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# A Strategy of Quantum Image Filtering in Frequency Domain

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**Abstract.** In the field of quantum image processing, the method for quantum image representation has been a hot issue. However the computational complexity of the whole preparation for the mature model (Flexible Representation of Quantum Images) is too high. In this paper we present a fast image storage method based on the transformation about two quantum image representation models. Then a filtering algorithm is proposed via using quantum Fourier transform. With low time complexity, the good performance of this filtering operation can be presented in the simulation experiments.

## Introduction

Quantum computation is the emerging interdisciplinary. Since Feynman proposed the first quantum computation model in 1982, the advantages of quantum information processing is becoming more and more apparent. Via using some peculiar quantum characteristics such as quantum entanglement and superposition, the speed of information processing is greatly accelerated, especially for some parallel computing tasks. Not only that, some difficult problems such as integer factorization that can not be solved in classical computation model can get the result using quantum algorithm [1].

In the real world, the application of image processing is more and more popular. Digital image processing has also become one of the most striking and promising researches, which has been widely used in many fields such as resource exploration, biomedicine, meteorology, and traffic. But with the increased image resolution and scale, the original image processing algorithms have encountered many challenges. Therefore some researchers devote to improve the efficiency of image processing via quantum mechanics.

In the field of quantum image processing, some researchers mainly focus on the storage model for quantum image such as Qubit Lattice [2], Entangled Image [3], Quantum Image Representation for Log-polar Images (QUALPI) [4], Flexible Representation of Quantum Images (FRQI) [5], and Novel Enhanced Quantum Representation of Digital Image Model (NEQR) [6]. At the same time, many quantum image processing algorithms based on these models have been proposed. In [7], there is a detailed analysis of these models and algorithms.

Because the presence of noise point will affect the result of quantum image processing and there are few researches focusing on the filtering process in quantum image. Therefore the restoration for noise pollution has been an urgent problem to be solved. In this paper, we propose an algorithm of quantum image filtering in frequency domain based on the model FRQI. Because the time complexity of the image storage process in this model is too high. We use a novel method via transformation from model NEQR into FRQI to reduce the time complexity. In the simulation result, the result of our filtering operation will be presented.

## Storage Method for Quantum Image

In fact, the model FRQI is one of the most popular representation method for quantum image. There are many quantum image processing algorithms based on this model such as image translation [8], edge extraction [9], and image watermarking algorithm [10,11]. More importantly, quantum Fourier transform that will be used in our filtering algorithm is easily carried out in this model. From this point of view, we choose FRQI to store the quantum image and achieve quantum image restoration from noise pollution.

In digital image, there are two kinds of information stored in the pixels, namely position information and color information. As for quantum image model FRQI, every pixel is represented by the basic state that is an entanglement of one quantum sequence and an independent qubit. A ground state of 2-dimensional qubit sequence is used to represent the position information, and the color information is indicated by the probability amplitude of that independent qubit. Finally, one image is stored in a quantum superposition state consisting of these basic states. For an image with size $2^n * 2^n$ and grays range $2^t$, the representation of FRQI model is as follows:

$$
\begin{aligned}
|I\rangle &= \frac{1}{2^n}\sum_{i=0}^{2^{2n}-1}|C_\theta\rangle \otimes |i\rangle \\
&= \frac{1}{2^n}\sum_{i=0}^{2^{2n}-1}(\cos\theta_i|0\rangle + \sin\theta_i|1\rangle)\otimes|i\rangle \\
&= \frac{1}{2^n}\sum_{x=0}^{2^n-1}\sum_{y=0}^{2^n-1}(\cos\theta_{xy}|0\rangle + \sin\theta_{xy}|1\rangle)\otimes|x\rangle|y\rangle \quad where\ \theta_i, \theta_{xy} \in [0, \frac{\pi}{2}]
\end{aligned} \tag{1}
$$

However, the time complexity of preparing the FRQI quantum image could be calculated $O(2^{4n})$ as discussed in [5]. Although the method of quantum image compression in [5] can be used, this preparation process is also quite complex. According to the discussion in [6], the whole preparation process for the NEQR quantum image will cost no more than $O(tn \times 2^{2n})$ with the same scale image. And the NEQR representation can also achieve a better compression ratio than FRQI as discussed in [7]. Therefore we first use the NEQR model to store the image. And then the FRQI quantum image will be obtain through a transformation from NEQR into FRQI.

