FLPI: representation of quantum images for log-polar coordinate

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ABSTRACT

We propose an effective method, flexible log-polar image (FLPI) to represent quantum images sampled in log-polar coordinate system. Each pixel is represented by three qubit sequences and the whole image is stored into a normalized quantum superposition state. If needed, a flexible qubit sequence can be added to represent multiple images. Through elementary operations, both arbitrary rotation transformation and similarity evaluation can be realized. We also design an image registration algorithm to recognize the angular difference between two images if one is rotated from the other. It is proven that the proposed algorithm could get conspicuous improvement in performance.

Key Words: Quantum Image, Log-Polar, Image Registration

1. INTRODUCTION

Quantum computation can make use of superposition state and entangled state to complete astonishing parallel tasks with astonishing speedup [1]. Some important theoretical and experimental results are very encouraging such as Shor's integer factoring algorithm[2] and Grover search algorithm[3].Applying quantum computing to image processing started with four quantum image representation models based on quantum mechanics, including Qubit Lattice[4], Entangled Image[5], Real Ket[6] and flexible representation of quantum image (FRQI)[7].Based on FRQI model, some simple geometric transformation and color operations can be completed respectively[8][9].Recently，the similarity assessment between two images was realized based on FRQI by a Hadamrd gate operation on a flexible strip to represent multiple images[10]. However, the FRQI model is designed in Cartesian coordinate system, which limits its extensibility to complex geometric transformation. And the similarity assessing method based on FRQI loses efficiency if the image is rotated.

In this paper, flexible log-polar image (FLPI), an improved method is designed based on FRQI to represent quantum images sampled in log-polar coordinate system. The whole image is stored by a normalized superposition state and operated simultaneously. In order to represent multiple images, an extra qubit sequence encoding the serial numbers on quantum circuit can be utilized to store them into one larger superposition state. Specially, with one additional qubit, the similarity value between two images is estimated according to the measurement results after a Hadamard operation on that qubit. Based on the two kinds of operations above, we design a novel image registration algorithm particularly for the situation that an image is rotated from another. Through this way we can determine the exact angular difference between the two images. The new method we proposed has conspicuous improvement in performance. It also could be a fundamental model for complex image processing and future analysis.

2. REPRESENTATION OF QUANTUM LOG-POLAR IMAGES

We will introduce the FLPI method to represent quantum images in log-polar coordinate system in this section. Through this way, the operation on the whole quantum image or multiple images can get completed simultaneously.

2.1Representation of single quantum image

The log-polar coordinate system is a common 2D sampling method to represent digital images as well as Cartesian coordinate. Different from the Cartesian coordinates, the log-polar coordinates have two notations as (ρ,θ) to denote the log radius and angular information of the each pixel. Inspired by the FRQI model[8], we use a normalized and equiprobable quantum superposition state in Eq.(1) to capture all the essential information about the colors and the positions of each pixel in an image. There are three quantum sequences in every ground state in form of tensor product. As for a typical FLPI image sampled in this way, we assume that the log radius and the angular orientations are $2^m \times 2^n$ respectively.

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$$\left|I\right\rangle = \frac{1}{\sqrt{2^{m+n}}} \sum_{\rho=0}^{2^m-1} \sum_{\theta=0}^{2^n-1} (\left|g(\rho,\theta)\right\rangle \otimes \left|\rho\right\rangle \otimes \left|\theta\right\rangle) \quad (1)$$

$$\left|g(\rho,\theta)\right\rangle = \cos\alpha_{\rho,\theta}\left|0\right\rangle + \sin\alpha_{\rho,\theta}\left|1\right\rangle,\ (\alpha_{\rho,\theta} \in [0,\frac{\pi}{2}]) \quad (2)$$

Here $\left|0\right\rangle$ and $\left|1\right\rangle$ are 2D computational basis quantum states, and $g(\rho,\theta)$ is the vector of angles denoting the gray scales of the pixel in Eq.(2).

