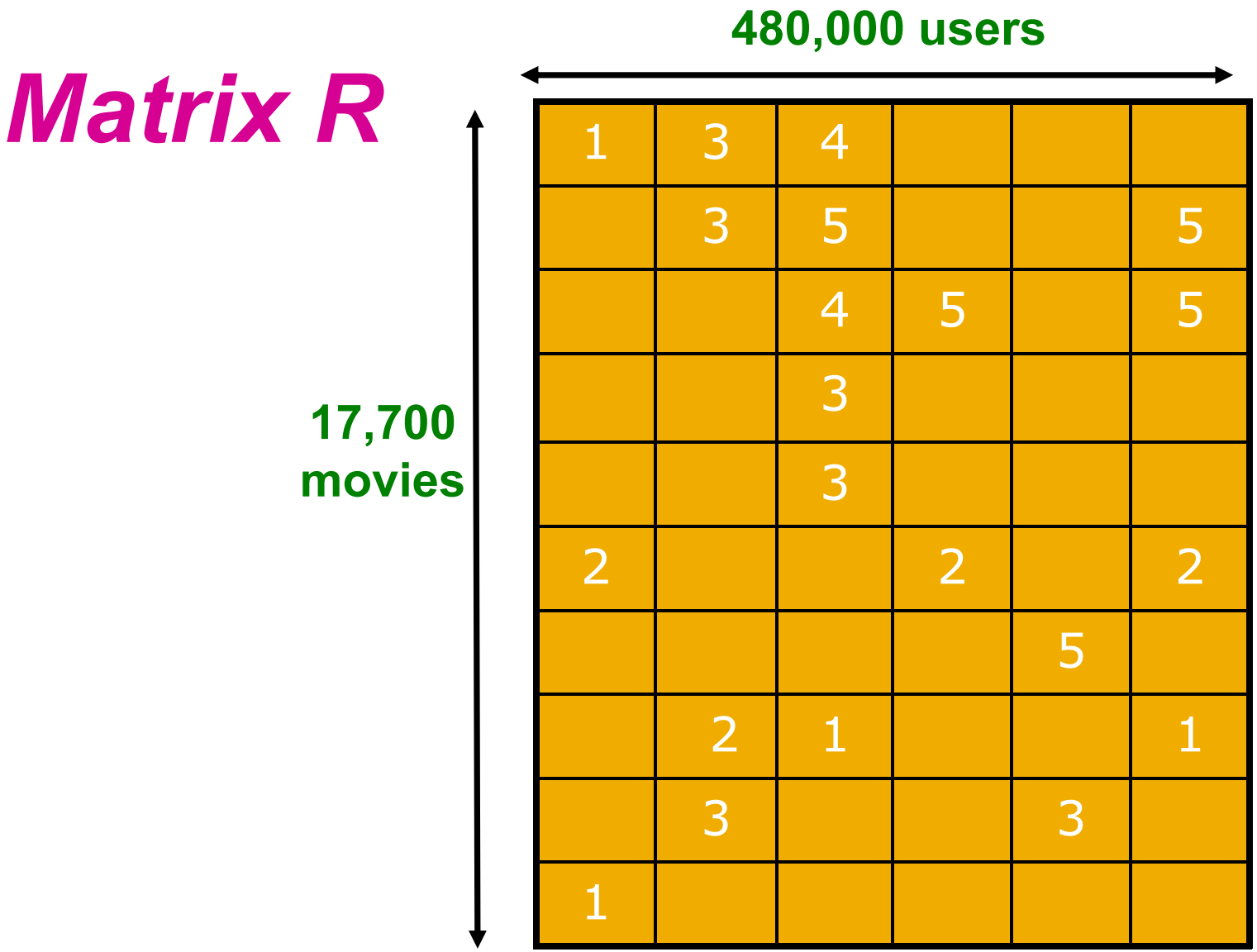
Information Storage and Retrieval

CSCE 670
Texas A&M University
Department of Computer Science & Engineering
Instructor: Prof. James Caverlee

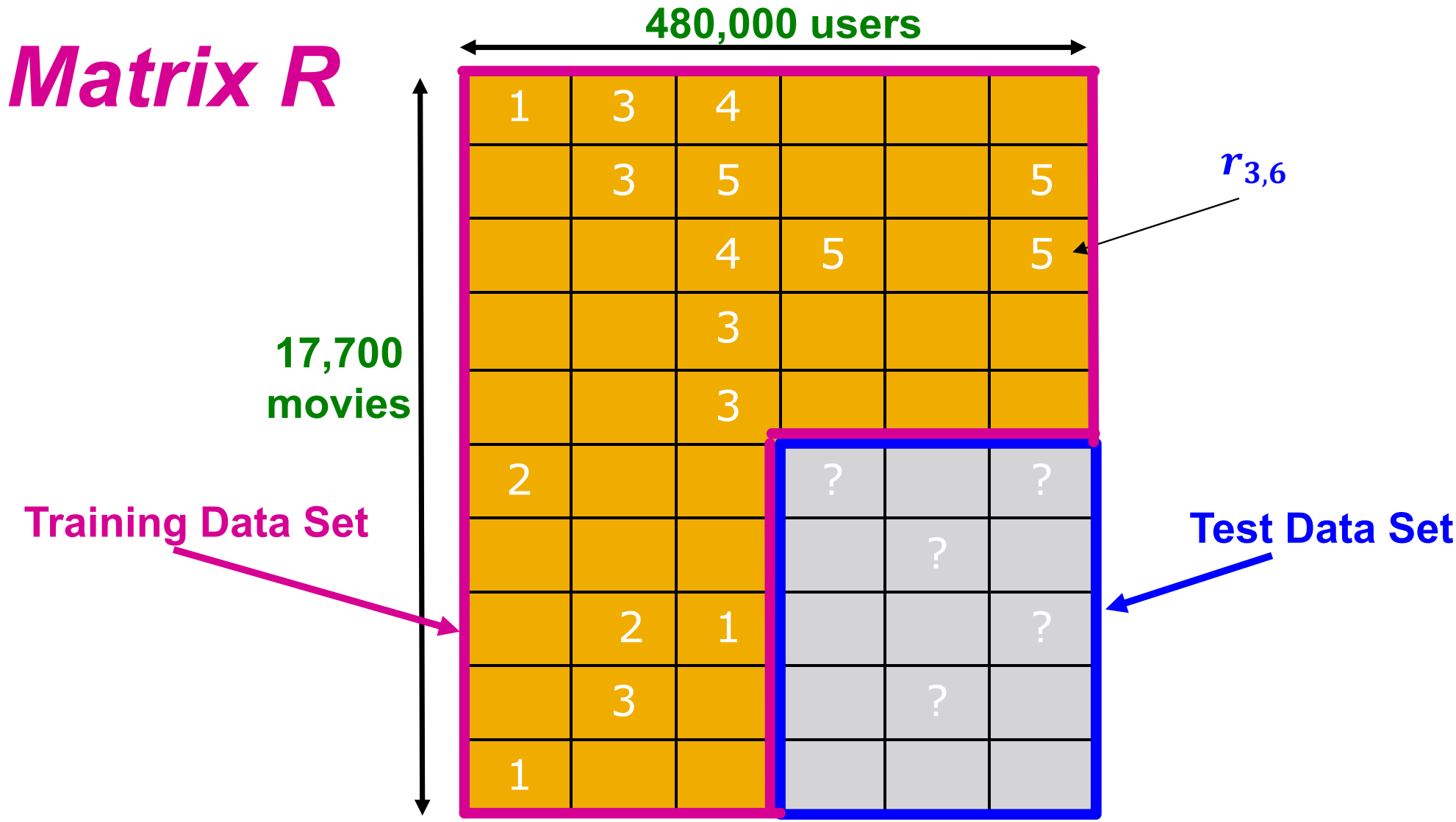
Latent Factor Models
20/22 March 2018

Slides adapted from Jure Leskovec, Yehuda Koren, and others

The Netflix Utility Matrix R

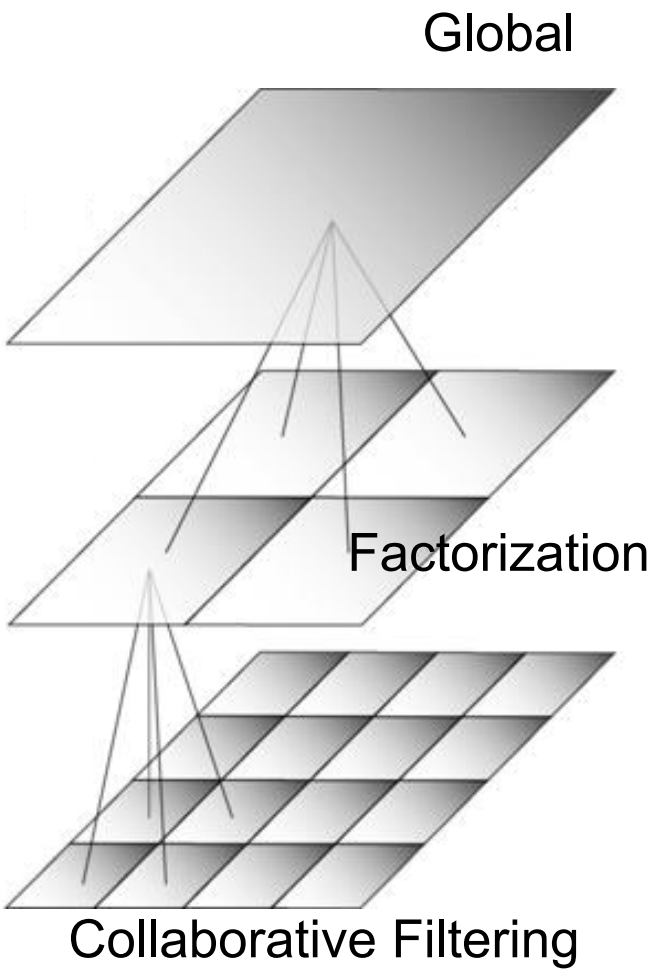


Utility Matrix R: Evaluation



BellKor Recommender System

- The winner of the Netflix Challenge
- Multi-scale modeling of the data
  - **Global:**
    - Overall deviations of users/movies
  - **Factorization:**
    - Addressing "regional" effects
  - **Collaborative Filtering:**
    - Extract local patterns



Modeling Local and Global Effects

- **Global:**
  - Mean movie rating: **3.7 stars**
  - *The Sixth Sense* is **0.5** stars above average
  - Joe rates **0.2** stars below average
  - => Baseline estimate: Joe will rate *The Sixth Sense* **4 stars**
- **Local Neighborhood (Collaborative Filtering)**
  - Joe didn't like related movie *Signs*
  - =>Final estimate: Joe will rate *The Sixth Sense* **3.8 stars**

Recap: Collaborative Filtering

- **Earliest and most popular collaborative filtering method**
- Derive unknown ratings from those of "**similar**" movies (item-item variant)
- Define **similarity measure s_{ij}** of items ***i*** and ***j***
- Select ***k***-nearest neighbors, compute the rating
  - ***N(i; x):*** items most similar to ***i*** that were rated by ***x***

$$\hat{r}_{xi} = \frac{\sum_{j \in N(i;x)} s_{ij} \cdot r_{xj}}{\sum_{j \in N(i;x)} s_{ij}}$$

s_{ij} similarity of items ***i*** and ***j***
r_{xj} rating of user ***x*** on item ***j***
N(i;x) set of items similar to item ***i*** that were rated by ***x***

Modeling Local and Global Effects

- **In practice we get better estimates if we model deviations:**

$$\hat{r}_{xi} = b_{xi} + \frac{\sum_{j \in N(i;x)} s_{ij} \cdot (r_{xj} - b_{xj})}{\sum_{j \in N(i;x)} s_{ij}}$$

baseline estimate for r_{xi}

$$b_{xi} = \mu + b_x + b_i$$

μ = overall mean rating
b_x = rating deviation of user x
= (*avg. rating of* user x) – μ
b_i = (*avg. rating of* movie i) – μ

Problems/Issues:

1) Similarity measures are "arbitrary

2) Pairwise similarities neglect interdependencies among users

3) Taking a weighted average can be restricting

Solution: Instead of s_{ij} use w_{ij} that we estimate directly from data

Idea: Interpolation Weights

- Use a **weighted sum** rather than **weighted avg.**:

$$\widehat{r_{xi}} = b_{xi} + \sum_{j \in N(i;x)} w_{ij}\left(r_{xj} - b_{xj}\right)$$

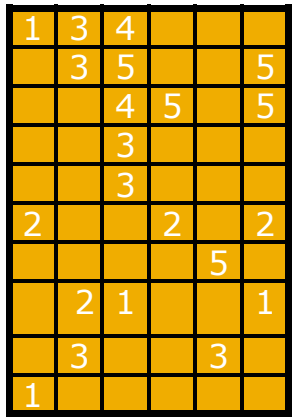
- **A few notes:**
  - $\boldsymbol{N(i; x)}$ … set of movies rated by user $\boldsymbol{x}$ that are similar to movie $\boldsymbol{i}$
  - $\boldsymbol{w_{ij}}$ is the interpolation weight (some real number)
    - We allow: $\boldsymbol{\sum_{j \in N(i,x)} w_{ij} \neq 1}$
  - $\boldsymbol{w_{ij}}$ models interaction between pairs of movies (it does not depend on user $\boldsymbol{x}$)

- $\widehat{r_{xi}} = b_{xi} + \sum_{j \in N(i,x)} w_{ij}\left(r_{xj} - b_{xj}\right)$
- **How to set w_{ij}?**
  - Remember, error metric is: $\frac{1}{|R|}\sqrt{\sum_{(i,x)\in R}(\hat{r}_{xi} - r_{xi})^2}$

    or equivalently **SSE:** $\sum_{(i,x)\in R}(\hat{r}_{xi} - r_{xi})^2$
  - Find w_{ij} that minimize **SSE** on **training data!**
    - Models relationships between item ***i*** and its neighbors ***j***
  - w_{ij} can be **learned/estimated** based on ***x*** and all other users that rated ***i***

Why is this a good idea?

