02**	5,10	0.4354±3.39e-02+	0.4568±2.74e-02+	**0.4950±1.53e-02**
	10,10	0.0515±1.46e-02+	0.0539±1.34e-02+	**0.0284±4.40e-03**	10,10	0.5122±1.82e-02+	0.5092±1.66e-02+	**0.5431±6.32e-03**
DF6	5,10	4.6934±4.39e-01+	4.5302±4.72e-01≈	**4.4190±3.78e-01**	5,10	0.0525±2.42e-02+	0.0662±4.02e-02≈	**0.0845±3.31e-02**
	10,10	4.0504±6.57e-01+	3.8270±5.02e-01+	**3.0309±7.18e-01**	10,10	0.1259±5.02e-02+	0.1662±5.38e-02+	**0.2229±7.71e-02**
DF7	5,10	0.0471±1.60e-02+	0.0346±1.33e-02+	**0.0254±4.25e-03**	5,10	0.4491±4.29e-03+	0.4535±2.77e-03+	**0.4564±1.54e-03**
	10,10	0.0242±7.17e-03+	0.0180±3.01e-03+	**0.0141±1.23e-03**	10,10	0.4537±3.03e-03+	0.4570±1.75e-03+	**0.4593±7.21e-04**
DF8	5,10	0.0096±3.73e-04+	0.0096±2.47e-04+	**0.0090±3.84e-04**	5,10	0.6174±6.74e-05+	**0.6175±8.37e-05**+	0.6172±8.73e-05
	10,10	0.0096±3.73e-04+	0.0096±7.38e-04+	**0.0083±4.43e-04**	10,10	0.6432±4.06e-04+	**0.6433±4.24e-04**+	0.6431±5.11e-04
DF9	5,10	0.1881±4.03e-02+	0.1837±5.13e-02+	**0.1305±2.15e-02**	5,10	0.3699±1.99e-02+	0.3734±2.32e-02+	**0.3984±1.76e-02**
	10,10	0.1726±6.59e-02+	0.1291±5.94e-02+	**0.0783±1.10e-02**	10,10	0.4097±2.51e-02+	0.4326±2.45e-02+	**0.4586±1.01e-02**
DF10	5,10	**0.1786±7.06e-03**−	0.1844±5.20e-03−	0.1890±7.33e-03	5,10	**0.5528±6.08e-03**−	0.5469±4.41e-03−	0.5443±6.80e-03
	10,10	**0.1559±6.22e-03**−	0.1592±4.17e-03−	0.1666±6.00e-03	10,10	**0.6264±5.03e-03**−	0.6225±4.34e-03−	0.6160±4.60e-03
DF11	5,10	0.0617±4.45e-04−	**0.0617±3.61e-04**−	0.0621±3.93e-04	5,10	**0.2944±5.96e-04**−	0.2944±5.49e-04−	0.2938±5.43e-04
	10,10	**0.0609±3.73e-04**−	0.0610±3.77e-04−	0.0613±4.33e-04	10,10	0.2906±5.59e-04−	**0.2906±6.08e-04**−	0.2899±6.95e-04
DF12	5,10	**0.1708±6.15e-03**−	0.1829±9.13e-03−	0.2088±9.71e-03	5,10	**0.8400±4.14e-03**−	0.8344±4.40e-03−	0.8191±5.80e-03
	10,10	**0.1520±7.88e-03**−	0.1599±6.02e-03−	0.1907±7.04e-03	10,10	**0.8629±5.44e-03**−	0.8568±4.73e-03−	0.8390±4.15e-03
DF13	5,10	0.2084±1.88e-02−	**0.2062±1.64e-02**−	0.2471±1.69e-02	5,10	0.5655±1.57e-02−	**0.5679±1.17e-02**−	0.5365±1.04e-02
	10,10	**0.1654±7.73e-03**−	0.1672±6.03e-03−	0.1956±6.41e-03	10,10	**0.5960±5.39e-03**−	0.5944±4.59e-03−	0.5740±4.81e-03
DF14	5,10	0.2620±1.28e-02≈	**0.2555±1.13e-02**≈	0.2568±1.03e-02	5,10	0.2866±1.17e-02≈	**0.2927±1.18e-02**≈	0.2885±1.19e-02