2.2 Representation of multiple quantum images

To take full advantage of quantum superposition state, we also attempt to use it to represent multiple quantum images for the purpose of similarity evaluation. A simple way is to give different quantum images their own serial numbers by adding an extra qubit sequence on the quantum circuit. As a result, the superposition state is extended to store more quantum images as Eq.(3) shows.

$$\left|L(u)\right\rangle = \frac{1}{2^{u/2}} \sum\nolimits_{j=0}^{2^u-1} \left|I_j\right\rangle \otimes \left|j\right\rangle \quad (3)$$

There are u qubits in the extra qubit sequence encoding the serial numbers of the images involved. So, we could have at most 2^u FLPI images stored. Here $\left|I_j\right\rangle$ is a FLPI image defined in Eq.(1) represented by $(m+n)$ qubits. As we could see, $\left|L(u)\right\rangle$ is also a normalized state. In particular, if there are two images involved, only one qubit is needed.

3. QUANTUM IMAGES REGISTRATION BASED ON SIMILARITY

Based on the representing method FLPI, we firstly discuss the realization of rotation transformation and the similarity estimation between two images. A novel image registration algorithm is also designed in this section. Particularly for the situation that an image is rotated from another, the proposed algorithm could exactly determine the angular difference between the two images.

To be more specific, with the original image rotated by all possible angular degrees, a rotation image database is set up. Then the similarity values between all the rotated original images and the target image would be measured. Finally, with the help of the Grover search algorithm, we could search for the largest similarity value and its index to determine the exact angular difference between the two images. Fig.1 shows the basic procedure of the proposed image registration algorithm with a simplified rotation database having eight rotated original images.

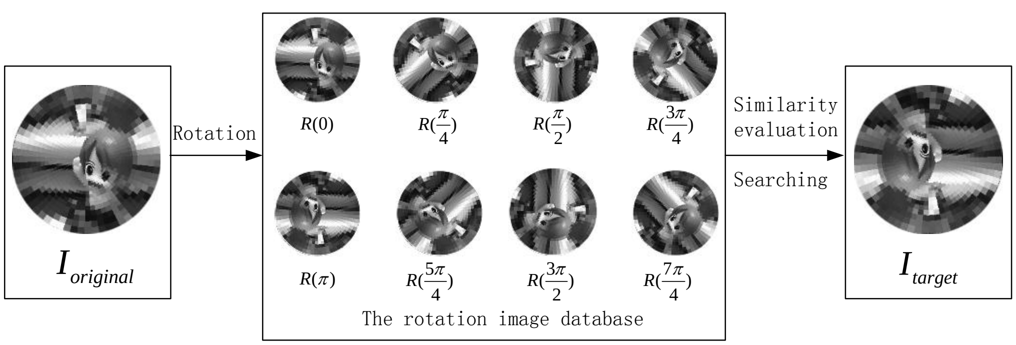

Fig1.The example procedure to determine the exact angular difference between the original and target images

3.1 Rotation transformation of quantum image

In this sector, we will discuss how to realize the arbitrary rotation transformation in order to set up the rotation image database. All that has to be done is a flexible shift operation on the qubit sequence encoding the information about θ of the original image. The θ of each pixel is expressed discretely by n qubits in Eq.(1) so that the shift operation can get disassembled into several sub-operations on different qubits. For example, the rotation $R(k)$ is represented as Eq.(4) where r_i is 0 or1, 0 means there is no change on that qubit.

$$R(k)=\sum_{i=0}^{n-1}R_i=\sum_{i=0}^{n-1}r_i\times 2^i \tag{4}$$

At most n times shift operations are needed to complete the whole rotation transformation, by 2^i degrees each time. As for the quantum rotation by 2^i degrees, Eq.(5) expresses this shift operation clearly and simply.

$$\begin{aligned}R_i\left|I\right\rangle &= R_i\Big(\frac{1}{\sqrt{2^{m+n}}}\sum_{\rho=0}^{2^m-1}\sum_{\theta=0}^{2^n-1}(\left|g(\rho,\theta)\right\rangle\otimes\left|\rho\right\rangle\otimes\left|\theta\right\rangle)\Big)\\ &=\frac{1}{\sqrt{2^{m+n}}}\sum_{\rho=0}^{2^m-1}\sum_{\theta=0}^{2^n-1}(\left|g(\rho,\theta)\right\rangle\otimes\left|\rho\right\rangle\otimes\left|(\theta_{n-1}\theta_{n-2}\cdots\theta_i+r_i)\bmod 2^i\right\rangle\otimes\left|\theta_{i-1}\theta_{i-2}\cdots\theta_0\right\rangle)\end{aligned} \tag{5}$$

One Not-Gate and $(n-i)$ Control-Not-Gate are used for this transformation. From the expressions above, we could conclude that the complexity of this sub-operation is approximate $O((n-i)^2)$ .That means, the total complexity of rotation $R(k)$ is no more than $O(n^3)$ .With this method, we could get the original image rotated by all the possible angular degrees, from one to (2^n-1) degrees easily and quickly. In other words, a rotation image database including 2^n log-polar images has been set up.