Recommendations via Optimization



- **Goal:** Make good recommendations
  - Quantify goodness using **RMSE:**
    Lower RMSE ⇒ better recommendations
  - Want to make good recommendations on items that user has not yet seen. Can't really do this!
  - **Let's set build a system such that it works well on known (user, item) ratings**
    And **hope** the system will also predict well the **unknown ratings**

Recommendations via Optimization

- **Idea: Let's set values *w* such that they work well on known (user, item) ratings**
- **How to find such values *w*?**
- **Idea:** Define an objective function and solve the optimization problem

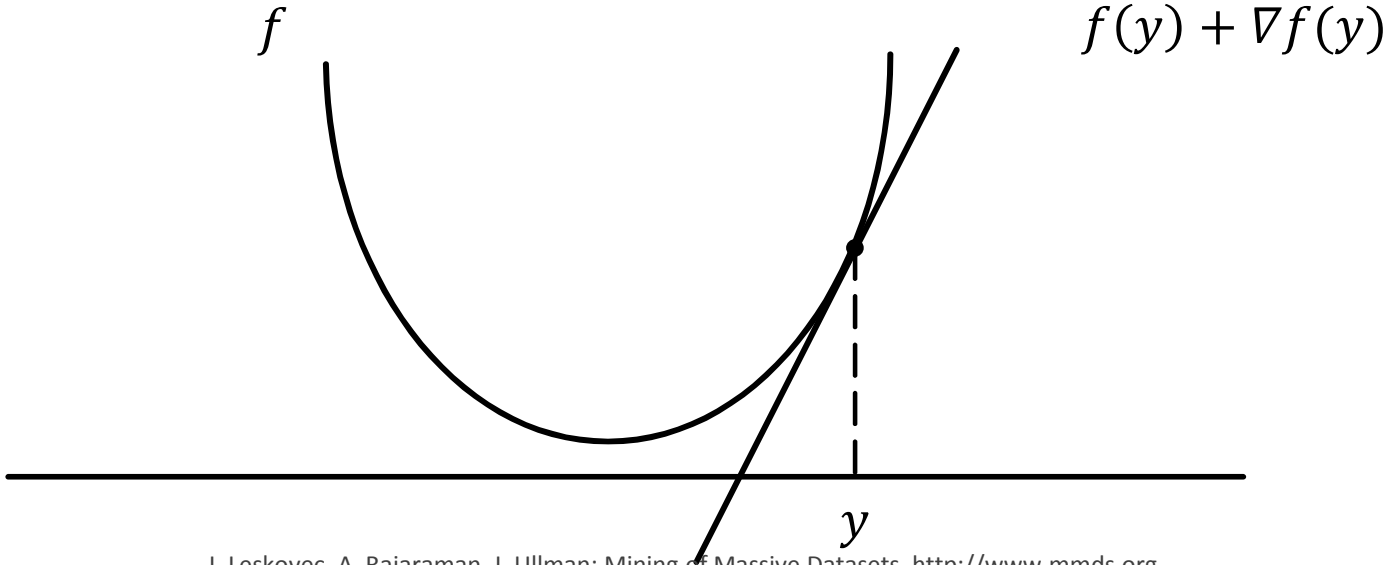
- Find $\boldsymbol{w_{ij}}$ that minimize **SSE** on **training data**!

$$J(w) = \sum_{x,i} \left(\underbrace{\left[b_{xi} + \sum_{j \in N(i;x)} w_{ij}\left(r_{xj} - b_{xj}\right) \right]}_{\text{Predicted rating}} - \underbrace{r_{xi}}_{\text{True rating}} \right)^2$$

- Think of $\boldsymbol{w}$ as a vector of numbers

Detour: Minimizing a function

- **A simple way to minimize a function $f(x)$:**
  - Compute the take a derivative ∇f
  - **Start at some point y and evaluate $\nabla f(y)$**
  - **Make a step in the reverse direction of the gradient: $y = y - \nabla f(y)$**
  - **Repeat until converged**



Interpolation Weights

- **We have the optimization problem, now what?**

$$J(w) = \sum_x \left(\left[b_{xi} + \sum_{j \in N(i;x)} w_{ij} \left(r_{xj} - b_{xj} \right) \right] - r_{xi} \right)^2$$

- **Gradient decent:**
  - **Iterate until convergence:** $\boldsymbol{w \leftarrow w - \eta \nabla_w J}$ η … learning rate
  - **where $\nabla_w J$ is the gradient (derivative evaluated on data):**

$$\nabla_w J = \left[\frac{\partial J(w)}{\partial w_{ij}} \right] = 2 \sum_{x,i} \left(\left[b_{xi} + \sum_{k \in N(i;x)} w_{ik} \left(r_{xk} - b_{xk} \right) \right] - r_{xi} \right) \left(r_{xj} - b_{xj} \right)$$

$$\textbf{for } j \in \{ N(i;x), \forall i, \forall x \}$$

$$\textbf{else } \frac{\partial J(w)}{\partial w_{ij}} = 0$$

  - **Note:** We fix movie i, go over all r_{xi}, for every movie $j \in N(i;x)$, we compute $\frac{\partial J(w)}{\partial w_{ij}}$

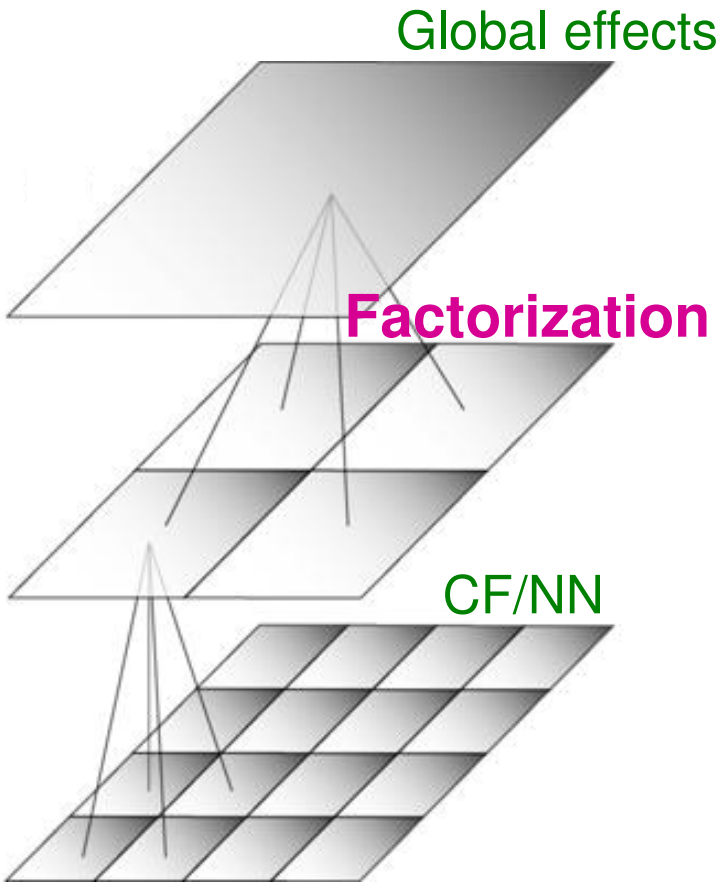
while $|w_{new} - w_{old}| > \varepsilon$:

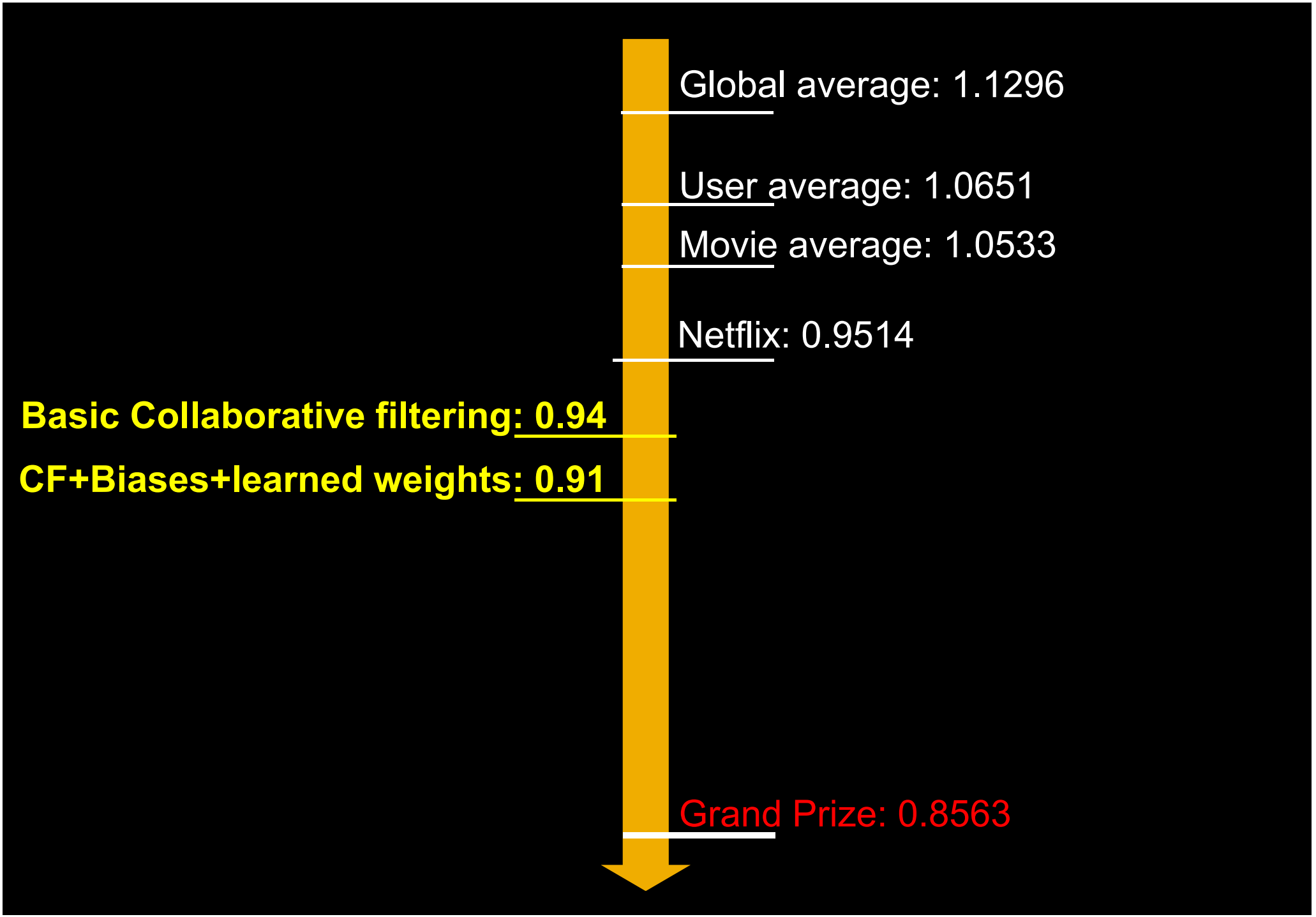
$w_{old} = w_{new}$

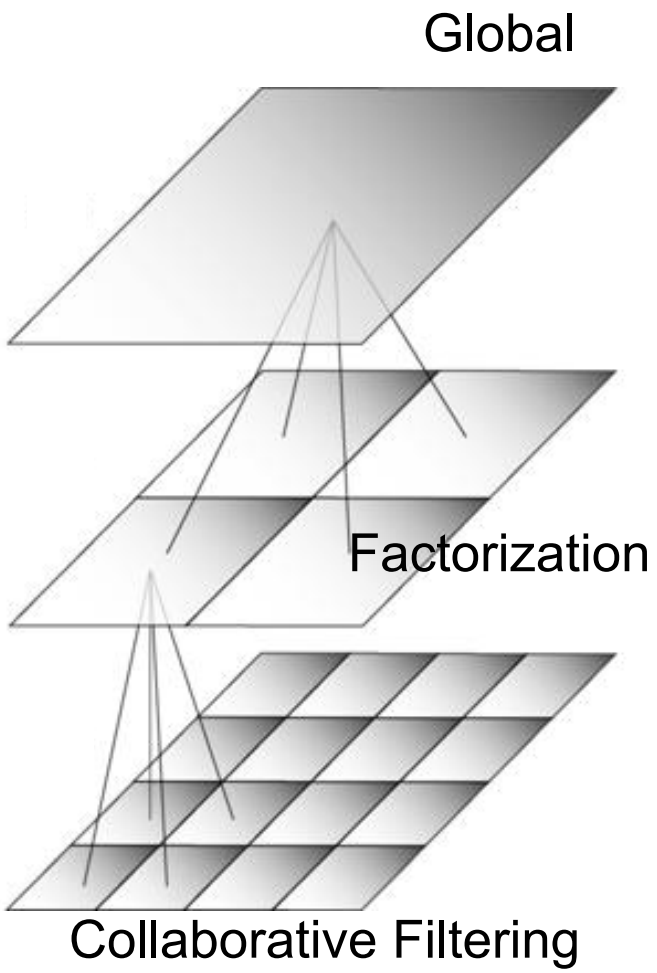
$w_{new} = w_{old} - \eta \cdot \nabla w_{old}$

Interpolation Weights

- **So far:** $\widehat{r_{xi}} = b_{xi} + \sum_{j \in N(i;x)} w_{ij}\left(r_{xj} - b_{xj}\right)$
  - Weights $\boldsymbol{w_{ij}}$ derived based on their role; **no use of an arbitrary similarity measure** ($\boldsymbol{w_{ij} \neq s_{ij}}$)
  - Explicitly account for interrelationships among the neighboring movies
- **Next: Latent factor model**
  - Extract "regional" correlations

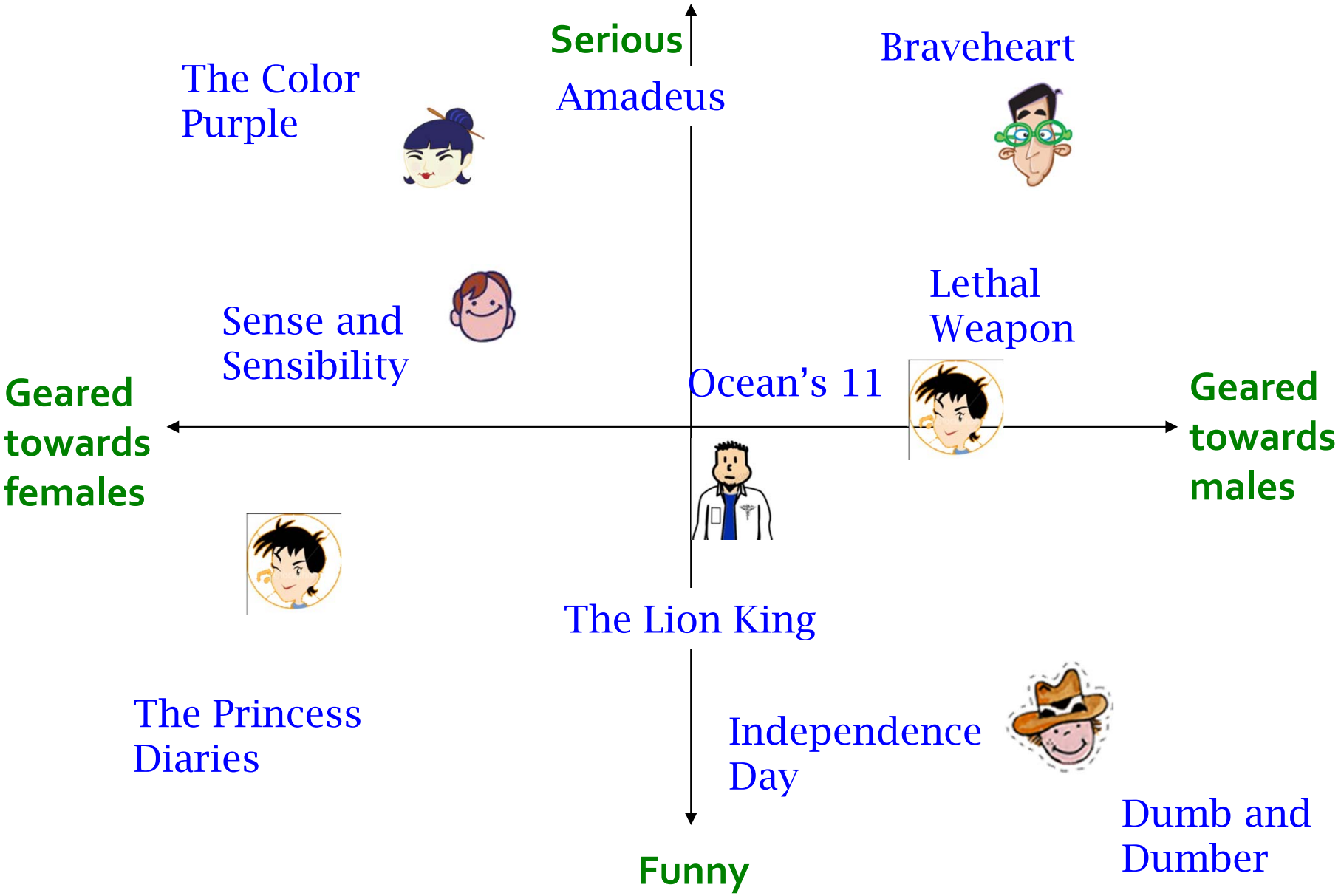



Global average: 1.1296
User average: 1.0651
Movie average: 1.0533
Netflix: 0.9514
Basic Collaborative filtering: 0.94
CF+Biases+learned weights: 0.91
Grand Prize: 0.8563