	10,10	0.2352±9.47e-03≈	**0.2345±1.01e-02**≈	0.2365±1.06e-02	10,10	0.3269±9.95e-03≈	**0.3271±1.03e-02**≈	0.3232±1.05e-02
$+/\approx/-$		17/3/8	14/6/8			15/4/9	14/4/10	
Average	Ranking	2.32	2	1.68		2.29	1.89	1.82

TABLE II
COMPARISON RESULTS BETWEEN CGCT$_{h1}$, CGCT$_{h2}$ AND CGCT-RM-MEDA, MEASURED BY THE MIGD AND MHV VALUES

Problem	n_t, τ_t	CGCT$_{h1}$	CGCT$_{h2}$	CGCT-RM-MEDA	n_t, τ_t	CGCT$_{h1}$	CGCT$_{h2}$	CGCT-RM-MEDA
DF1	5,10	0.0522±1.06e-02+	0.0670±1.77e-02+	**0.0342±8.72e-03**	5,10	0.4746±1.19e-02+	0.4619±1.32e-02+	**0.4981±1.03e-02**
	10,10	0.0258±5.44e-03+	0.0291±5.62e-03+	**0.0183±3.78e-03**	10,10	0.5068±7.22e-03+	0.5025±7.37e-03+	**0.5180±4.88e-03**
DF2	5,10	0.0406±6.60e-03+	0.0472±7.96e-03+	**0.0337±2.59e-03**	5,10	0.6621±8.72e-03+	0.6536±9.42e-03+	**0.6714±3.90e-03**
	10,10	**0.0361±2.53e-03**−	0.0400±2.79e-03−	0.0470±4.49e-03	10,10	**0.6679±3.71e-03**−	0.6623±4.11e-03−	0.6547±4.75e-03
DF3	5,10	0.0237±2.22e-03+	0.0194±1.05e-03+	**0.0160±8.28e-04**	5,10	0.4796±2.81e-03+	0.4863±1.51e-03+	**0.4914±1.14e-03**
	10,10	0.0164±6.37e-04+	0.0137±7.82e-04+	**0.0106±4.06e-04**	10,10	0.4897±9.52e-04+	0.4939±1.20e-03+	**0.4984±6.51e-04**
DF4	5,10	0.1378±9.10e-03+	0.1296±7.56e-03+	**0.1185±3.49e-03**	5,10	0.6806±6.11e-03+	0.6916±7.00e-03+	**0.7026±2.06e-03**
	10,10	0.1199±5.60e-03+	0.1165±2.58e-03+	**0.1144±1.66e-03**	10,10	0.7030±4.07e-03+	0.7099±2.70e-03+	**0.7153±1.04e-03**
DF5	5,10	0.1069±2.14e-02+	0.1150±2.78e-02+	**0.0665±1.63e-02**	5,10	0.4473±1.72e-02+	0.4410±2.34e-02+	**0.4950±1.53e-02**
	10,10	0.0636±9.51e-03+	0.0682±1.76e-02+	**0.0284±4.40e-03**	10,10	0.4928±1.21e-02+	0.4881±2.12e-02+	**0.5431±6.32e-03**
DF6	5,10	5.9382±3.96e-01+	5.5151±3.76e-01+	**4.4190±3.78e-01**	5,10	0.0073±6.54e-03+	0.0125±8.89e-03+	**0.0845±3.31e-02**
	10,10	6.0173±4.16e-01+	4.9760±4.73e-01+	**3.0309±7.18e-01**	10,10	0.0102±8.32e-03+	0.0417±4.08e-02+	**0.2229±7.71e-02**
DF7	5,10	0.0325±7.72e-03+	0.0508±1.35e-02+	**0.0254±4.25e-03**	5,10	0.4524±1.93e-03+	0.4448±3.36e-03+	**0.4564±1.54e-03**
	10,10	0.0224±3.57e-03+	0.0224±3.00e-03+	**0.0141±1.23e-03**	10,10	0.4540±2.20e-03+	0.4541±1.48e-03+	**0.4593±7.21e-04**
DF8	5,10	0.0111±4.54e-04+	0.0103±2.76e-04+	**0.0090±3.84e-04**	5,10	0.6155±4.33e-04+	0.6162±2.26e-04+	**0.6172±8.73e-05**
	10,10	0.0104±6.19e-04+	0.0099±4.19e-04+	**0.0083±4.43e-04**	10,10	0.6414±5.93e-04+	0.6421±4.20e-04+	**0.6431±5.11e-04**
