CSCI 567 Machine Learning

Homework #5

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Question 1.1 Answer :

$$\text{Define } \alpha_t(j) = P(Z_t = s_j, x_{1:t}) \text{ then,}$$
$$\alpha_1(s_1) = P(Z_1 = s_1) \times b_{1A} = 0.28$$
$$\alpha_1(s_2) = P(Z_1 = s_2) \times b_{2A} = 0.06$$
$$\alpha_2(s_1) = (a_{11} \times \alpha_1(s_1) + a_{21} \times \alpha_1(s_2)) \times b_{1G} = 0.0992$$
$$\alpha_2(s_2) = (a_{12} \times \alpha_1(s_1) + a_{22} \times \alpha_1(s_2)) \times b_{2G} = 0.0184$$
$$\alpha_3(s_1) = (a_{11} \times \alpha_2(s_1) + a_{21} \times \alpha_2(s_2)) \times b_{1C} = 0.008672$$
$$\alpha_3(s_2) = (a_{12} \times \alpha_2(s_1) + a_{22} \times \alpha_2(s_2)) \times b_{2C} = 0.009264$$
$$\alpha_4(s_1) = (a_{11} \times \alpha_3(s_1) + a_{21} \times \alpha_3(s_2)) \times b_{1G} = 0.00425728$$
$$\alpha_4(s_2) = (a_{12} \times \alpha_3(s_1) + a_{22} \times \alpha_3(s_2)) \times b_{2G} = 0.00145856$$
$$\alpha_5(s_1) = (a_{11} \times \alpha_4(s_1) + a_{21} \times \alpha_4(s_2)) \times b_{1T} = 0.0003989248$$
$$\alpha_5(s_2) = (a_{12} \times \alpha_4(s_1) + a_{22} \times \alpha_4(s_2)) \times b_{2T} = 0.0005179776$$
$$\alpha_6(s_1) = (a_{11} \times \alpha_5(s_1) + a_{21} \times \alpha_5(s_2)) \times b_{1A} = 0.000210532352$$
$$\alpha_6(s_2) = (a_{12} \times \alpha_5(s_1) + a_{22} \times \alpha_5(s_2)) \times b_{2A} = 0.000078114304$$
$$P(\boldsymbol{X}_{1:6} = \mathbf{O}_{1:6}; \boldsymbol{\Theta}) = \alpha_6(s_1) + \alpha_6(s_2) = 0.000288646656$$

Question 1.2 Answer :

$$\delta_1(s_1) = b_{1A}P(Z_1 = s_1) = 0.28,\ \delta_1(s_2) = b_{2A}P(Z_1 = s_2) = 0.06$$
$$\delta_2(s_1) = \max\{b_{1G}a_{11}\delta_1(s_1), b_{1G}a_{21}\delta_1(s_2)\} = \max\{0.0896, 0.0096\} = 0.0896$$
$$\delta_2(s_2) = \max\{b_{2G}a_{12}\delta_1(s_1), b_{2G}a_{22}\delta_1(s_2)\} = \max\{0.0112, 0.0072\} = 0.0112$$
$$\delta_3(s_1) = \max\{b_{1C}a_{11}\delta_2(s_1), b_{1C}a_{21}\delta_2(s_2)\} = \max\{0.007168, 0.000448\} = 0.007168$$
$$\delta_3(s_2) = \max\{b_{2C}a_{12}\delta_2(s_1), b_{2C}a_{22}\delta_2(s_2)\} = \max\{0.005376, 0.002016\} = 0.005376$$
$$\delta_4(s_1) = \max\{b_{1G}a_{11}\delta_3(s_1), b_{1G}a_{21}\delta_3(s_2)\} = \max\{0.00229376, 0.00086016\} = 0.00229376$$
$$\delta_4(s_2) = \max\{b_{2G}a_{12}\delta_3(s_1), b_{2G}a_{22}\delta_3(s_2)\} = \max\{0.00028672, 0.00064512\} = 0.00064512$$
$$\delta_5(s_1) = \max\{b_{1T}a_{11}\delta_4(s_1), b_{1T}a_{21}\delta_4(s_2)\} = \max\{0.0001835, 0.0000258048\} = 0.0001835$$
$$\delta_5(s_2) = \max\{b_{2T}a_{12}\delta_4(s_1), b_{2T}a_{22}\delta_4(s_2)\} = \max\{0.0001376256, 0.0001161216\} = 0.0001376256$$
$$\delta_6(s_1) = \max\{b_{1A}a_{11}\delta_5(s_1), b_{1A}a_{21}\delta_5(s_2)\} = \max\{5.8720256E-5, 2.2020096eE-5\} = 5.8720256E-5$$
$$\delta_6(s_2) = \max\{b_{2A}a_{12}\delta_5(s_1), b_{2A}a_{22}\delta_5(s_2)\} = \max\{7.340032E-6, 1.6515072E-5\} = 1.6515072E-5$$
$$z_6^* = s1,\ z_5^* = s1,\ z_4^* = s1,\ z_3^* = s1,\ z_2^* = s1,\ z_1^* = s1$$

Question 1.3 Answer :

$$x^* = \text{argmax}_x P(X_7 = x | \boldsymbol{X}_{1:6} = \mathbf{O}_{1:6}; \boldsymbol{\Theta}) = \text{argmax}_x P(X_7 = x, \boldsymbol{X}_{1:6} = \mathbf{O}_{1:6}; \boldsymbol{\Theta})$$

$$P(X_7 = x, \boldsymbol{X}_{1:6} = \mathbf{O}_{1:6}; \boldsymbol{\Theta}) = \alpha_7(s_1) + \alpha_7(s_2)$$

$$= (a_{11} \times \alpha_6(s_1) + a_{21} \times \alpha_6(s_2)) \times b_{1x} + (a_{12} \times \alpha_6(s_1) + a_{22} \times \alpha_6(s_2)) \times b_{2x}$$

$$= (0.8 \times 0.000058720256 + 0.4 \times 0.000016515072) \times b_{1x} + (0.2 \times 0.000058720256 + 0.6 \times 0.000016515072) \times b_{2x}$$

$$\text{because,} (0.8 \times 0.000058720256 + 0.4 \times 0.000016515072) > (0.2 \times 0.000058720256 + 0.6 \times 0.000016515072), so,\ b_{1x} > b_{2x}$$

$$\text{obviously,} x^* = A \text{ or } G$$