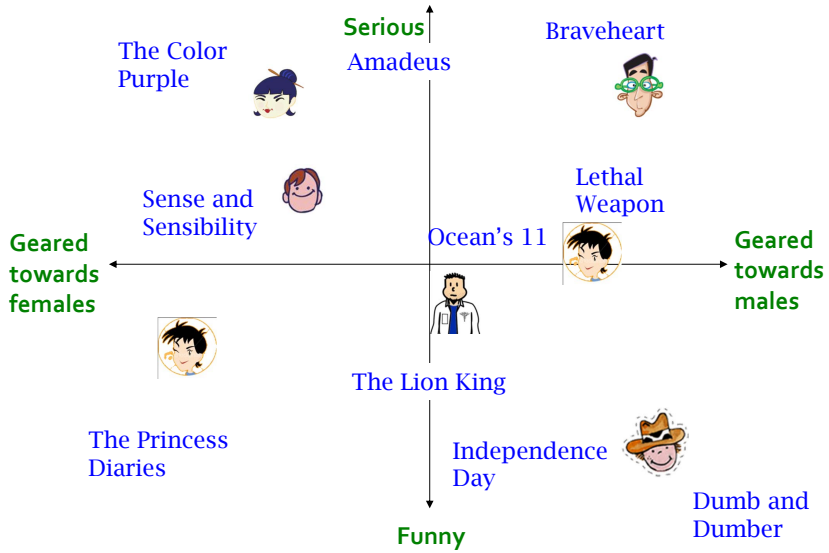


Latent Factor Model: Extract "regional" correlations

Latent Factor Models



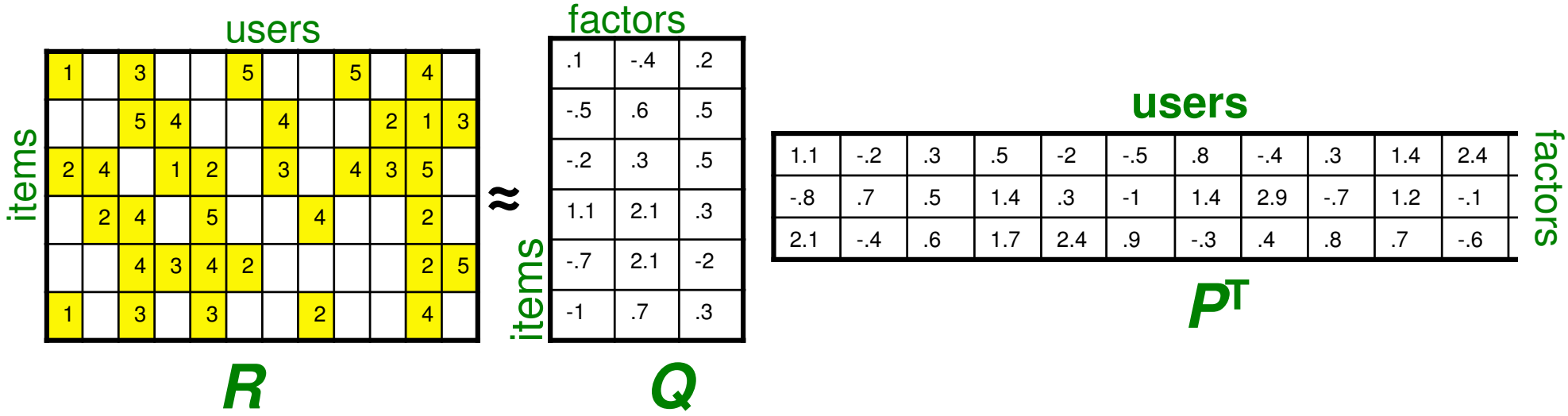
- How would you map from high dimensional features (ratings) to lower dimensional "latent factors"?
- Or … how would you map many individual users to smaller "latent" groups?
- Or … how would you map many individual items to smaller "latent" groups?



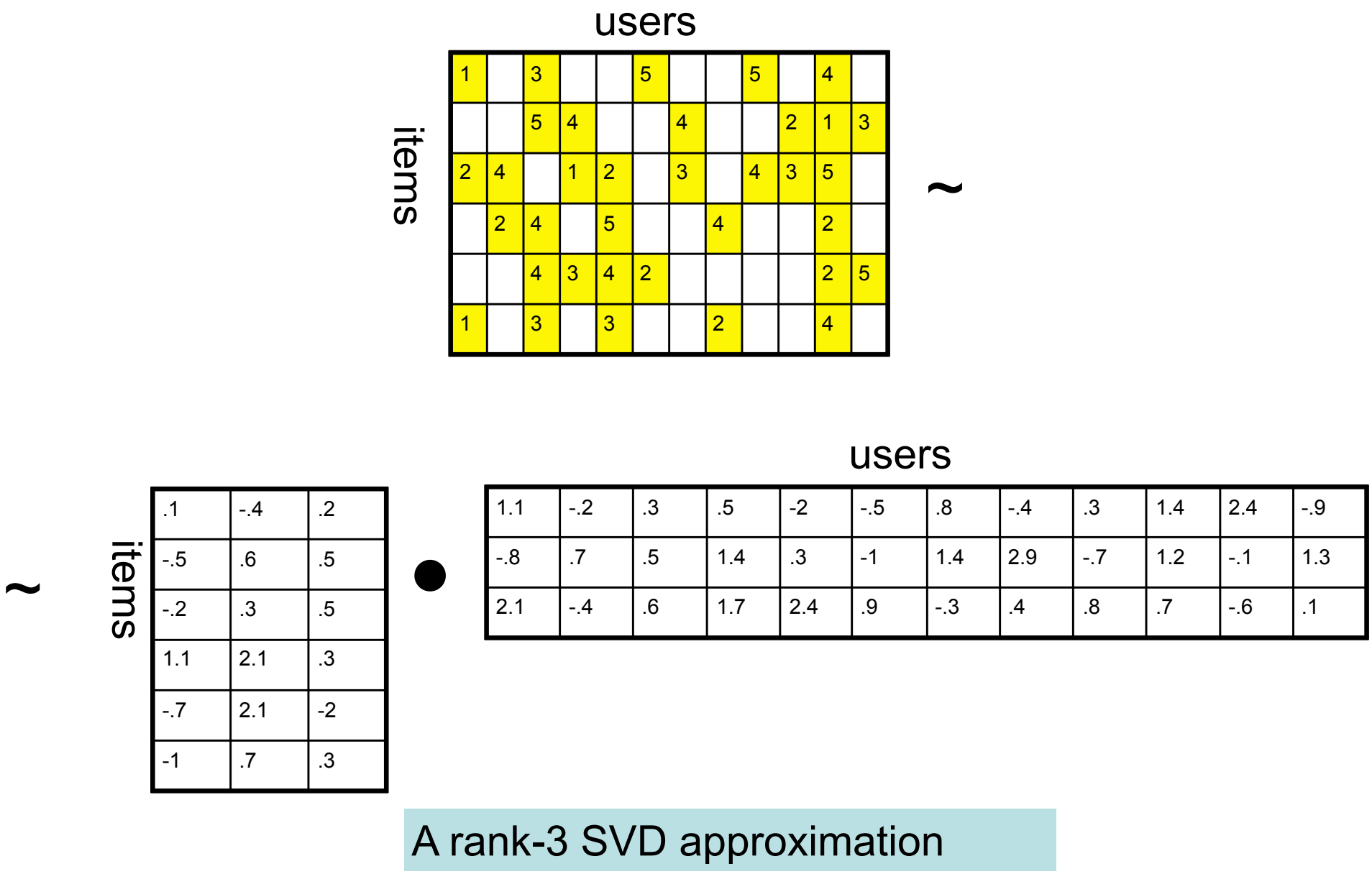
Latent factor models

SVD: $A = U \Sigma V^T$

- "SVD" on Netflix data: $\mathbf{R} \approx \boldsymbol{Q} \cdot \boldsymbol{P}^T$

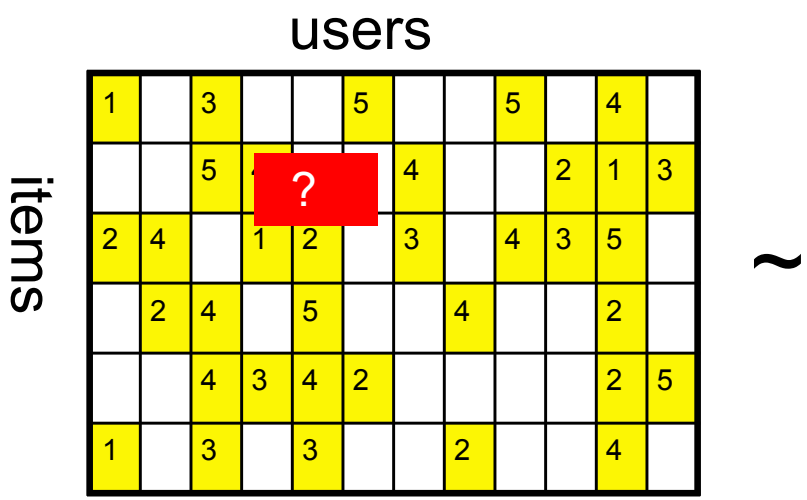


Latent factor models



A rank-3 SVD approximation

Estimate unknown ratings as inner-products of factors:



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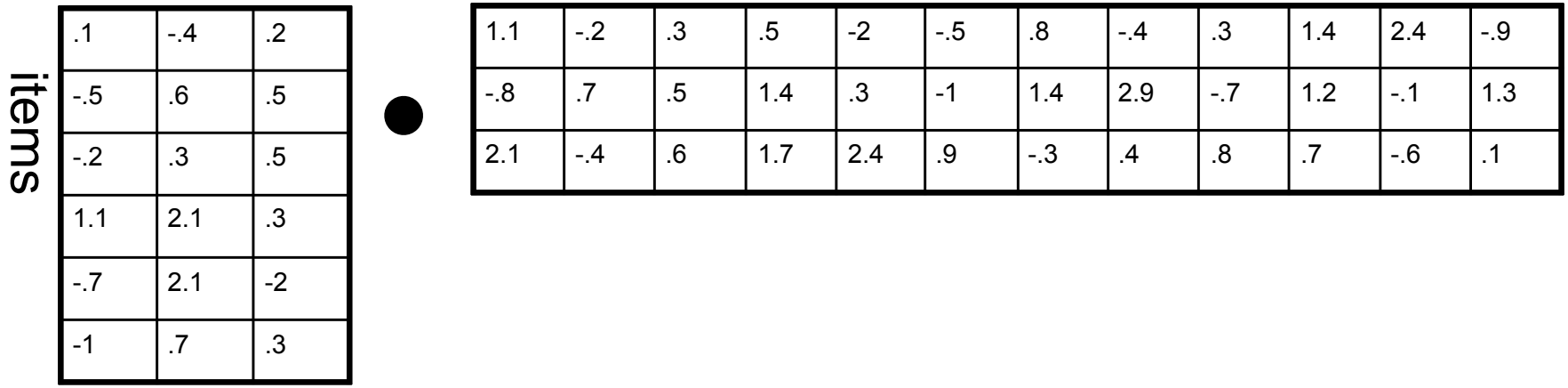


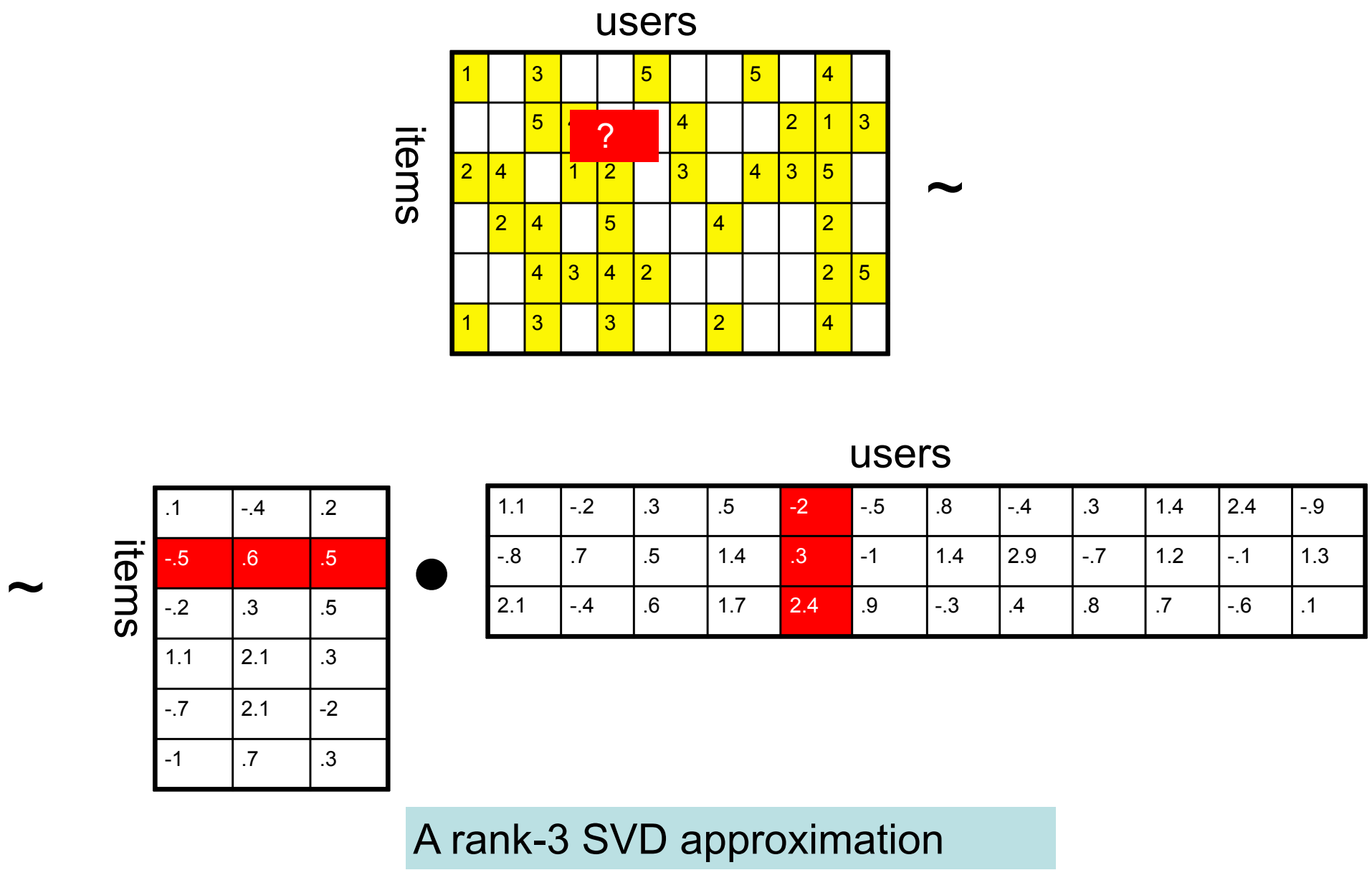
.1	-.4	.2
-.5	.6	.5
-.2	.3	.5
1.1	2.1	.3
-.7	2.1	-2
-1	.7	.3