DF9	5,10	0.2268±5.58e-02+	0.1785±4.25e-02+	**0.1305±2.15e-02**	5,10	0.3308±2.72e-02+	0.3646±2.72e-02+	**0.3984±1.76e-02**
	10,10	0.1649±4.95e-02+	0.1090±3.31e-02+	**0.0783±1.10e-02**	10,10	0.4001±2.12e-02+	0.4367±1.85e-02+	**0.4586±1.01e-02**
DF10	5,10	0.1864±5.98e-03≈	**0.1572±5.09e-03**−	0.1890±7.33e-03	5,10	0.5459±5.71e-03≈	**0.5715±4.97e-03**−	0.5443±6.80e-03
	10,10	0.1633±5.60e-03−	**0.1369±6.58e-03**−	0.1666±6.00e-03	10,10	0.6194±4.73e-03−	**0.6458±6.85e-03**−	0.6160±4.60e-03
DF11	5,10	0.0659±1.25e-03+	0.0646±9.62e-04+	**0.0621±3.93e-04**	5,10	0.2891±1.39e-03+	0.2906±9.74e-04+	**0.2938±5.43e-04**
	10,10	0.0632±6.39e-04+	0.0624±3.89e-04+	**0.0613±4.33e-04**	10,10	0.2872±7.81e-04+	0.2885±5.68e-04+	**0.2899±6.95e-04**
DF12	5,10	0.2060±8.75e-03≈	**0.1737±7.00e-03**−	0.2088±9.71e-03	5,10	0.8223±4.39e-03≈	**0.8369±3.88e-03**−	0.8191±5.80e-03
	10,10	0.1903±7.60e-03≈	**0.1665±5.75e-03**−	0.1907±7.04e-03	10,10	0.8435±3.51e-03≈	**0.8511±3.43e-03**−	0.8390±4.15e-03
DF13	5,10	**0.2333±1.82e-02**−	0.2666±2.32e-02+	0.2471±1.69e-02	5,10	**0.5437±1.41e-02**−	0.5231±1.94e-02+	0.5365±1.04e-02
	10,10	**0.1872±7.62e-03**−	0.2024±1.35e-02≈	0.1956±6.41e-03	10,10	**0.5779±5.36e-03**−	0.5669±9.84e-03≈	0.5740±4.81e-03
DF14	5,10	0.2617±1.34e-02≈	**0.2533±1.25e-02**≈	0.2568±1.03e-02	5,10	0.2821±1.35e-02≈	**0.2905±1.50e-02**≈	0.2885±1.19e-02
	10,10	0.2436±1.06e-02+	**0.2351±1.22e-02**≈	0.2365±1.06e-02	10,10	0.3182±1.01e-02+	**0.3264±1.21e-02**≈	0.3232±1.05e-02
$+/\approx/-$		20/4/4	20/3/5			19/4/5	21/2/5	
Average	Ranking	2.39	2.11	1.5		2.43	2.07	1.5

TABLE III

Comparison results between CGCT_{c1}, CGCT_{c2} and CGCT-RM-MEDA, measured by the MIGD values

Problem	n_t, τ_t	CGCT_{c1}	CGCT-RM-MEDA	CGCT_{c2}	CGCT-RM-MEDA	CGCT_{c1}	CGCT_{c2}
DF1	5,10	0.2059±7.22e-02+	**0.0342±8.72e-03**	0.0927±1.89e-02+	**0.0342±8.72e-03**	0.2059±7.22e-02+	**0.0927±1.89e-02**
	10,10	0.1115±1.95e-02+	**0.0183±3.78e-03**	0.0270±6.25e-03+	**0.0183±3.78e-03**	0.1115±1.95e-02+	**0.0270±6.25e-03**
DF2	5,10	0.0744±2.02e-02+	**0.0337±2.59e-03**	0.0705±5.06e-03+	**0.0337±2.59e-03**	0.0744±2.02e-02≈	**0.0705±5.06e-03**
	10,10	0.0919±1.03e-02+	**0.0470±4.49e-03**	0.0474±3.37e-03≈	**0.0470±4.49e-03**	0.0919±1.03e-02+	**0.0474±3.37e-03**
