Guidelines and Rules (please read carefully):

- There are 7 problems in this assignment. Please turn in: (i) codes, and (ii) a lab report to the Gradescope.
- You will use MATLAB to run the computer simulations. The codes should run without problems on the instructor's computer.
- For mathematical part of the questions, please show process of the solution in the report. Answers with only results will not earn full grade.
- For the simulation part, in your report, please clearly state how you set up the simulations, your ideas and the necessary intermediate results and figures/tables you got from simulations.
- Penalty for late submission: 10% penalty if late within one day; Not acceptable (0 grade) if late more than one day.
- You may discuss coding/debugging issues with your classmates. You cannot, however, share code. It is a violation of ethical policies if you request code from someone else, or if you give code to someone else.
- Main reference for lab session: *Introduction to Probability by Charles M. Grinstead and J. Laurie Snell*, free open book at: http://www.dartmouth.edu/~chance/teaching_aids/books_articles/probability_book/amsbook.mac.pdf

1. Please use Example 2.3 in the Grinstead's open book to simulate the Buffon's Needle experiments. Please calculate, simulate and store the estimated number of π using 100, 1000, 10000, 100000 trials, and plot the estimated numbers versus trial numbers to visualize the convergence (better use logarithmic coordinate for the number of trials).

2. Consider n people who are attending a class and n is smaller than 200. We assume that every person has an equal probability of being born on any day during the year, independently of everyone else, and ignore the additional complication presented by leap years (i.e., nobody is born on February 29, we have 365 days for one year). What is P(n), the probability that for all n people each person has a distinct birthday? Please calculate and simulate the P(n) and plot the P(n) against n to visualize the trend.

3. Draw the top 7 cards from a well-shuffled standard 52-card deck (note in a standard deck of 52 cards, there is a total of 4 Kings see http://en.wikipedia.org/wiki/Standard_52-card_deck). Find the probability that the 7 cards include exactly 3 Kings. Please solve it mathematically and use computer simulation to prove your solution.

4. Please use simulation to estimate the area under the graph of $y = 1/(x + 1)$ in the unit square (x in [0,1], y in [0,1]) in the same way as in Fig 2.3 of the Grinstead book. Calculate the true value of this area and use your simulation results to estimate the value of log2. How accurate is your estimation?

5. If X is a geometric random variable with $p = 0.25$, what is the probability that $X \geq 4$? Verify your result by performing a computer simulation.

6. Use binomial random variable to approximate a Poisson random variable, plot the Probability mass function (PMF):
   a) Compare the PMFs for Poisson(1) and Binomial(100,0.01) random variables.
      - Poisson(1) refers to a Poisson distribution with $\lambda = 1$.
      - Binomial(100,0.01) refers to a Binomial distribution with $n = 100,\ p = 0.01$.
   b) Generate realizations of a Poisson random variable by using a binomial approximation.

7. Calculate the following probabilities and use computer simulation to verify the results: if $X \sim \mathrm{N}(\mu,\ \sigma^2)$, find $P(X > \mu + a\sigma)$ for $a = 1,2,3$, where σ is the standard deviation. Note that you may select arbitrary values of μ and σ in your simulation.