•

1.1	-.2	.3	.5	-2	-.5	.8	-.4	.3	1.4	2.4	-.9
-.8	.7	.5	1.4	.3	-1	1.4	2.9	-.7	1.2	-.1	1.3
2.1	-.4	.6	1.7	2.4	.9	-.3	.4	.8	.7	-.6	.1

A rank-3 SVD approximation

Estimate unknown ratings as inner-products of factors:



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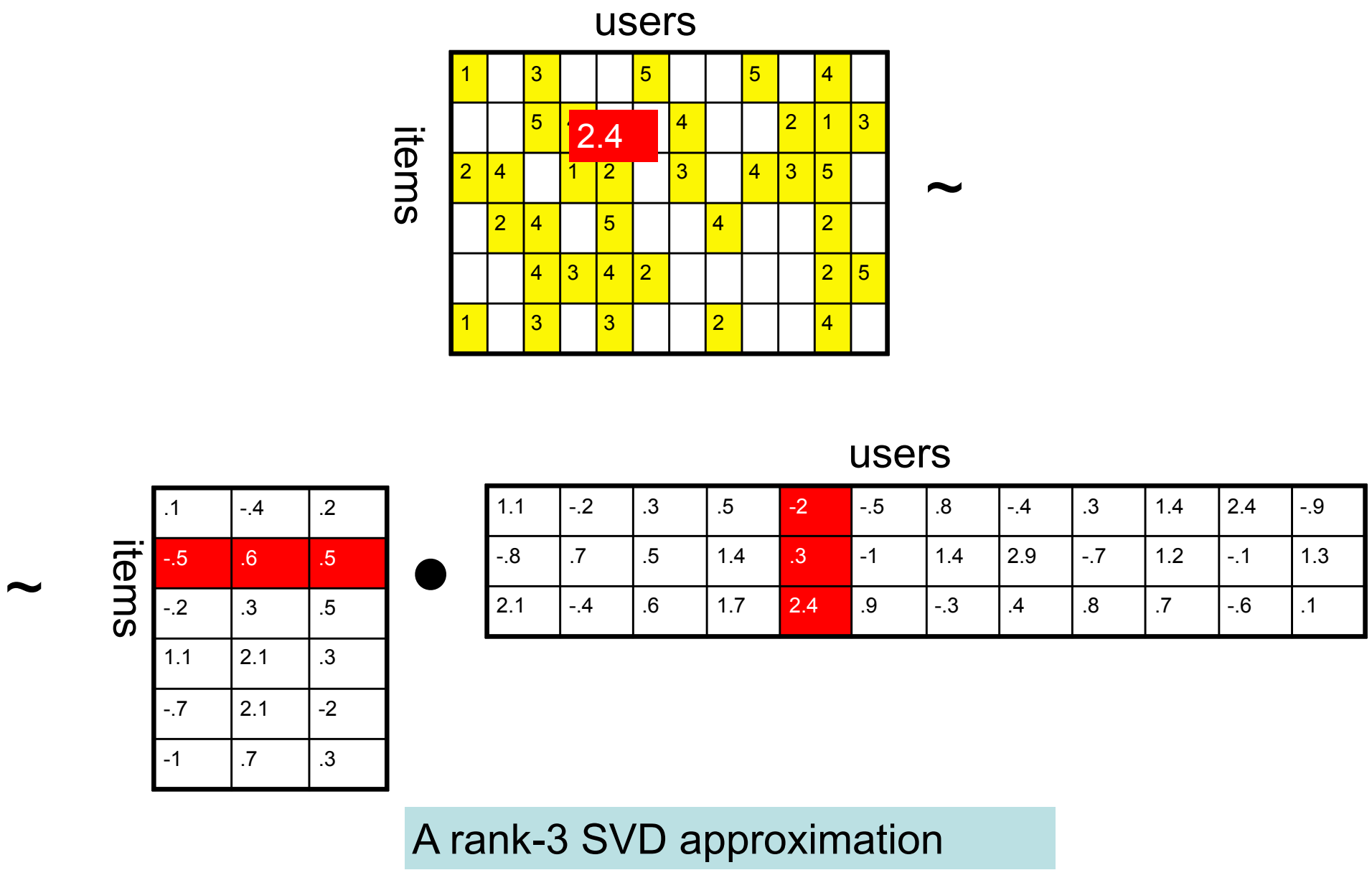


SVD

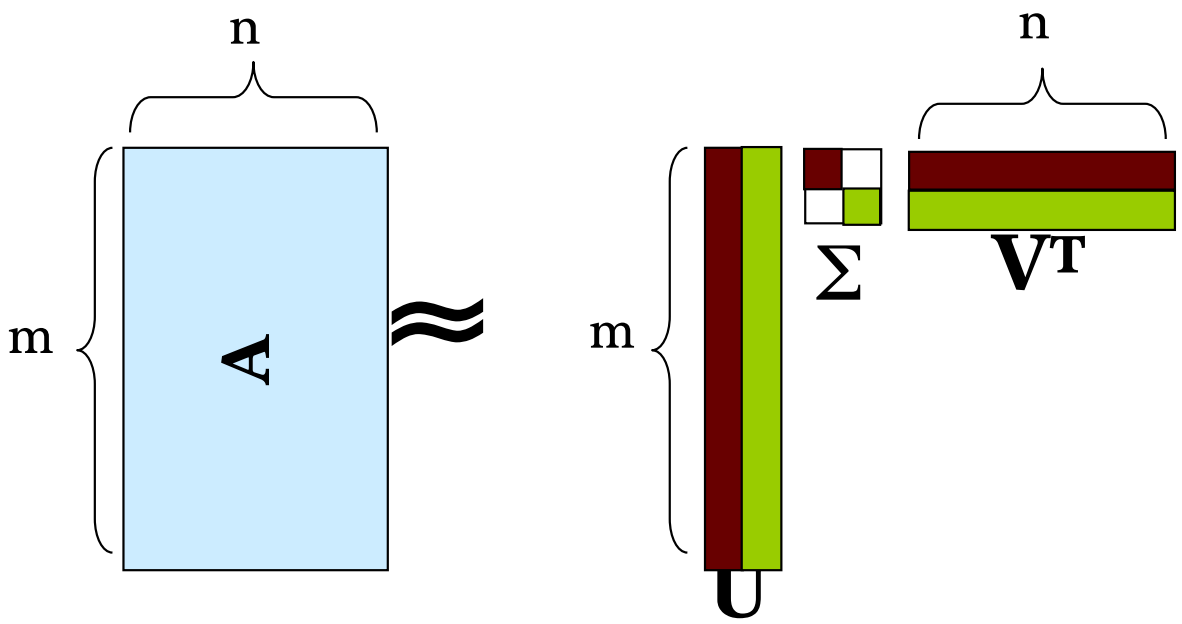
SVD - Definition

$$\mathbf{A}_{[m \times n]} = \mathbf{U}_{[m \times r]} \Sigma_{[r \times r]} (\mathbf{V}_{[n \times r]})^T$$

- **A**: **Input data matrix**
  - *m* x *n* matrix (e.g., *m* documents, *n* terms)
- **U**: **Left singular vectors**
  - *m* x *r* matrix (*m* documents, *r* concepts)
- Σ: **Singular values**
  - *r* x *r* diagonal matrix (strength of each 'concept') (*r* : rank of the matrix **A**)
- **V**: **Right singular vectors**
  - *n* x *r* matrix (*n* terms, *r* concepts)

SVD

$$\mathbf{A} \approx \mathbf{U}\mathbf{\Sigma}\mathbf{V}^T = \sum_i \sigma_i \mathbf{u}_i \circ \mathbf{v}_i^T$$



SVD - Properties

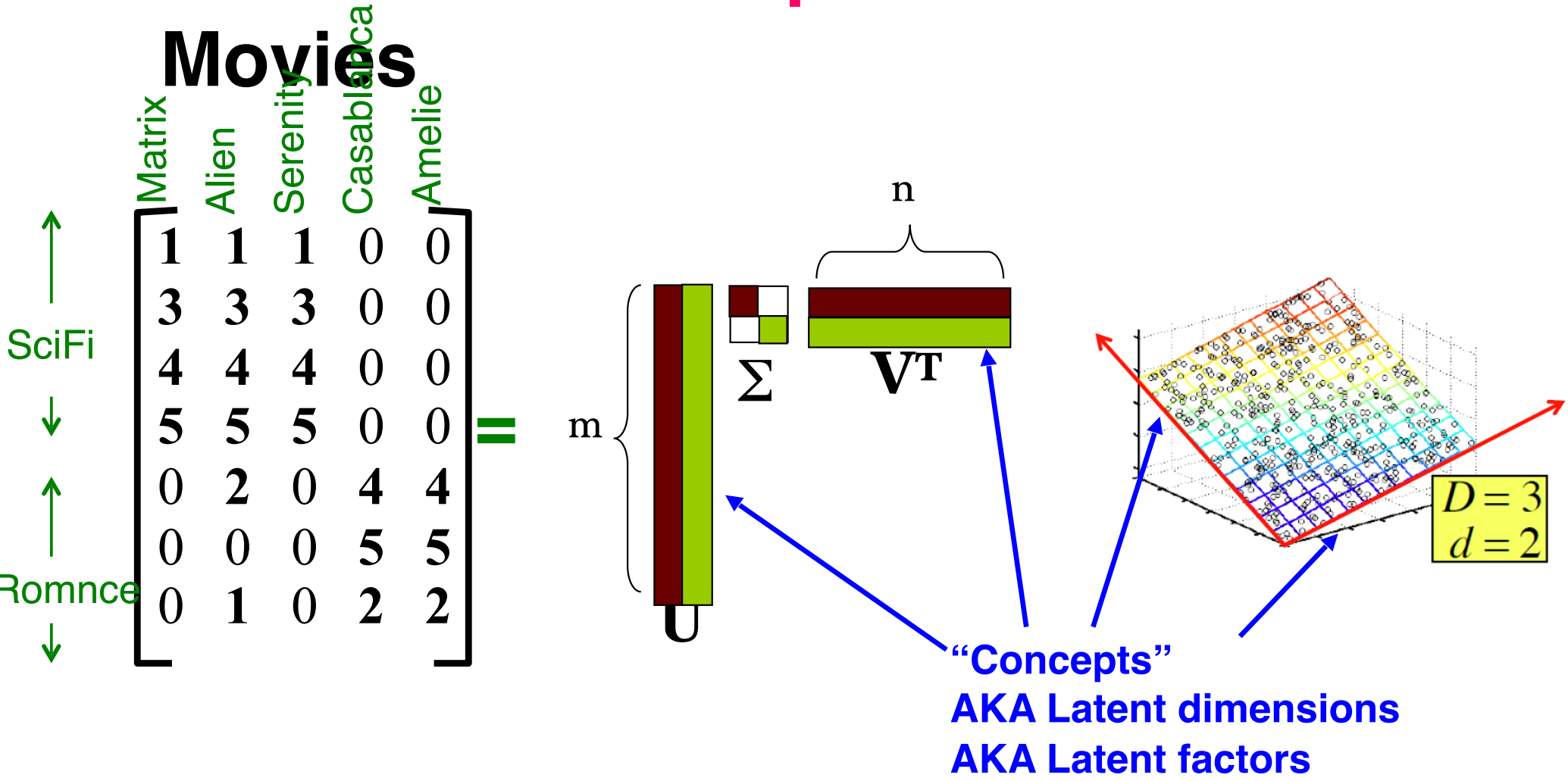
It is **always** possible to decompose a real matrix $\boldsymbol{A}$ into $\boldsymbol{A = U \Sigma V^T}$, where

- $\boldsymbol{U}, \Sigma, \boldsymbol{V}$: unique
- $\boldsymbol{U}, \boldsymbol{V}$: column orthonormal
  - $\boldsymbol{U^T U = I}; \boldsymbol{V^T V = I}$ ($\boldsymbol{I}$: identity matrix)
  - (Columns are orthogonal unit vectors)
- Σ: diagonal
  - Entries (**singular values**) are positive, and sorted in decreasing order ($\boldsymbol{\sigma_1 \geq \sigma_2 \geq \ldots \geq 0}$)

Nice proof of uniqueness: http://www.mpi-inf.mpg.de/~bast/ir-seminar-ws04/lecture2.pdf