DF3	5,10	0.0166±1.32e-03+	**0.0160±8.28e-04**	0.0195±1.37e-03+	**0.0160±8.28e-04**	**0.0166±1.32e-03**−	0.0195±1.37e-03
	10,10	0.0120±8.29e-04+	**0.0106±4.06e-04**	0.0120±5.96e-04+	**0.0106±4.06e-04**	**0.0120±8.29e-04**≈	0.0120±5.96e-04
DF4	5,10	0.1352±1.08e-02+	**0.1185±3.49e-03**	0.1286±8.41e-03+	**0.1185±3.49e-03**	0.1352±1.08e-02+	**0.1286±8.41e-03**
	10,10	0.1246±7.21e-03+	**0.1144±1.66e-03**	0.1156±5.09e-03≈	**0.1144±1.66e-03**	0.1246±7.21e-03+	**0.1156±5.09e-03**
DF5	5,10	0.3761±1.77e-01+	**0.0665±1.63e-02**	0.0888±1.92e-02+	**0.0665±1.63e-02**	0.3761±1.77e-01+	**0.0888±1.92e-02**
	10,10	0.1177±3.05e-02+	**0.0284±4.40e-03**	0.0297±3.54e-03≈	**0.0284±4.40e-03**	0.1177±3.05e-02+	**0.0297±3.54e-03**
DF6	5,10	5.5040±4.83e-01+	**4.4190±3.78e-01**	**4.3328±4.14e-01**≈	4.4190±3.78e-01	5.5040±4.83e-01+	**4.3328±4.14e-01**
	10,10	4.7241±6.48e-01+	**3.0309±7.18e-01**	3.2540±5.71e-01≈	**3.0309±7.18e-01**	4.7241±6.48e-01+	**3.2540±5.71e-01**
DF7	5,10	0.1275±4.02e-02+	**0.0254±4.25e-03**	0.0267±7.07e-03≈	**0.0254±4.25e-03**	0.1275±4.02e-02+	**0.0267±7.07e-03**
	10,10	0.0357±8.41e-03+	**0.0141±1.23e-03**	0.0149±1.43e-03+	**0.0141±1.23e-03**	0.0357±8.41e-03+	**0.0149±1.43e-03**
DF8	5,10	0.0099±2.26e-04+	**0.0090±3.84e-04**	0.0092±4.00e-04≈	**0.0090±3.84e-04**	0.0099±2.26e-04+	**0.0092±4.00e-04**
	10,10	0.0097±2.97e-04+	**0.0083±4.43e-04**	0.0087±4.96e-04+	**0.0083±4.43e-04**	0.0097±2.97e-04+	**0.0087±4.96e-04**
DF9	5,10	0.2525±1.01e-01+	**0.1305±2.15e-02**	0.2971±6.34e-02+	**0.1305±2.15e-02**	**0.2525±1.01e-01**−	0.2971±6.34e-02
	10,10	0.2511±1.37e-01+	**0.0783±1.10e-02**	0.1080±2.33e-02+	**0.0783±1.10e-02**	0.2511±1.37e-01+	**0.1080±2.33e-02**
DF10	5,10	**0.1473±4.52e-03**−	0.1890±7.33e-03	**0.1876±6.44e-03**≈	0.1890±7.33e-03	**0.1473±4.52e-03**−	0.1876±6.44e-03
	10,10	**0.1273±5.84e-03**−	0.1666±6.00e-03	**0.1630±6.09e-03**−	0.1666±6.00e-03	**0.1273±5.84e-03**−	0.1630±6.09e-03
DF11	5,10	0.0647±1.13e-03+	**0.0621±3.93e-04**	0.0622±4.96e-04≈	**0.0621±3.93e-04**	0.0647±1.13e-03+	**0.0622±4.96e-04**
	10,10	0.0631±7.94e-04+	**0.0613±4.33e-04**	**0.0612±3.48e-04**≈	0.0613±4.33e-04	0.0631±7.94e-04+	**0.0612±3.48e-04**
DF12	5,10	**0.1446±6.26e-03**−	0.2088±9.71e-03	**0.2058±7.74e-03**≈	0.2088±9.71e-03	**0.1446±6.26e-03**−	0.2058±7.74e-03
	10,10	**0.1327±4.53e-03**−	0.1907±7.04e-03	0.1951±8.59e-03+	**0.1907±7.04e-03**	**0.1327±4.53e-03**−	0.1951±8.59e-03