3.2 Similarity evaluation based on Hadamard operation

Now we have got every image in the rotation database represented as $\left|I_{ori}^k\right\rangle$ which means the original image is rotated by k degrees. Then the similarity between each of them and the target image should be evaluated for further the procedure. The rotated image $\left|I_{ori}^k\right\rangle$ would get jointed by the target image with one additional qubit, forming a larger superposition state. With the famous Hadamard gate applied to the additional qubit on the circuit, the quantum state can be shown as follows in Eq.(6) and Eq.(7).

$$\begin{aligned}H\left|L_k(1)\right\rangle &=\frac{1}{\sqrt{2}}(\left|I_{ori}^k\right\rangle\otimes H\left|0\right\rangle+\left|I_{t\arg et}\right\rangle\otimes H\left|1\right\rangle)\\ &=\frac{1}{2}[\left|I_{ori}^k\right\rangle\otimes(\left|0\right\rangle+\left|1\right\rangle)+\left|I_{t\arg et}\right\rangle\otimes(\left|0\right\rangle-\left|1\right\rangle)]\\ &=\frac{1}{2}[(\left|I_{ori}^k\right\rangle+\left|I_{t\arg et}\right\rangle)\otimes(\left|0\right\rangle+(\left|I_{ori}^k\right\rangle-\left|I_{t\arg et}\right\rangle)\otimes\left|1\right\rangle]\end{aligned} \tag{6}$$

$$\begin{aligned}H\left|L_k(1)\right\rangle &=\frac{1}{2^{\frac{m+n}{2}+1}}\{\sum_{\rho=0}^{2^m-1}\sum_{\theta=0}^{2^n-1}[(\cos\alpha_{\rho,\theta}^k+\cos\beta_{\rho,\theta})\left|0\right\rangle+(\sin\alpha_{\rho,\theta}^k+\beta_{\rho,\theta})\left|1\right\rangle]\left|\rho\right\rangle\left|\theta\right\rangle\}\otimes\left|0\right\rangle\\ &+\frac{1}{2^{\frac{m+n}{2}+1}}\{\sum_{\rho=0}^{2^m-1}\sum_{\theta=0}^{2^n-1}[(\cos\alpha_{\rho,\theta}^k-\cos\beta_{\rho,\theta})\left|0\right\rangle+(\sin\alpha_{\rho,\theta}^k-\beta_{\rho,\theta})\left|1\right\rangle]\left|\rho\right\rangle\left|\theta\right\rangle\}\otimes\left|1\right\rangle\end{aligned} \tag{7}$$

Then the measurement on the additional qubit is needed because the probability distribution of the measurement results is strongly connected with the similarity between these two images. From Eq.(7) we know that the probability of getting the basis state $\left|0\right\rangle$ is shown in Eq.(8).

$$\begin{aligned}\Pr_k(\left|0\right\rangle) &=\Big(\frac{1}{2^{\frac{m+n}{2}+1}}\Big)^2\sum_{\rho=0}^{2^m-1}\sum_{\theta=0}^{2^n-1}[(\cos\alpha_{\rho,\theta}^k+\cos\beta_{\rho,\theta})^2+(\sin\alpha_{\rho,\theta}^k+\sin\beta_{\rho,\theta})^2]\\ &=\frac{1}{2^{m+n+1}}\sum_{\rho=0}^{2^m-1}\sum_{\theta=0}^{2^n-1}[1+\cos(\alpha_{\rho,\theta}^k-\beta_{\rho,\theta})]\end{aligned} \tag{8}$$

It ranges from 0.5 to 1 and increases as the difference of two pixels on the same position decreases. Hence, we use the probability of $|0\rangle$ to represent the similarity value of two quantum images. With this operation of similarity evaluation applied to our rotation image database, we could get every similarity value between the rotated original image and the target image, represented by $\Pr_k(|0\rangle)$, here k ranges from 0 to (2^n-1).