SVD – Example: Users-to-Movies

- **A = U Σ V^T - example: Users to Movies**



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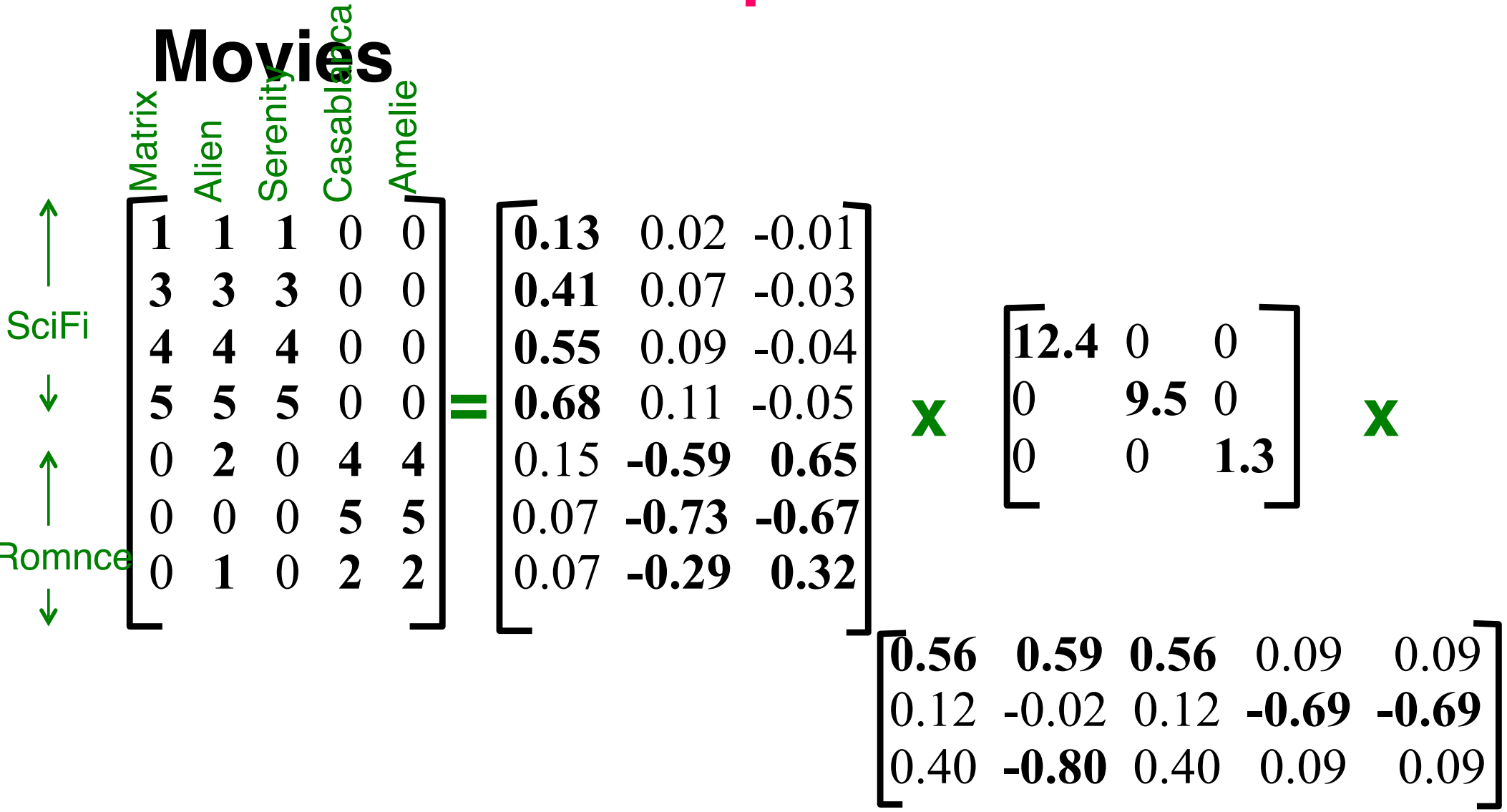

SVD – Example: Users-to-Movies

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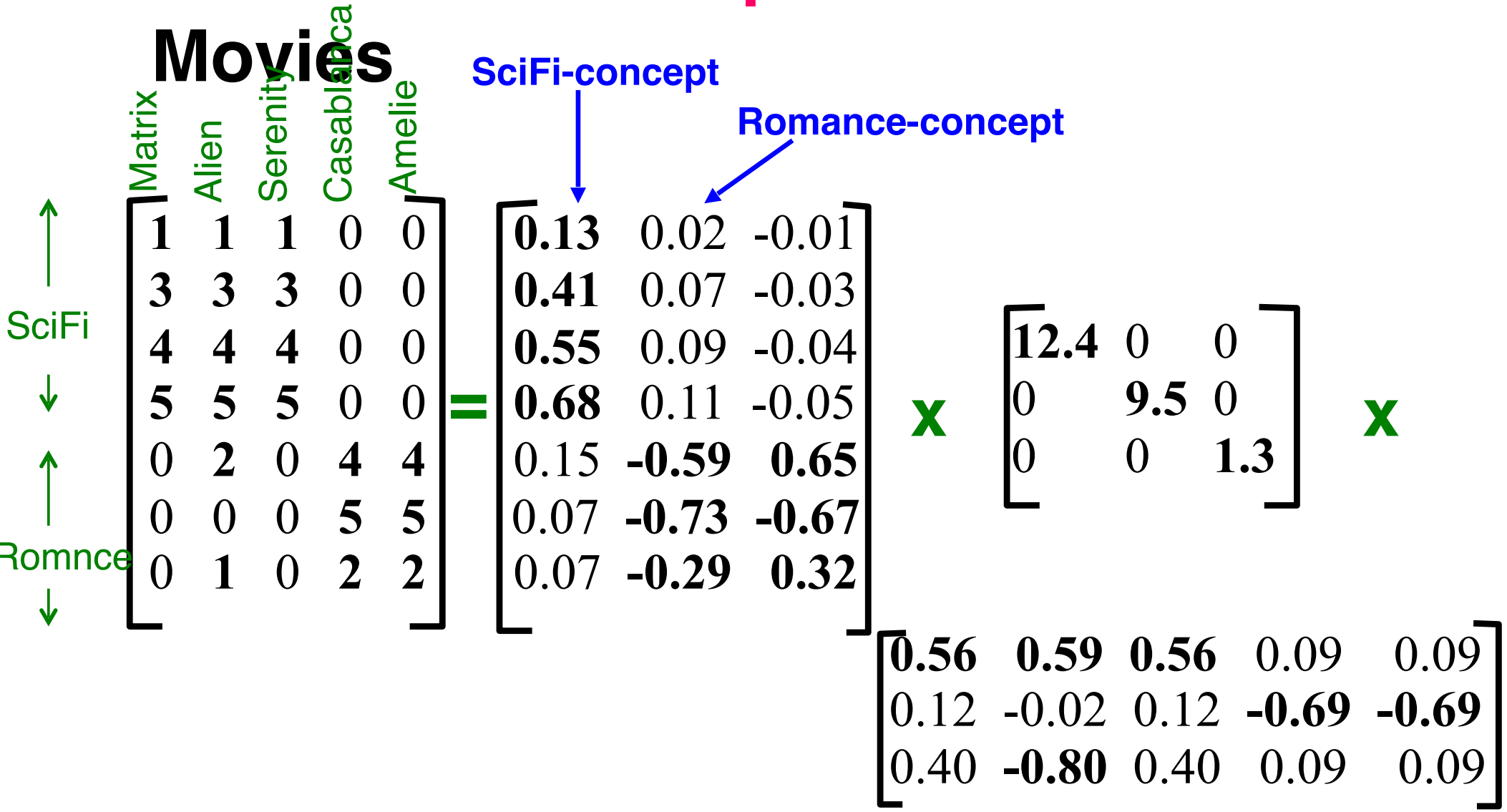
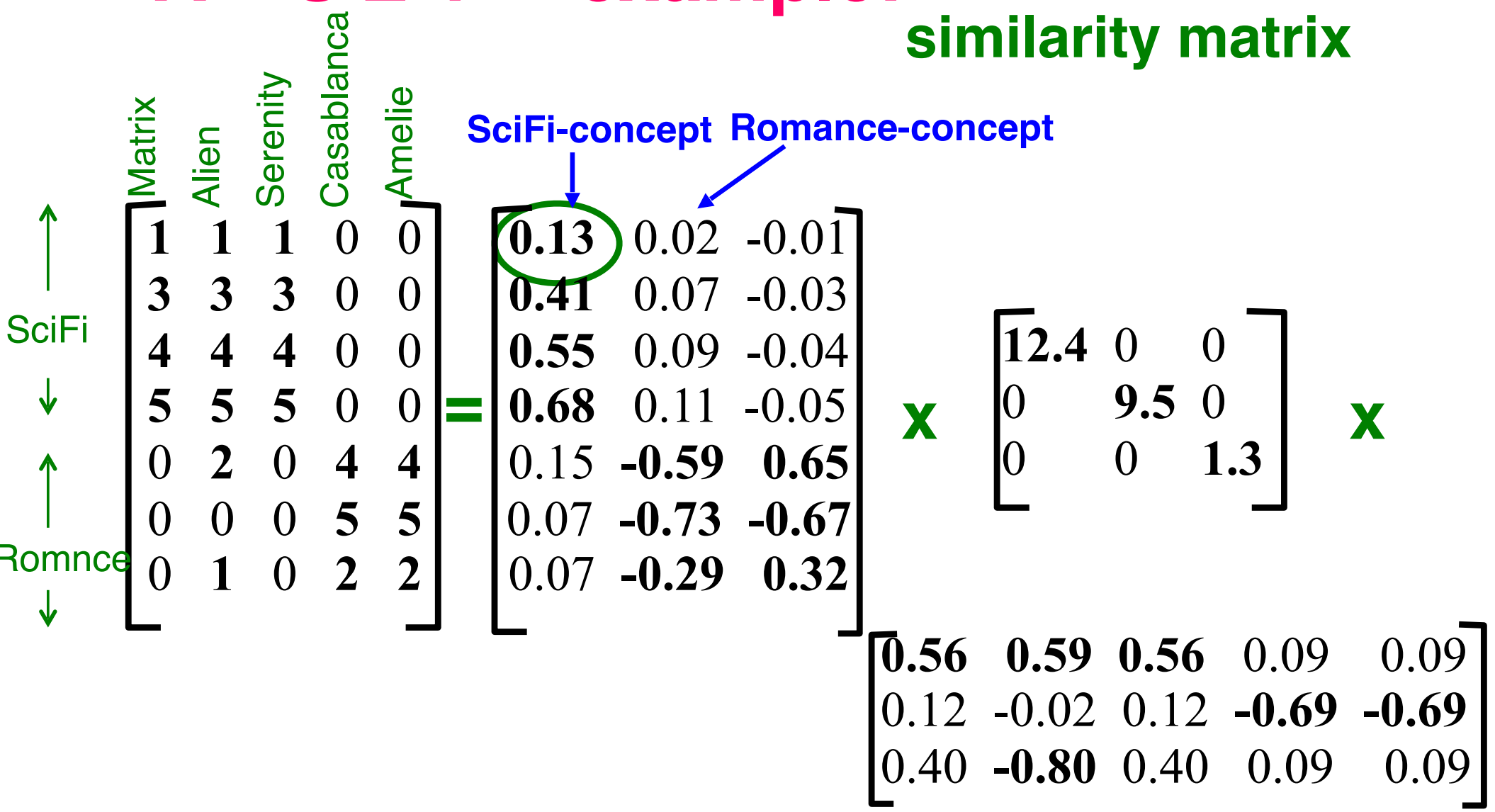


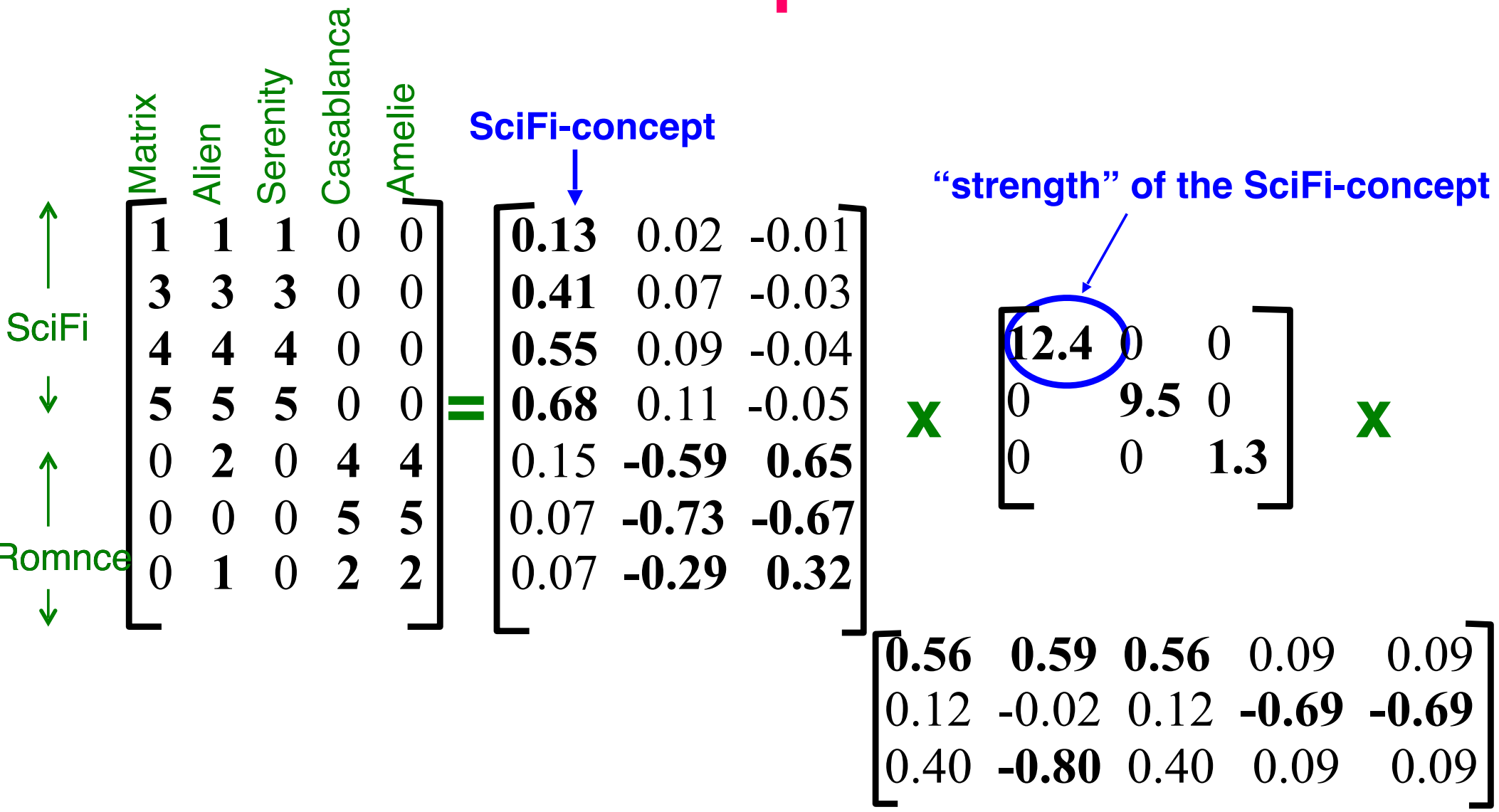
SVD – Example: Users-to-Movies

- **A = U Σ V^T - example:** ***U*** **is "user-to-concept similarity matrix**



SVD – Example: Users-to-Movies

- $A = U \Sigma V^T$ - example:



SVD – Example: Users-to-Movies

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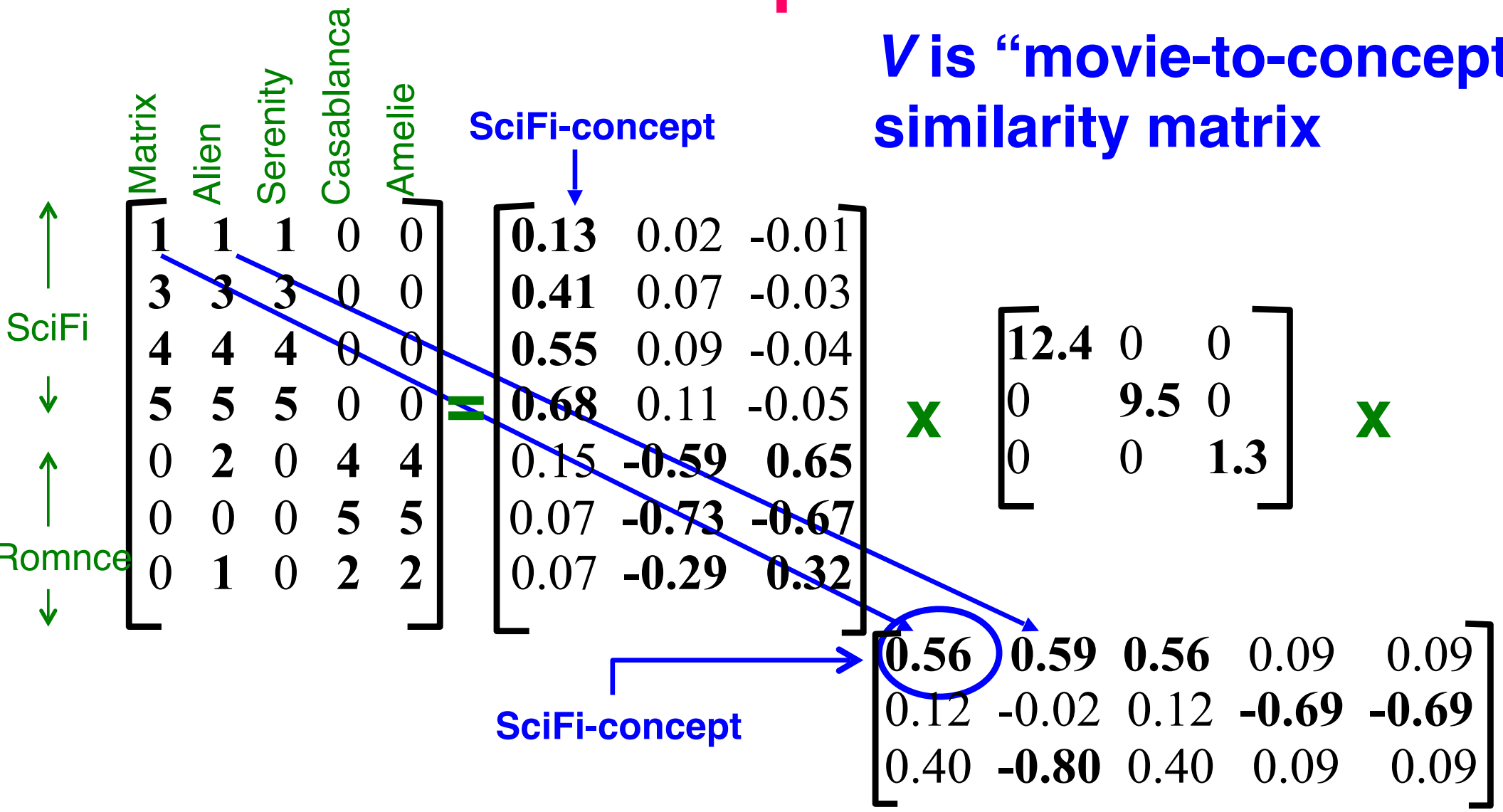

SVD - Interpretation #1

'**movies**', '**users**' and '**concepts**':

- ***U***: user-to-concept similarity matrix
- ***V***: movie-to-concept similarity matrix
- Σ: its diagonal elements:
  'strength' of each concept

Dimensionality Reduction with SVD

SVD - Interpretation #2

More details

- Q: How exactly is dim. reduction done?

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 3 & 3 & 3 & 0 & 0 \\ 4 & 4 & 4 & 0 & 0 \\ 5 & 5 & 5 & 0 & 0 \\ 0 & 2 & 0 & 4 & 4 \\ 0 & 0 & 0 & 5 & 5 \\ 0 & 1 & 0 & 2 & 2 \end{bmatrix} = \begin{bmatrix} 0.13 & 0.02 & -0.01 \\ 0.41 & 0.07 & -0.03 \\ 0.55 & 0.09 & -0.04 \\ 0.68 & 0.11 & -0.05 \\ 0.15 & -0.59 & 0.65 \\ 0.07 & -0.73 & -0.67 \\ 0.07 & -0.29 & 0.32 \end{bmatrix} \times \begin{bmatrix} 12.4 & 0 & 0 \\ 0 & 9.5 & 0 \\ 0 & 0 & 1.3 \end{bmatrix} \times \begin{bmatrix} 0.56 & 0.59 & 0.56 & 0.09 & 0.09 \\ 0.12 & -0.02 & 0.12 & -0.69 & -0.69 \\ 0.40 & -0.80 & 0.40 & 0.09 & 0.09 \end{bmatrix}$$

SVD - Interpretation #2

More details

- Q: How exactly is dim. reduction done?
- A: Set smallest singular values to zero

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 3 & 3 & 3 & 0 & 0 \\ 4 & 4 & 4 & 0 & 0 \\ 5 & 5 & 5 & 0 & 0 \\ 0 & 2 & 0 & 4 & 4 \\ 0 & 0 & 0 & 5 & 5 \\ 0 & 1 & 0 & 2 & 2 \end{bmatrix} = \begin{bmatrix} 0.13 & 0.02 & -0.01 \\ 0.41 & 0.07 & -0.03 \\ 0.55 & 0.09 & -0.04 \\ 0.68 & 0.11 & -0.05 \\ 0.15 & -0.59 & 0.65 \\ 0.07 & -0.73 & -0.67 \\ 0.07 & -0.29 & 0.32 \end{bmatrix} \times \begin{bmatrix} 12.4 & 0 & 0 \\ 0 & 9.5 & 0 \\ 0 & 0 & 1.3 \end{bmatrix} \times \begin{bmatrix} 0.56 & 0.59 & 0.56 & 0.09 & 0.09 \\ 0.12 & -0.02 & 0.12 & -0.69 & -0.69 \\ 0.40 & -0.80 & 0.40 & 0.09 & 0.09 \end{bmatrix}$$

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- Q: How exactly is dim. reduction done?
- A: Set smallest singular values to zero

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 3 & 3 & 3 & 0 & 0 \\ 4 & 4 & 4 & 0 & 0 \\ 5 & 5 & 5 & 0 & 0 \\ 0 & 2 & 0 & 4 & 4 \\ 0 & 0 & 0 & 5 & 5 \\ 0 & 1 & 0 & 2 & 2 \end{bmatrix} \approx \begin{bmatrix} 0.13 & 0.02 \\ 0.41 & 0.07 \\ 0.55 & 0.09 \\ 0.68 & 0.11 \\ 0.15 & -0.59 \\ 0.07 & -0.73 \\ 0.07 & -0.29 \end{bmatrix} \times \begin{bmatrix} 12.4 & 0 \\ 0 & 9.5 \end{bmatrix} \times \begin{bmatrix} 0.56 & 0.59 & 0.56 & 0.09 & 0.09 \\ 0.12 & -0.02 & 0.12 & -0.69 & -0.69 \end{bmatrix}$$

SVD - Interpretation #2

More details

- Q: How exactly is dim. reduction done?
- A: Set smallest singular values to zero

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 3 & 3 & 3 & 0 & 0 \\ 4 & 4 & 4 & 0 & 0 \\ 5 & 5 & 5 & 0 & 0 \\ 0 & 2 & 0 & 4 & 4 \\ 0 & 0 & 0 & 5 & 5 \\ 0 & 1 & 0 & 2 & 2 \end{bmatrix} \approx \begin{bmatrix} 0.92 & 0.95 & 0.92 & 0.01 & 0.01 \\ 2.91 & 3.01 & 2.91 & -0.01 & -0.01 \\ 3.90 & 4.04 & 3.90 & 0.01 & 0.01 \\ 4.82 & 5.00 & 4.82 & 0.03 & 0.03 \\ 0.70 & 0.53 & 0.70 & 4.11 & 4.11 \\ -0.69 & 1.34 & -0.69 & 4.78 & 4.78 \\ 0.32 & 0.23 & 0.32 & 2.01 & 2.01 \end{bmatrix}$$

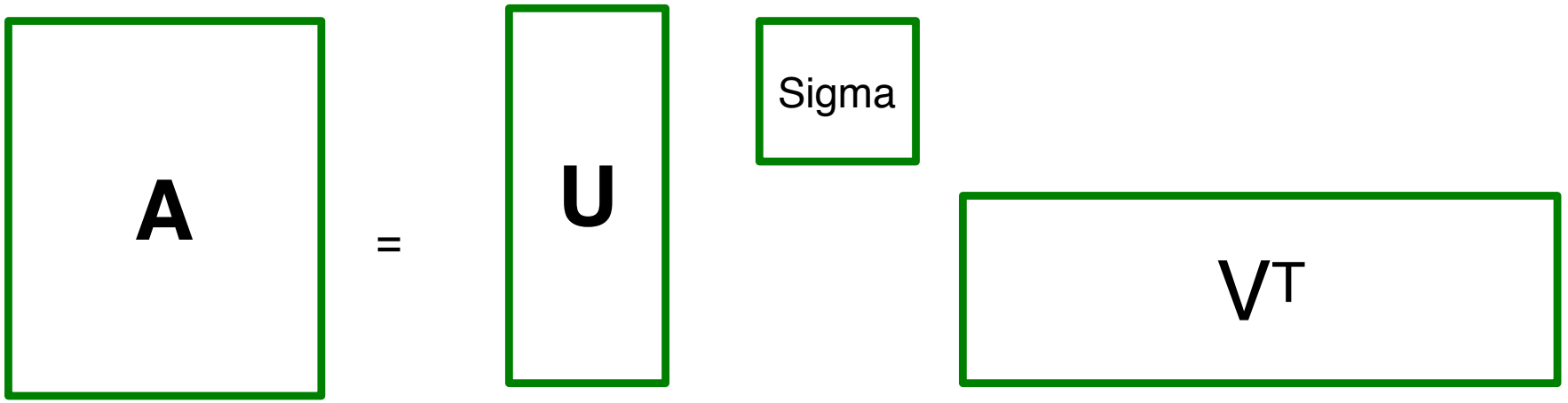
Frobenius norm:

$$\|M\|_F = \sqrt{\Sigma_{ij} M_{ij}^2}$$

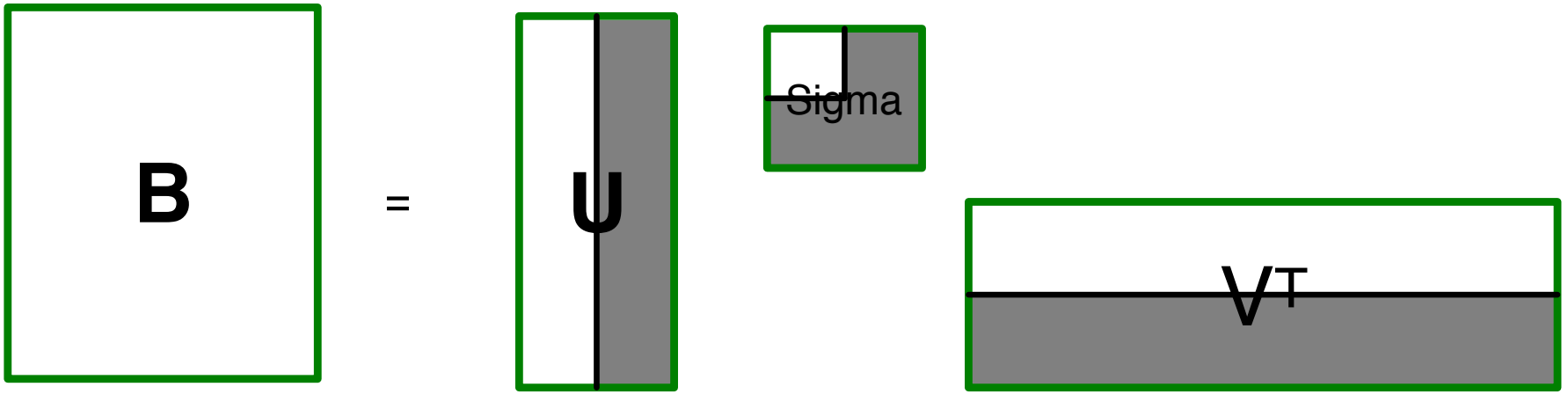
$$\|A-B\|_F = \sqrt{\Sigma_{ij} (A_{ij}-B_{ij})^2}$$

is "small"

SVD – Best Low Rank Approx.



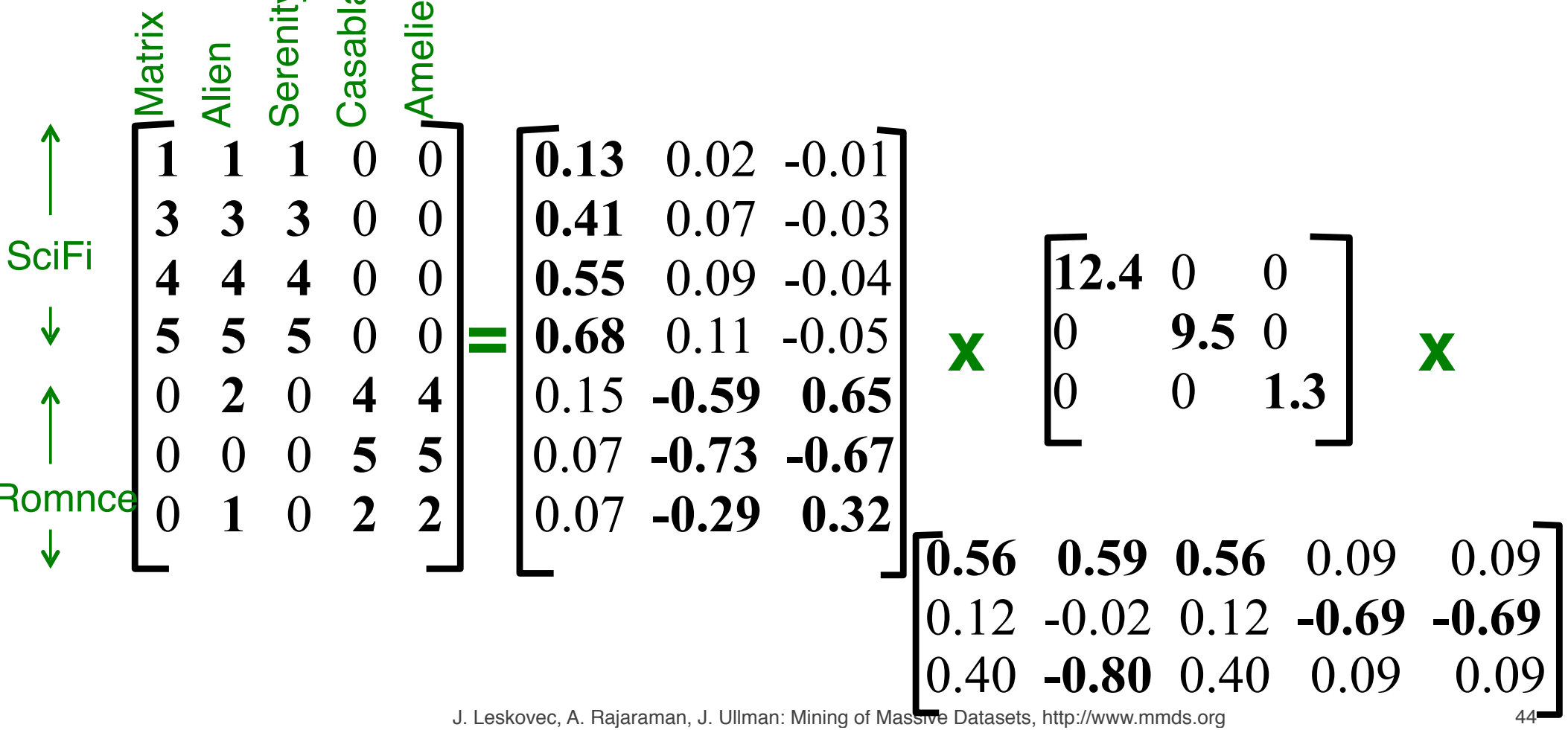
B is best approximation of A



Example of SVD & Conclusion

Case study: How to query?