DF13	5,10	0.2800±4.73e-02+	**0.2471±1.69e-02**	0.2713±2.03e-02+	**0.2471±1.69e-02**	0.2800±4.73e-02≈	**0.2713±2.03e-02**
	10,10	**0.1743±1.29e-02**−	0.1956±6.41e-03	0.1981±6.36e-03≈	**0.1956±6.41e-03**	**0.1743±1.29e-02**−	0.1981±6.36e-03
DF14	5,10	**0.2487±9.67e-03**−	0.2568±1.03e-02	0.2583±1.11e-02≈	**0.2568±1.03e-02**	**0.2487±9.67e-03**−	0.2583±1.11e-02
	10,10	**0.2297±1.02e-02**−	0.2365±1.06e-02	0.2383±1.16e-02≈	**0.2365±1.06e-02**	**0.2297±1.02e-02**−	0.2383±1.16e-02
$+/\approx/-$		21/0/7		13/14/1		16/3/9	
Average	Ranking	1.75	1.25	1.82	1.18	1.64	1.36

TABLE IV

Comparison results between CGCT_{c1}, CGCT_{c2} and CGCT-RM-MEDA, measured by the MHV values

Problem	n_t, τ_t	CGCT_{c1}	CGCT-RM-MEDA	CGCT_{c2}	CGCT-RM-MEDA	CGCT_{c1}	CGCT_{c2}
DF1	5,10	0.3741±2.95e-02+	**0.4981±1.03e-02**	0.4346±1.73e-02+	**0.4981±1.03e-02**	0.3741±2.95e-02+	**0.4346±1.73e-02**
	10,10	0.4197±1.58e-02+	**0.5180±4.88e-03**	0.5054±7.77e-03+	**0.5180±4.88e-03**	0.4197±1.58e-02+	**0.5054±7.77e-03**
DF2	5,10	0.6215±2.12e-02+	**0.6714±3.90e-03**	0.6150±6.62e-03+	**0.6714±3.90e-03**	**0.6215±2.12e-02**≈	0.6150±6.62e-03
	10,10	0.5944±1.30e-02+	**0.6547±4.75e-03**	0.6506±4.39e-03≈	**0.6547±4.75e-03**	0.5944±1.30e-02+	**0.6506±4.39e-03**
DF3	5,10	0.4904±1.91e-03+	**0.4914±1.14e-03**	0.4863±1.95e-03+	**0.4914±1.14e-03**	**0.4904±1.91e-03**−	0.4863±1.95e-03
	10,10	0.4968±9.28e-04+	**0.4984±6.51e-04**	0.4965±8.18e-04+	**0.4984±6.51e-04**	**0.4968±9.28e-04**≈	0.4965±8.18e-04
DF4	5,10	0.6920±4.92e-03+	**0.7026±2.06e-03**	0.6849±6.40e-03+	**0.7026±2.06e-03**	**0.6920±4.92e-03**+	0.6849±6.40e-03
	10,10	0.7025±4.44e-03+	**0.7153±1.04e-03**	0.7084±3.80e-03≈	**0.7153±1.04e-03**	0.7025±4.44e-03+	**0.7084±3.80e-03**
DF5	5,10	0.3623±3.74e-02+	**0.4950±1.53e-02**	0.4682±2.04e-02+	**0.4950±1.53e-02**	0.3623±3.74e-02+	**0.4682±2.04e-02**
	10,10	0.4374±2.96e-02+	**0.5431±6.32e-03**	0.5410±5.29e-03≈	**0.5431±6.32e-03**	0.4374±2.96e-02+	**0.5410±5.29e-03**
DF6	5,10	0.0182±1.98e-02+	**0.0845±3.31e-02**	0.0699±3.24e-02≈	**0.0845±3.31e-02**	0.0182±1.98e-02+	**0.0699±3.24e-02**
	10,10	0.0879±4.14e-02+	**0.2229±7.71e-02**	0.2020±7.68e-02≈	**0.2229±7.71e-02**	0.0879±4.14e-02+	**0.2020±7.68e-02**
DF7	5,10	0.4337±4.16e-03+	**0.4564±1.54e-03**	**0.4565±1.52e-03**≈	0.4564±1.54e-03	0.4337±4.16e-03+	**0.4565±1.52e-03**