Compared with the FRQI, the NEQR model utilized a qubit sequence to represent the gray value of pixels instead of probability amplitude of a single qubit. The formulation of NEQR representation for an image with size $2^n * 2^n$ and gray range $2^t$ is as follows:

$$
\begin{aligned}
|I\rangle &= \frac{1}{2^n}\sum_{x=0}^{2^n-1}\sum_{y=0}^{2^n-1}|f(x,y)\rangle|x\rangle|y\rangle \\
&= \frac{1}{2^n}\sum_{x=0}^{2^n-1}\sum_{y=0}^{2^n-1}\bigotimes_{i=0}^{k-1}|C_{xy}^i\rangle|x\rangle|y\rangle \qquad where \quad f(x,y)\in[0,2^{k-1}]
\end{aligned} \tag{2}
$$

Because the quantum Fourier transform that is utilized in the subsequent filtering algorithm is easy to carry out in FRQI representation. We eventually store the image in this model. Suppose that we have the image stored in NEQR using the method in [6], a conversion circuit shown in Figure 1 can transform this model into FRQI representation.

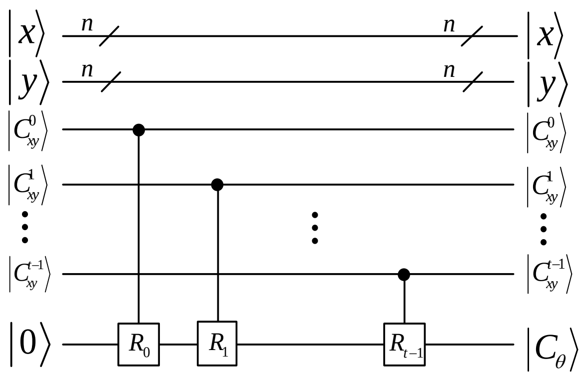

Figure 1. The circuit of transformation from NEQR into FRQI.

From the Figure 1, the controlled rotation matric $R_i$ is unitary operator shown in (3). It means that the qubit representing the pixel value in FRQI is rotated by the angle $2^i\alpha$ about $\hat{y}$ axis using the circuit constructed in [5]. By this way, the discrete value represented by a qubit sentence from NEQR can be transformed into a continuous number represented by one qubit in FRQI. Because only trotation gates is needed, the complexity of this procedure is $O(1)$.

$$
\begin{aligned}
&R_i = \begin{pmatrix} \cos(2^i\alpha) & -\sin(2^i\alpha) \\ \sin(2^i\alpha) & \cos(2^i\alpha) \end{pmatrix} \\
&\quad \alpha = (\frac{\pi}{2})/2^t \quad i \in [0, \text{t-1}]
\end{aligned} \tag{3}
$$

Through above model transformation, the image is finally stored in the FRQI representation. Because the preparation process of NEQR quantum image will cost approximately $O(tn\times 2^{2n})$. The computational complexity of this whole preparation is $O(tn\times 2^{2n})$ which is below the original result.

## Quantum Image Filtering Algorithm in Frequency Domain

In fact, spectrum map reveals the gradient distribution of original image. Each pixel value reveals the intensity of the frequency and the frequency is represented by the coordinate value. Such as the spectrum map shown in Figure 2, the low frequency information which determine the basic structure of original image is relatively bright part in the center and the high frequency information which determine the edge and detail of original image is the darker parts on the peripheries.

In the field of digital image processing, there are some mature filtering methods based on frequency domain, such as high-pass filtering, Wiener filtering and so on. In order to carry out these filtering method using quantum mechanics, we need first obtain the spectrum map by quantum Fourier transform (QFT), and then carry out the relevant operation on the frequency domain. At last, processed image can be obtained by inverse QFT.

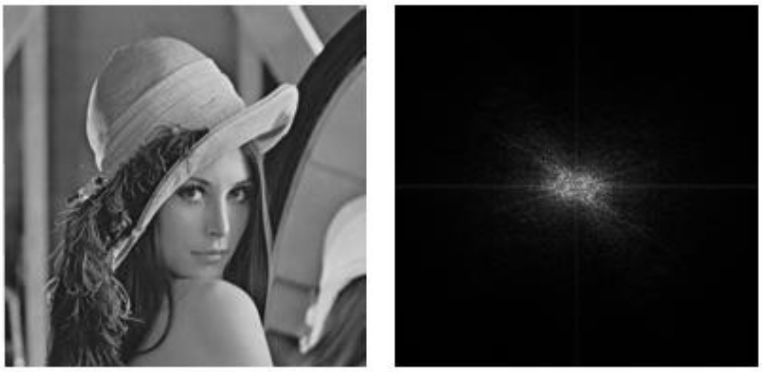

Figure 2. The left is the original image Lena and the right is its spectrum map.