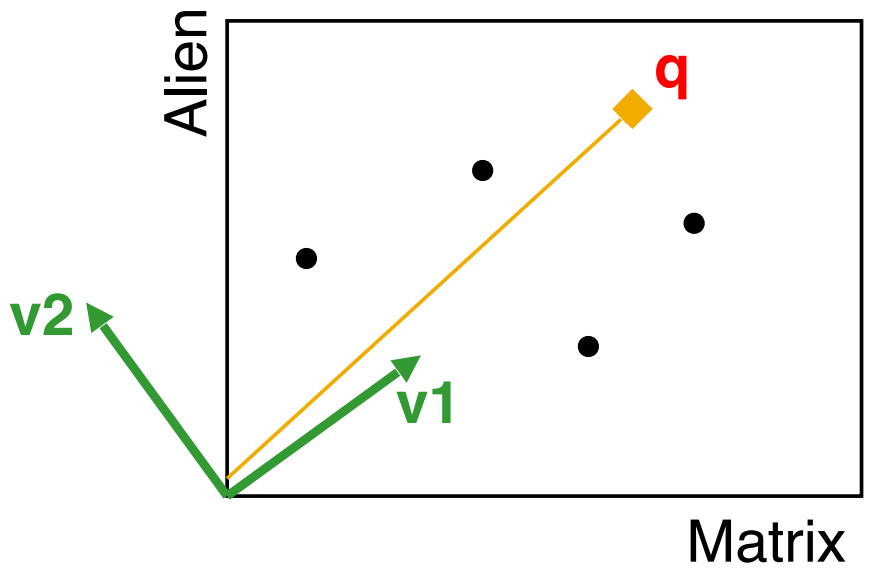
- Q: Find users that like 'Matrix'
- A: Map query into a 'concept space' – how?



Case study: How to query?

- **Q: Find users that like 'Matrix'**
- **A: Map query into a 'concept space' – how?**

$$q = \begin{bmatrix} \text{Matrix} & \text{Alien} & \text{Serenity} & \text{Casablanca} & \text{Amelie} \\ 5 & 0 & 0 & 0 & 0 \end{bmatrix}$$



Project into concept space:
Inner product with each 'concept' vector $\mathbf{v}_i$

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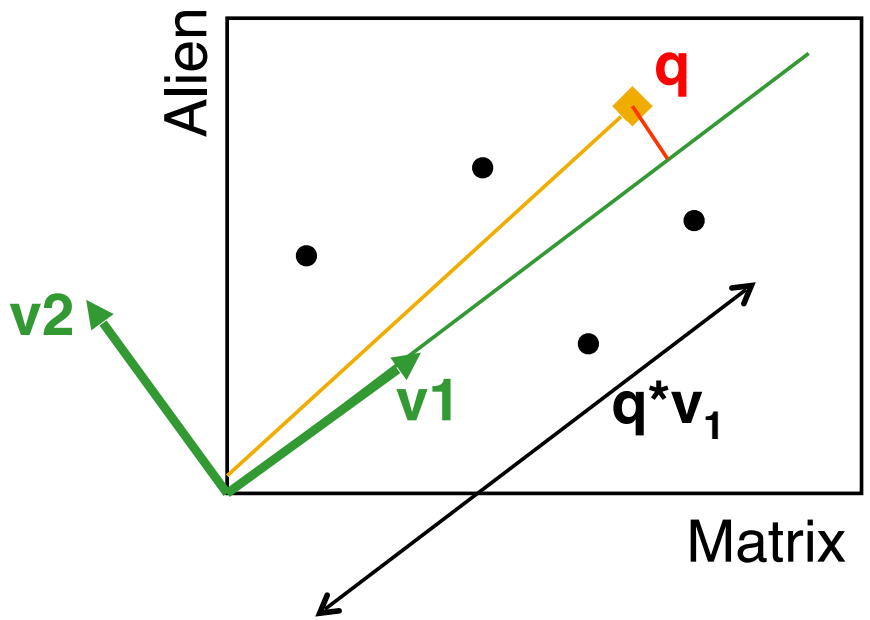


Case study: How to query?

Compactly, we have:

$$q_{concept} = q\ V$$

E.g.:

$$q = \begin{bmatrix} \text{Matrix} & \text{Alien} & \text{Serenity} & \text{Casablanca} & \text{Amelie} \\ 5 & 0 & 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} 0.56 & 0.12 \\ 0.59 & -0.02 \\ 0.56 & 0.12 \\ 0.09 & -0.69 \\ 0.09 & -0.69 \end{bmatrix} = \begin{bmatrix} \text{SciFi-concept} \\ \downarrow \\ 2.8 & 0.6 \end{bmatrix}$$

movie-to-concept similarities (V)

Case study: How to query?

- How would the user *d* that rated ('Alien', 'Serenity') be handled?

  $d_{concept} = d\ V$

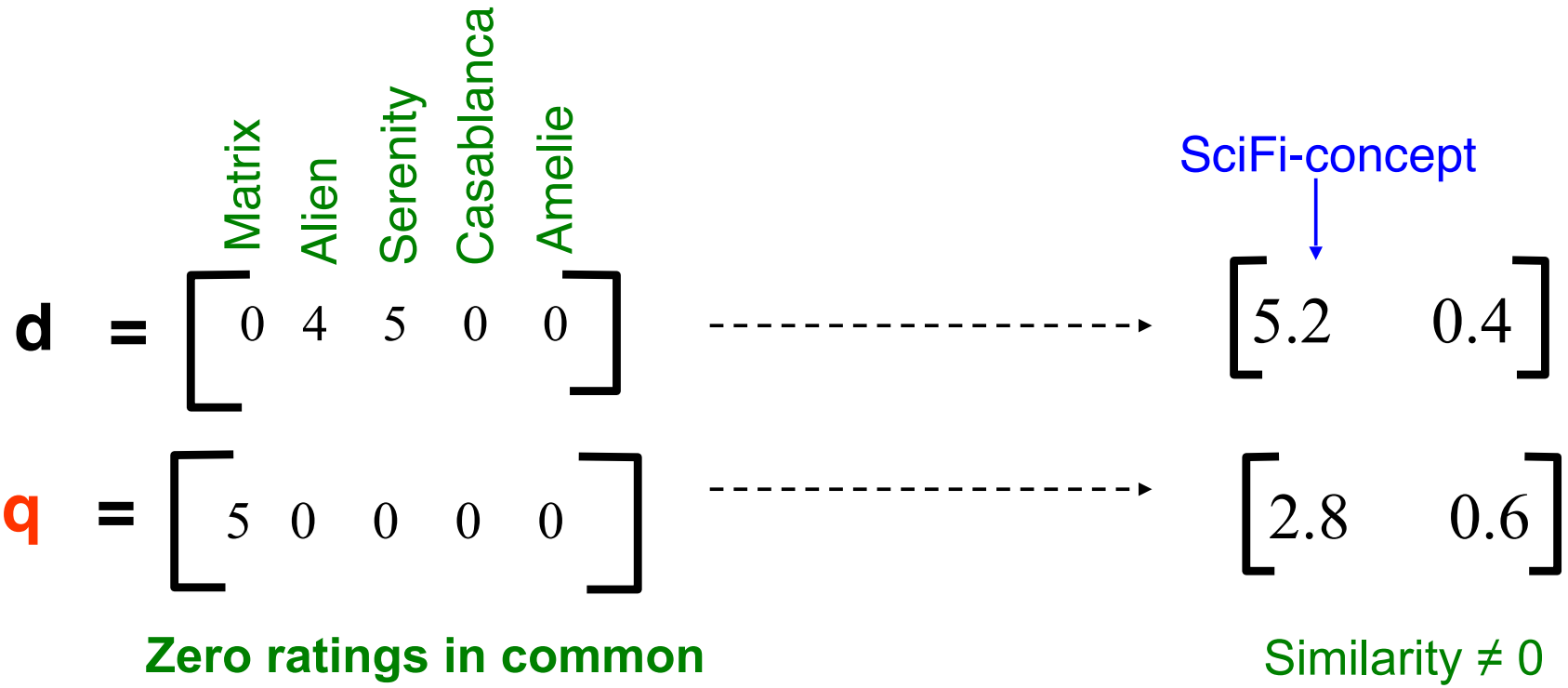
E.g.:

$$q = \begin{bmatrix} \text{Matrix} & \text{Alien} & \text{Serenity} & \text{Casablanca} & \text{Amelie} \\ 0 & 4 & 5 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} 0.56 & 0.12 \\ 0.59 & -0.02 \\ 0.56 & 0.12 \\ 0.09 & -0.69 \\ 0.09 & -0.69 \end{bmatrix} = \begin{bmatrix} \text{SciFi-concept} \\ \downarrow \\ 5.2 & 0.4 \end{bmatrix}$$

movie-to-concept similarities (V)

Case study: How to query?

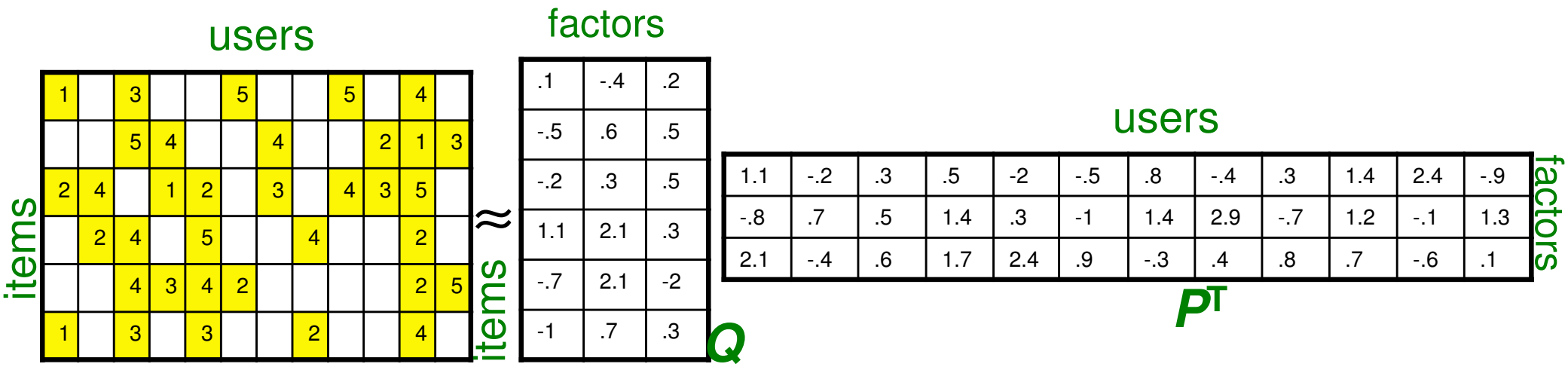
- **Observation:** User ***d*** that rated ('*Alien*', '*Serenity*') will be **similar** to user **q** that rated ('*Matrix*'), although ***d*** and **q** have **zero ratings in common**!



SVD

Our goal is to find P and Q such that:

$$\min_{P,Q} \sum_{(i,x)\in R} \left(r_{xi} - q_i \cdot p_x\right)^2$$



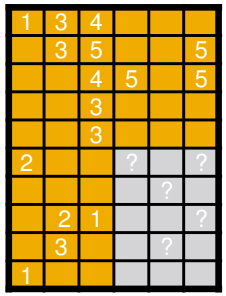
1		3			5			5		4	
		5	4			4			2	1	3
2	4		1	2		3		4	3	5	
	2	4		5			4			2	
		4	3	4	2					2	5
1		3		3			2			4	

.1	-.4	.2
-.5	.6	.5
-.2	.3	.5
1.1	2.1	.3
-.7	2.1	-2
-1	.7	.3

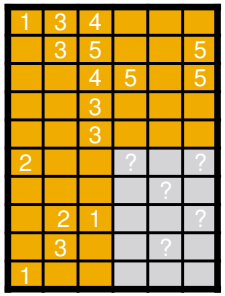
1.1	-.2	.3	.5	-2	-.5	.8	-.4	.3	1.4	2.4	-.9
-.8	.7	.5	1.4	.3	-1	1.4	2.9	-.7	1.2	-.1	1.3
2.1	-.4	.6	1.7	2.4	.9	-.3	.4	.8	.7	-.6	.1

- **Want to minimize SSE for unseen test data**
- **Idea: Minimize SSE on training data**
  - Want large ***k*** (# of factors) to capture all the signals
  - But, **SSE** on **test data** begins to rise for ***k*** > 2

- This is a classical example of **overfitting:**
  - With too much freedom (too many free parameters) the model starts fitting noise
    - That is it fits too well the training data and thus **not generalizing** well to unseen test data