3.3 Improved Grover search algorithm for quantum image registration

The final step of our image registration algorithm is to search for the largest similarity value and determine the exact angular difference between the two images. It has been proved that the Grover search algorithm can be utilized to search for the extreme value of a number array with the time complexity of $O\sqrt{N}$ [11].As mentioned before, we use the notation $\Pr_k(|0\rangle)$ to represent the similarity value between the rotated original image by k degrees and the target image. For the sake of applying the improved Grover search algorithm, we encode them into one superposition state $|Search_0\rangle$, which has 2^n basis state items, as Eq.(9) shows.

$$|Search_0\rangle = \frac{1}{\sqrt{2^n}}\sum_{k=0}^{2^n-1}|\Pr_k(|0\rangle)\rangle, \quad (k=0,1...2^n-1) \tag{9}$$

There are three main steps to search for the largest $\Pr_k(|0\rangle)$.

(a)Choose a threshold index $0 \le s \le 2^n - 1$ uniformly in random, the corresponding basis state item is $\Pr_s(|0\rangle)$.

(b) Apply the quantum Grover search algorithm for all the items that is larger than $\Pr_s(|0\rangle)$.

(c) Observe the results, if not empty, choose one basis state of them in random, $\Pr_l(|0\rangle)$ set it as the new threshold and apply the search again, or output the final search result as $\Pr_s(|0\rangle)$.

To finish the step (b), two flexible operators are needed, which are defined by Eq.(10), (11) and (12).

$$\hat{C}(\Pr_k(|0\rangle)) = \begin{cases} \Pr_k(|0\rangle), & \Pr_k(|0\rangle) < \Pr_s(|0\rangle)) \\ -\Pr_k(|0\rangle), & \Pr_k(|0\rangle) >= \Pr_s(|0\rangle)) \end{cases} \tag{10}$$

$$\hat{D} \equiv 2\hat{P} - \hat{I} = \frac{2}{2^n}\begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & 1 & \cdots & 1 \\ & & \vdots & \\ 1 & 1 & \cdots & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ & & \ddots & \\ 0 & 0 & \cdots & 1 \end{bmatrix} \tag{11}$$

$$\hat{P} = |Search_0\rangle\langle Search_0| \tag{12}$$

$$(\hat{D}\hat{C})^m|Search_0\rangle \approx \frac{1}{\sqrt{t}}\sin(\frac{2m+1}{\sqrt{2^n}})\sum_{i=0}^{t-1}|\Pr_i(|0\rangle)\rangle + \frac{1}{\sqrt{2^n-t}}\cos(\frac{2m+1}{\sqrt{2^n}})\sum_{j=t}^{2^n-1}|\Pr_j(|0\rangle)\rangle \tag{13}$$

The function of these two operators is to recognize and magnify the amplitude of the marked items as Eq.(13) shows. After repeating the operator $\hat{D}\hat{C}$, we will get all the basic state items larger than the threshold $\Pr_s(|0\rangle)$. Then we will make sure that the largest $\Pr_k(|0\rangle)$ could stick out. With the largest similarity value searching result, the rotation index is obtained which means the exact angular difference between the original and target images with probability nearly 1.

The whole algorithm includes three major steps, setting up the rotation database, evaluating the similarity values and search for the registration result. Firstly, the arbitrary rotation transformation has the time complexity of $O(n^3)$.We only need to measure the additional qubit without any more operation in the procedure of similarity evaluation. Finally, the improved Grover search algorithm has about a time complexity of $O\sqrt{2^n}$. Therefore, the whole algorithm has a time consumption of $O\sqrt{2^n}$, which has a conspicuous speedup in performance.

4. CONCLUSION AND FUTURE WORK

The merging of quantum mechanics and digital image processing has been proven effective and it is enchanting to explore more ways to develop more practical application than image registration. FLPI is specially designed for log-polar quantum images with all the pixels of the image stored and operated simultaneously in form of quantum superposition state. By this way, we could get an image rotated arbitrarily and estimate the similarity value between two images within only one operation. With the help of an improved Grover search algorithm utilized to obtain the largest similarity value, the angular difference between two images can be recognized if one is rotated from the other. It is proven that the new algorithm can provide conspicuous improvement in performance. As for future work, there could be more exploitation such as object recognition, multicolor transformation, etc. In particular, FLPI could get improved to represent 3D images for the purpose of complex object recognition on quantum radar.

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