Sometimes, the noise points of image may be concentrated in a certain frequency segment of spectrum map. Under this circumstance, we can get the spectrum image with the help of quantum Fourier transform (QFT) firstly. Then through filtering out this part of the frequency, the noise points are removed. Finally, we will get the restored image by inverse quantum Fourier transform.

*Step 1:* Quantum Fourier Transform (QFT). In [5], there is a mature method for QFT in FRQI. Therefore we can easily get the spectrum $|F\rangle$ of the original image by this way. The result of this procedure is as follow.

$$
\begin{aligned}
|F\rangle &= \frac{1}{2^n}\sum_{i=0}^{2^{2n}-1}(\cos\theta_i|0\rangle + \sin\theta_i|1\rangle)\otimes QFT(|i\rangle) \\
&= \frac{1}{2^n}\left[\sum_{l=0}^{2^{2n}-1} c_l|0l\rangle + \sum_{l=0}^{2^{2n}-1} s_l|1l\rangle\right]
\end{aligned} \tag{4}
$$

where

$$c_l = \frac{1}{2^n}\sum_{l=0}^{2^{2n}-1} e^{2\pi jil/2^{2n}} \cos\theta_i,$$
$$s_l = \frac{1}{2^n}\sum_{l=0}^{2^{2n}-1} e^{2\pi jil/2^{2n}} \sin\theta_i,$$
$$l = 0,1,\cdots 2^{2n}-1$$

*Step 2:* Frequency splitting. The usual method of frequency domain filtering is to remove pixels in a specific frequency range. The selection of the frequency band can be determined by the machine adaptively, also by user pre-defined threshold. Because the quantum circuit designed with current technology is dependent on the choice of the threshold T, the adaptive method is not suitable for the frequency domain filtering. Therefore according to a pre-input thresholds T, the pixel point in a special range will be removed through comparing the pixel with thresholds. We can get the processed spectrum map using the same method in (5).

$$\xi = \frac{1}{2^n}\{\sum_{\substack{0\le x\le 2^n-1\\ 0\le y\le 2^n-1\\ \theta\le T}} \left(\cos\theta_{xy}|0\rangle + \sin\theta_{xy}|1\rangle\right)\otimes|x\rangle|y\rangle + \sum_{\substack{0\le x\le 2^n-1\\ 0\le y\le 2^n-1\\ \theta\ge T}} |0\rangle\otimes|x\rangle|y\rangle\ \} \tag{5}$$

*Step 3:* Inverse Quantum Fourier Transform (IQFT). Because quantum computation is a reversible operation. The circuit for the IQFT in this procedure can be achieved with the similar method in [5]. Through inverse quantum Fourier transformation in the final spectrum map from step2, the image filtering in (6) will be finished.

$$\begin{aligned}|I_{filtered}\rangle &= IQFT\left(\frac{1}{2^n}\left[\sum_{l=0}^{2^{2n}-1} c_l|0l\rangle + \sum_{l=0}^{2^{2n}-1} s_l|1l\rangle\right]\right)\\ &= \frac{1}{2^n}\sum_{i=0}^{2^{2n}-1}(\cos\theta_i'|0\rangle + \sin\theta_i'|1\rangle)\end{aligned} \tag{6}$$

where

$$c_l = \frac{1}{2^n}\sum_{l=0}^{2^{2n}-1} e^{2\pi jil/2^{2n}} \cos\theta_i',$$
$$s_l = \frac{1}{2^n}\sum_{l=0}^{2^{2n}-1} e^{2\pi jil/2^{2n}} \sin\theta_i',$$
$$l = 0,1,\cdots 2^{2n}-1$$

In fact, all procedures of this algorithm is completed here. And the recovered image $|I_{filtered}\rangle$ is stored in FRQI model which can be used for other operations such as feature point extraction.

## Algorithm Analysis

QFT and IQFT are classified for the same type of operation in [5] and the complexity of this type of operation is indicated based on the number of simple quantum gates used in the corresponding quantum circuit. According to the discussion in [5], these two processes have the same computational complexity $O(n^2)$. Therefore if we neglect the time consumption in quantum measurements in step 2, the time complexity of this whole quantum image processing is $O(n^2)$.

