CSCI 567 Machine Learning

Homework #1

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Question 1.1 Answer :

The Rank of the design matrix X is equal to or larger than $D+1$, where the D is the dimensionality of $\boldsymbol{w}$.

Question 1.2 Answer :

$$RSS\left(\tilde{\boldsymbol{w}}\right) = \sum_{n}\left[y_n - \left(b + \sum_{d} w_d x_{nd}\right)\right]^2$$

$$if \quad \frac{\partial RSS\left(\tilde{\boldsymbol{w}}\right)}{\partial b} = 0,$$

$$then, \quad \sum_{n}\left[y_n - \left(b + \sum_{d} w_d x_{nd}\right)\right] = 0$$

$$\sum_{n} b = \sum_{n} y_n - \sum_{n}\sum_{d} w_d x_{nd}$$

$$b = \frac{1}{N}\sum_{n} y_n - \sum_{d}\left(w_d \frac{1}{N}\sum_{n} x_{nd}\right)$$

$$under\ the\ condition: \quad \frac{1}{N}\sum_{n} x_{nd} = 0 \qquad \forall d = 1, 2, ..., D.$$

$$we\ can\ get: \sum_{d}\left(w_d \frac{1}{N}\sum_{n} x_{nd}\right) = 0$$

$$then, \quad b = \frac{1}{N}\sum_{n} y_n$$

Question 2.1 Answer :

$$\min_b \varepsilon(b) = \min_b -\sum_n \left\{ y_n \log \sigma(b) + (1 - y_n) \log[1 - \sigma(b)] \right\}$$

$$\frac{\partial \varepsilon(b)}{\partial b} = -\sum_n \left\{ y_n [1 - \sigma(b)] - (1 - y_n) \sigma(b) \right\} = 0$$

$$so, \quad \sum_n \sigma(b) = \sum_n y_n$$

$$b^* = \log \frac{\sum y_n}{N - \sum y_n}$$