	10,10	0.4478±3.45e-03+	**0.4593±7.21e-04**	0.4589±8.37e-04+	**0.4593±7.21e-04**	0.4478±3.45e-03+	**0.4589±8.37e-04**
DF8	5,10	**0.6173±5.28e-05**+	0.6172±8.73e-05	0.6166±4.85e-04≈	**0.6172±8.73e-05**	**0.6173±5.28e-05**+	0.6166±4.85e-04
	10,10	**0.6432±1.05e-04**+	0.6431±5.11e-04	0.6418±7.15e-04+	**0.6431±5.11e-04**	**0.6432±1.05e-04**+	0.6418±7.15e-04
DF9	5,10	0.3457±3.52e-02+	**0.3984±1.76e-02**	0.2820±1.94e-02+	**0.3984±1.76e-02**	**0.3457±3.52e-02**−	0.2820±1.94e-02
	10,10	0.3786±2.99e-02+	**0.4586±1.01e-02**	0.4285±2.05e-02+	**0.4586±1.01e-02**	0.3786±2.99e-02+	**0.4285±2.05e-02**
DF10	5,10	**0.5826±5.12e-03**−	0.5443±6.80e-03	0.5434±5.14e-03≈	**0.5443±6.80e-03**	**0.5826±5.12e-03**−	0.5434±5.14e-03
	10,10	**0.6569±6.33e-03**−	0.6160±4.60e-03	**0.6186±5.12e-03**−	0.6160±4.60e-03	**0.6569±6.33e-03**−	0.6186±5.12e-03
DF11	5,10	0.2905±1.34e-03+	**0.2938±5.43e-04**	0.2937±6.79e-04≈	**0.2938±5.43e-04**	0.2905±1.34e-03+	**0.2937±6.79e-04**
	10,10	0.2874±1.06e-03+	**0.2899±6.95e-04**	**0.2902±5.87e-04**≈	0.2899±6.95e-04	0.2874±1.06e-03+	**0.2902±5.87e-04**
DF12	5,10	**0.8541±3.59e-03**−	0.8191±5.80e-03	**0.8203±4.46e-03**≈	0.8191±5.80e-03	**0.8541±3.59e-03**−	0.8203±4.46e-03
	10,10	**0.8728±2.86e-03**−	0.8390±4.15e-03	**0.8390±3.91e-03**+	0.8390±4.15e-03	**0.8728±2.86e-03**−	0.8390±3.91e-03
DF13	5,10	0.5200±2.64e-02+	**0.5365±1.04e-02**	0.5198±1.33e-02+	**0.5365±1.04e-02**	**0.5200±2.64e-02**≈	0.5198±1.33e-02
	10,10	**0.5858±1.01e-02**−	0.5740±4.81e-03	0.5723±5.01e-03≈	**0.5740±4.81e-03**	**0.5858±1.01e-02**−	0.5723±5.01e-03
DF14	5,10	**0.3016±1.22e-02**−	0.2885±1.19e-02	0.2884±1.13e-02≈	**0.2885±1.19e-02**	**0.3016±1.22e-02**−	0.2884±1.13e-02
	10,10	**0.3323±1.13e-02**−	0.3232±1.05e-02	**0.3253±1.07e-02**≈	0.3232±1.05e-02	**0.3323±1.13e-02**−	0.3253±1.07e-02
$+/\approx/-$		19/1/8		14/14/0		13/3/12	
Average	Ranking	1.68	1.32	1.79	1.21	1.46	1.54

TABLE V

COMPARISON RESULTS BETWEEN CGCT$_{ar}$ AND CGCT-RM-MEDA, MEASURED BY THE MIGD AND MHV VALUES

Problem	n_t, τ_t	CGCT$_{ar}$	CGCT-RM-MEDA	n_t, τ_t	CGCT$_{ar}$	CGCT-RM-MEDA
DF1	5,10	0.0586±1.04e-02+	**0.0342±8.72e-03**	5,10	0.4713±9.59e-03+	**0.4981±1.03e-02**
	10,10	0.0196±4.25e-03≈	**0.0183±3.78e-03**	10,10	0.5166±5.11e-03≈	**0.5180±4.88e-03**
DF2	5,10	0.0770±1.01e-02+	**0.0337±2.59e-03**	5,10	0.6145±7.60e-03+	**0.6714±3.90e-03**