In our simulation experiment, a classical computer with processor of Inter Core I5 CPU is used to code. We utilized Matlab program to process image with different densities and types of noise pollution. After the application of our quantum image filtering method, the processing results will be obtained as follow.

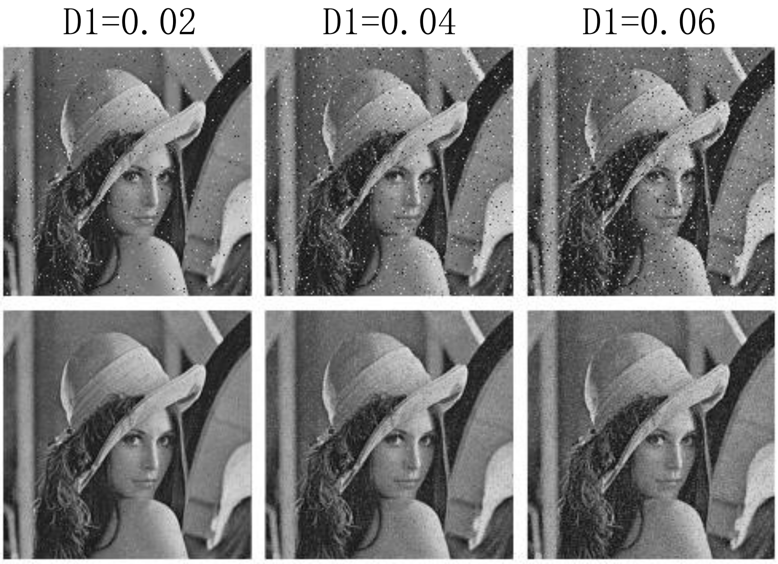


Figure 3. Simulation experiments of quantum image restoration. D1 denotes the density of salt & pepper noise.

In order to test the performance of our quantum image filtering algorithm. Two types of noise pollution salt & pepper noise and Gaussian noise are added in the original Lena image respectively. The filtered images shown in the second line in Figure 3 and Figure 4 is the results after using our MATLAB simulation program.

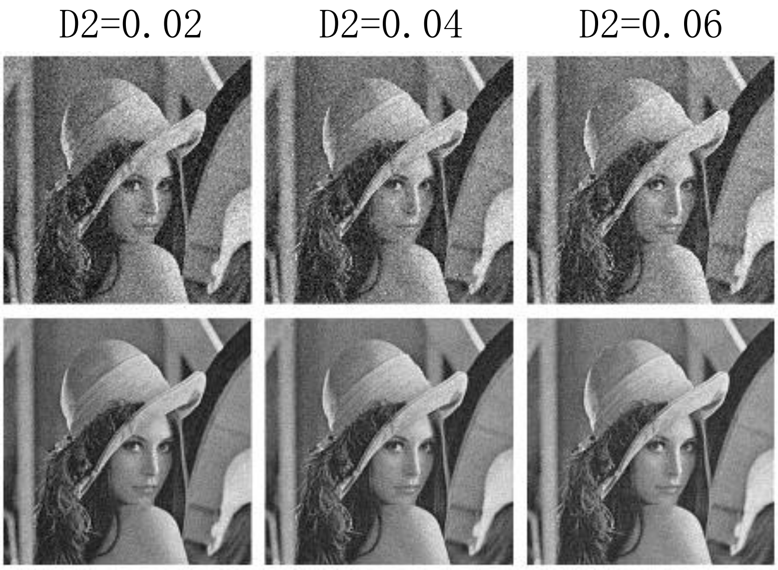


Figure 4. Simulation experiments of quantum image restoration. D2 denotes the density of Gaussian noise.

## Conclusion and Future Work

In this paper, a noise-restoration algorithm for quantum image in frequency domain has been proposed. The core of this method is quantum Fourier transformation (QFT) based on the quantum image representation model FRQI. Via using some basic quantum gates, the computational complexity of this algorithm is appropriate $O\left(n^2\right)$ without consideration of quantum measurement.

Although QFT is easy to carry out in the model FRQI, the preparation process of it is too complex. Therefore we first store the image in model NEQR using an efficient method to get the quantum image. Then through a transformation quantum circuit, the image represented by FRQI will be obtained. The computational complexity of whole preparation is greatly reduced compared with the original process.

The restoration for noise pollution is an essential problem in the field of image processing. In frequency domain, we only make an attempt in this paper. In the future, we will continue to research in other filtering methods, and construct a quantum image noise recovery system.

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