Guidelines and Rules for Lab problems (please read carefully):

- There are 5 problems in this assignment. Please turn in: (i) codes, and (ii) a lab report.
- You will use MATLAB to run the computer simulations. The codes should run without problems on the instructor's computer.
- For mathematical part of the questions, please show process of the solution in the report. Answers with only results will not earn full grade.
- Both documents should be submitted to Gradescope**.** Check for the correct version of your files after you upload them. PDF format is preferred for the lab report. Include your **name and NetID** in all filenames, including codes and reports.
- Penalty for late submission: 10% penalty if late within one day; Not acceptable (0 grade) if late more than one day.
- You may discuss coding/debugging issues with your classmates. You cannot, however, share code. It is a violation of ethical policies if you request code from someone else, or if you give code to someone else.
- Main reference for lab session: *Introduction to Probability by Charles M. Grinstead and J. Laurie Snell*, free open book at: http://www.dartmouth.edu/~chance/teaching_aids/books_articles/probability_book/amsbook.mac.pdf

1. For the joint PMF shown in Table 1, determine the correlation coefficient. Next use a computer simulation to generate realizations of the random vector (X, Y) and estimate the correlation coefficient as

$$\hat{\rho}_{X,Y} = \frac{\frac{1}{M}\sum_{m=1}^{M} x_m y_m - \bar{x}\bar{y}}{\sqrt{(\frac{1}{M}\sum_{m=1}^{M} x_m^2 - \bar{x}^2)(\frac{1}{M}\sum_{m=1}^{M} y_m^2 - \bar{y}^2)}}$$

where

$$\bar{x} = \frac{1}{M}\sum_{m=1}^{M} x_m, \qquad \bar{y} = \frac{1}{M}\sum_{m=1}^{M} y_m$$

and (x_m, y_m) is the mth realization.

	j=0	j=1
i=0	1/8	1/8
i=1	1/4	1/2

Table 1: Joint PMF values.

2. Given two geometric RVs $X \sim \text{geom}(p)$, $Y \sim \text{geom}(p)$, and X and Y are independent, show that the PMF of $Z = X + Y$ is given by

$$p_Z[k] = p^2(k-1)(1-p)^{k-2}, \qquad k = 2,3,\ldots$$

To avoid errors, use the discrete unit step sequence. Next, for $p = 1/2$ generate realizations of Z by first generating realizations of X, then generating realizations of Y and adding each pair of realizations together. Estimate the PMF of Z and compare it to the true PMF.

3. Consider the nonlinear transformation

$$W = X^2 + 5Y^2$$

$$Z = -5X^2 + Y^2$$

Write a computer program to plot in the $x - y$ plane the points (x_i, y_j) for $x_i = 0.95 + (i - 1)/100$ for $i = 1, 2, \ldots, 11$. $y_j = 1.95 + (j-1)/100$ for $j = 1, 2, \ldots, 11$. Next transform all these points into the $w - z$ plane using the given nonlinear transformation. What kind of figure do you see? Next calculate the area of the figure (you can use a rough approximation based on the computer generated figure output) and figure output) and finally take the ratio of the areas of the figures in the two planes. Does this ratio agree with the Jacobian factor

$$\left|\det\left(\frac{\partial(w,z)}{\partial(x,y)}\right)\right|$$

When evaluated at $x = 1, y = 2$?

4. If $X_i \sim \mathcal{N}(1,1), i = 1, 2, \ldots, N$ are independent identically distributed (I.I.D.) random variables, plot a realizations of the sample mean random variable versus N. Should the realization converge, and if so, to what value?

5. Let $X_{1_i} \sim \mathcal{U}(0,2)$ for $i = 1, 2, \ldots, N$ be I.I.D. random variables and let $X_{2_i} \sim \mathcal{N}(1,4)$ for $i = 1, 2, \ldots, N$ be another set of I.I.D random variables. If the sample mean random variable is formed for each set of I.I.D. random variables, which one should converge faster?

Implement a computer simulation to check your results.