	10,10	0.0495±4.14e-03+	**0.0470±4.49e-03**	10,10	0.6504±4.76e-03+	**0.6547±4.75e-03**
DF3	5,10	0.0171±1.14e-03+	**0.0160±8.28e-04**	5,10	0.4897±1.61e-03+	**0.4914±1.14e-03**
	10,10	0.0122±5.62e-04+	**0.0106±4.06e-04**	10,10	0.4963±8.22e-04+	**0.4984±6.51e-04**
DF4	5,10	0.1310±7.47e-03+	**0.1185±3.49e-03**	5,10	0.6834±5.58e-03+	**0.7026±2.06e-03**
	10,10	**0.1140±5.79e-03**≈	0.1144±1.66e-03	10,10	0.7055±4.14e-03≈	**0.7153±1.04e-03**
DF5	5,10	0.0987±2.58e-02+	**0.0665±1.63e-02**	5,10	0.4698±1.51e-02+	**0.4950±1.53e-02**
	10,10	0.0462±6.24e-03+	**0.0284±4.40e-03**	10,10	0.5167±8.76e-03+	**0.5431±6.32e-03**
DF6	5,10	4.6094±4.79e-01+	**4.4190±3.78e-01**	5,10	0.0651±3.54e-02+	**0.0845±3.31e-02**
	10,10	3.7521±7.48e-01+	**3.0309±7.18e-01**	10,10	0.1579±9.56e-02+	**0.2229±7.71e-02**
DF7	5,10	0.0266±5.20e-03≈	**0.0254±4.25e-03**	5,10	0.4561±1.67e-03≈	**0.4564±1.54e-03**
	10,10	0.0153±1.58e-03+	**0.0141±1.23e-03**	10,10	0.4586±1.09e-03+	**0.4593±7.21e-04**
DF8	5,10	0.0093±3.16e-04+	**0.0090±3.84e-04**	5,10	0.6165±3.35e-04+	**0.6172±8.73e-05**
	10,10	0.0089±4.02e-04+	**0.0083±4.43e-04**	10,10	0.6416±8.35e-04+	**0.6431±5.11e-04**
DF9	5,10	0.1546±1.80e-02+	**0.1305±2.15e-02**	5,10	0.3732±1.37e-02+	**0.3984±1.76e-02**
	10,10	0.0856±7.15e-03+	**0.0783±1.10e-02**	10,10	0.4483±8.20e-03+	**0.4586±1.01e-02**
DF10	5,10	**0.1867±8.61e-03**≈	0.1890±7.33e-03	5,10	**0.5461±6.10e-03**≈	0.5443±6.80e-03
	10,10	**0.1655±6.80e-03**≈	0.1666±6.00e-03	10,10	**0.6165±5.88e-03**≈	0.6160±4.60e-03
DF11	5,10	0.0623±5.21e-04≈	**0.0621±3.93e-04**	5,10	0.2935±7.00e-04≈	**0.2938±5.43e-04**
	10,10	0.0614±4.37e-04≈	**0.0613±4.33e-04**	10,10	0.2899±5.88e-04≈	**0.2899±6.95e-04**
DF12	5,10	0.2094±8.84e-03≈	**0.2088±9.71e-03**	5,10	0.8185±4.08e-03≈	**0.8191±5.80e-03**
	10,10	**0.1901±6.03e-03**≈	0.1907±7.04e-03	10,10	**0.8422±3.13e-03**≈	0.8390±4.15e-03
DF13	5,10	**0.2213±1.05e-02**−	0.2471±1.69e-02	5,10	**0.5562±8.15e-03**−	0.5365±1.04e-02
	10,10	**0.1863±7.92e-03**−	0.1956±6.41e-03	10,10	**0.5798±5.95e-03**−	0.5740±4.81e-03
DF14	5,10	0.2594±1.52e-02≈	**0.2568±1.03e-02**	5,10	0.2870±1.34e-02≈	**0.2885±1.19e-02**
	10,10	**0.2364±1.07e-02**≈	0.2365±1.06e-02	10,10	**0.3248±1.13e-02**≈	0.3232±1.05e-02
$+/\approx/-$		15/11/2			16/9/3	
Average	Ranking	1.75	1.25		1.79	1.21