- **To solve overfitting we introduce regularization:**

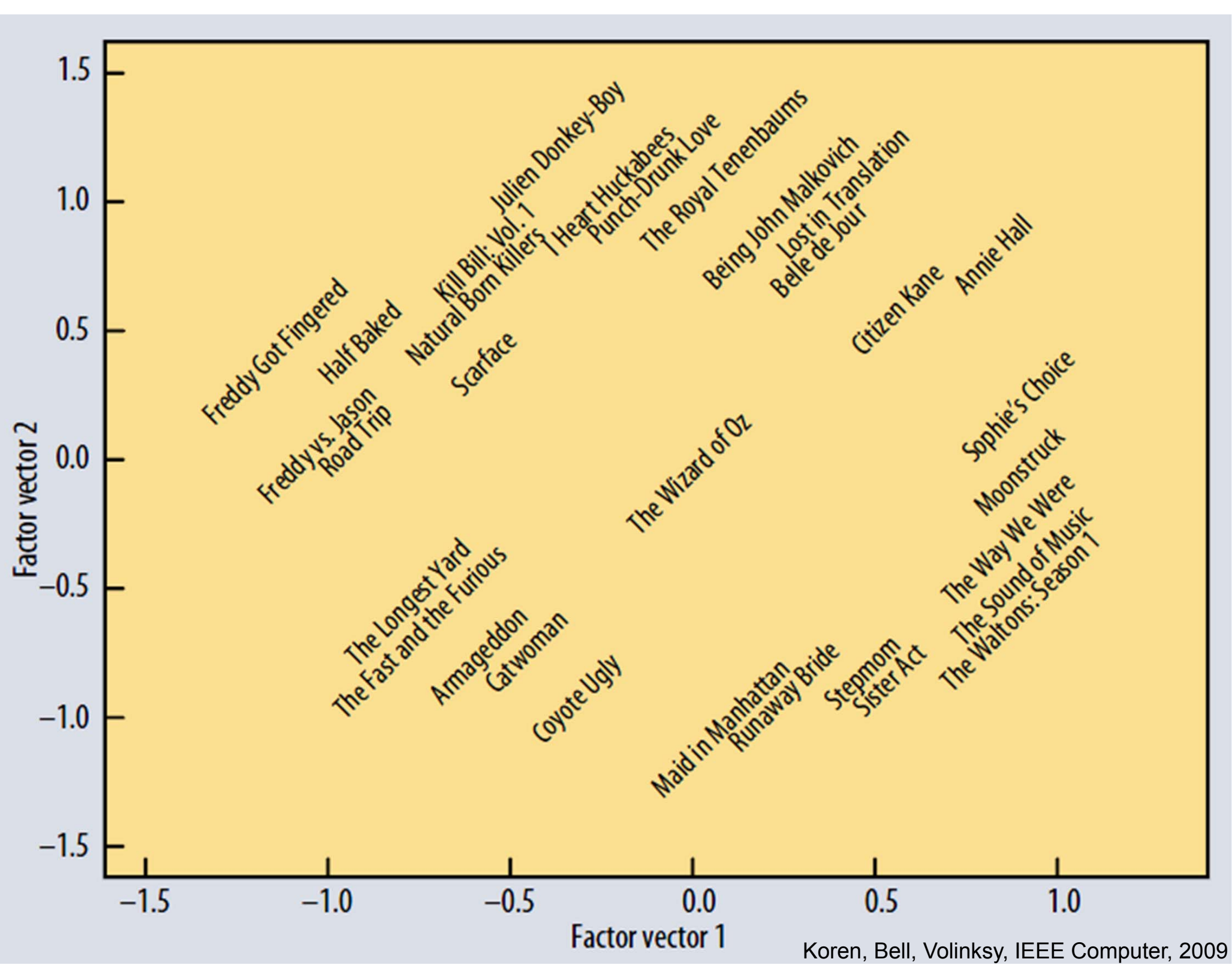


  - Allow rich model where there are sufficient data
  - Shrink aggressively where data are scarce

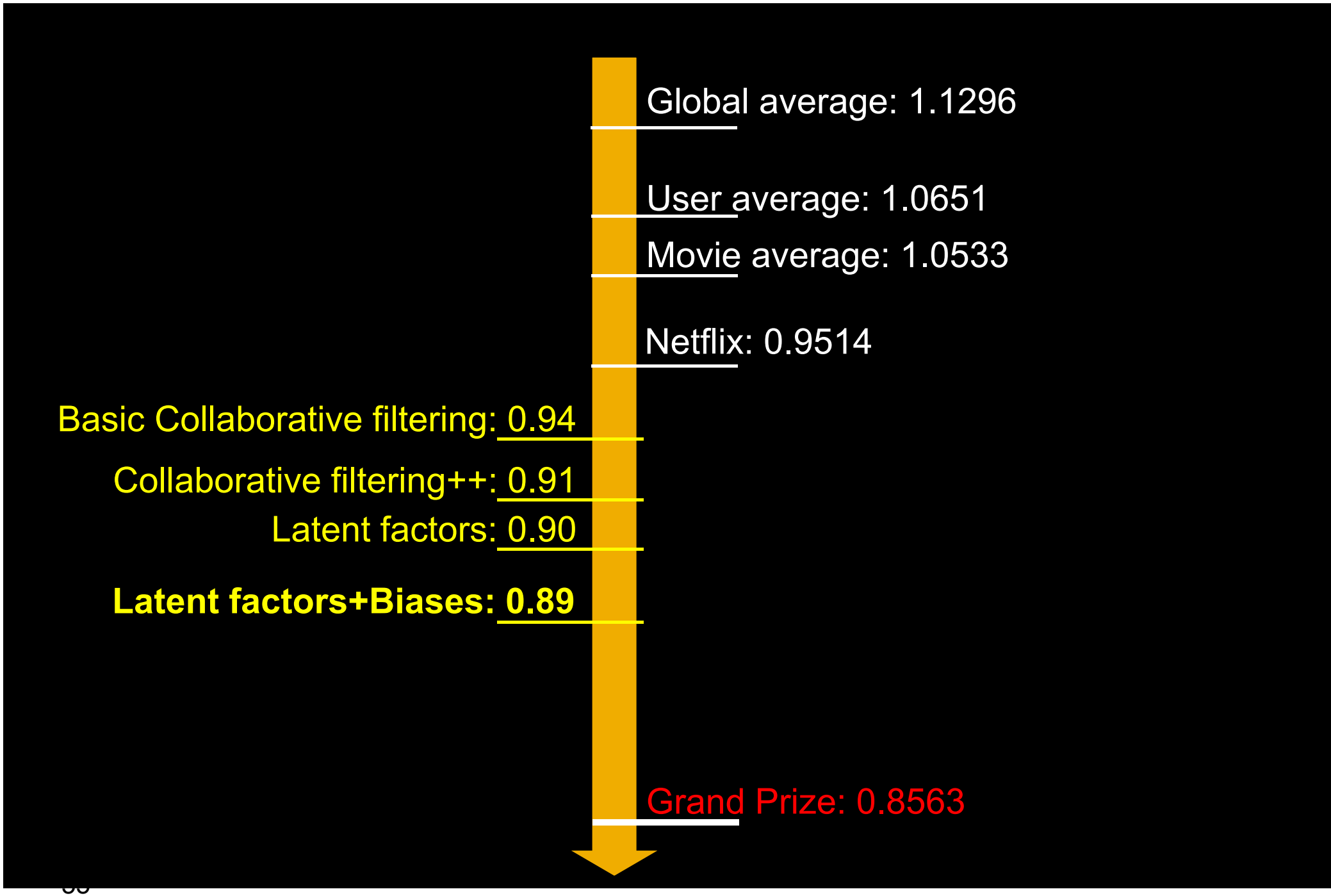
$$\min_{P,Q} \sum_{training} \underbrace{(r_{xi} - q_i p_x)^2}_{\text{“error”}} + \underbrace{\left[\lambda_1 \sum_x \|p_x\|^2 + \lambda_2 \sum_i \|q_i\|^2 \right]}_{\text{“length”}}$$

λ_1, λ_2 … user set regularization parameters

Note: We do not care about the “raw” value of the objective function, but we care in P,Q that achieve the minimum of the objective



Koren, Bell, Volinksy, IEEE Computer, 2009


Global average: 1.1296
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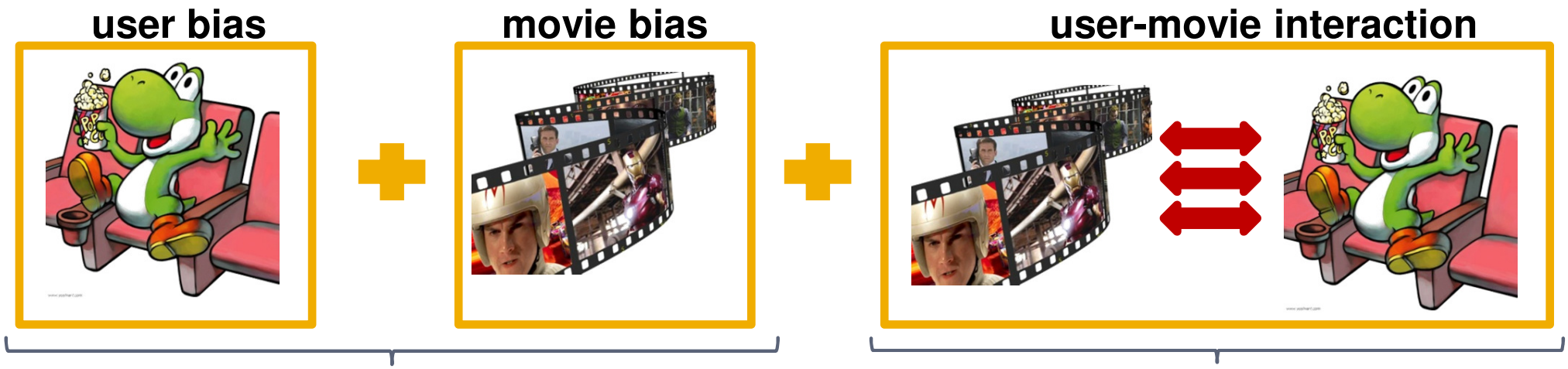
Movie average: 1.0533
Netflix: 0.9514
Basic Collaborative filtering: 0.94
Collaborative filtering++: 0.91
Latent factors: 0.90
Latent factors+Biases: 0.89
Grand Prize: 0.8563

Modeling Biases and Interactions



Baseline predictor

- Separates users and movies
- Benefits from insights into user's behavior
- Among the main practical contributions of the competition

User-Movie interaction

- Characterizes the matching between users and movies
- Attracts most research in the field
- Benefits from algorithmic and mathematical innovations

- μ = overall mean rating
- b_x = bias of user x
- b_i = bias of movie i

Baseline Predictor

- We have expectations on the rating by user ***x*** of movie ***i***, even without estimating ***x***'s attitude towards movies like ***i***

- Rating scale of user ***x***
- Values of other ratings user gave recently (day-specific mood, anchoring, multi-user accounts)

- (Recent) popularity of movie ***i***
- Selection bias; related to number of ratings user gave on the same day ("frequency")

Putting it all together

$$r_{xi} = \mu + b_x + b_i + q_i \cdot p_x$$

- μ: Overall mean rating
- b_x: Bias for user ***x***
- b_i: Bias for movie ***i***
- $q_i \cdot p_x$: User-Movie interaction

- **Example:**
  - Mean rating: $\mu = 3.7$
  - You are a critical reviewer: your ratings are 1 star lower than the mean: $b_x = -1$
  - Star Wars gets a mean rating of *0.5* higher than average movie: $b_i = +0.5$
  - Predicted rating for you on Star Wars:
    $= 3.7 - 1 + 0.5 = 3.2$

Fitting the new model

- **Solve:**

$$\min_{Q,P} \sum_{(x,i)\in R} \left(r_{xi} - (\mu + b_x + b_i + q_i\, p_x)\right)^2$$

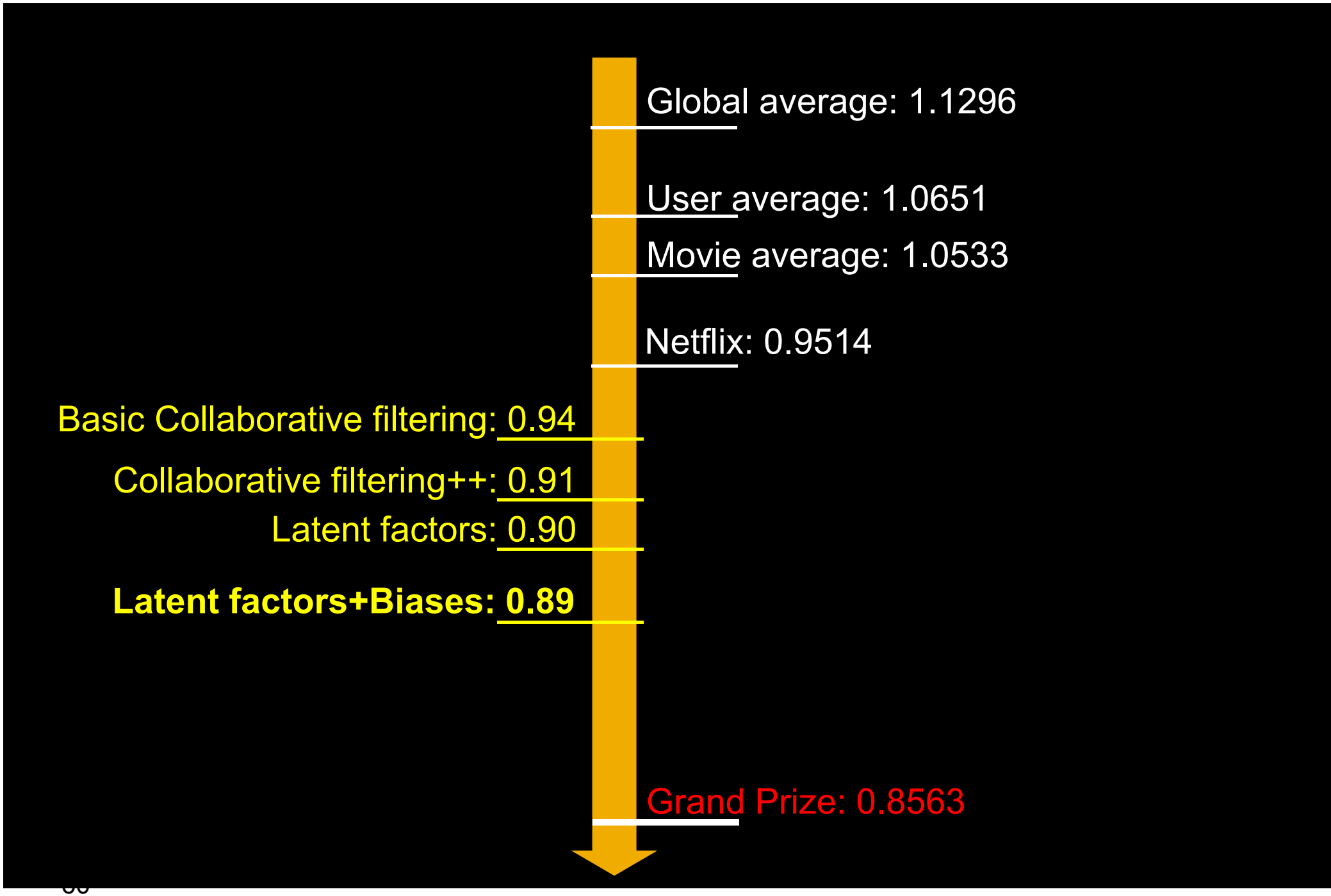
goodness of fit

$$+\left(\lambda_1 \sum_i \|q_i\|^2 + \lambda_2 \sum_x \|p_x\|^2 + \lambda_3 \sum_x \|b_x\|^2 + \lambda_4 \sum_i \|b_i\|^2\right)$$

regularization

λ is selected via grid-search on a validation set

- **Stochastic gradient decent to find parameters**
  - **Note:** Both biases $\boldsymbol{b_x}$, $\boldsymbol{b_i}$ as well as interactions $\boldsymbol{q_i}$, $\boldsymbol{p_x}$ are treated as parameters (we estimate them)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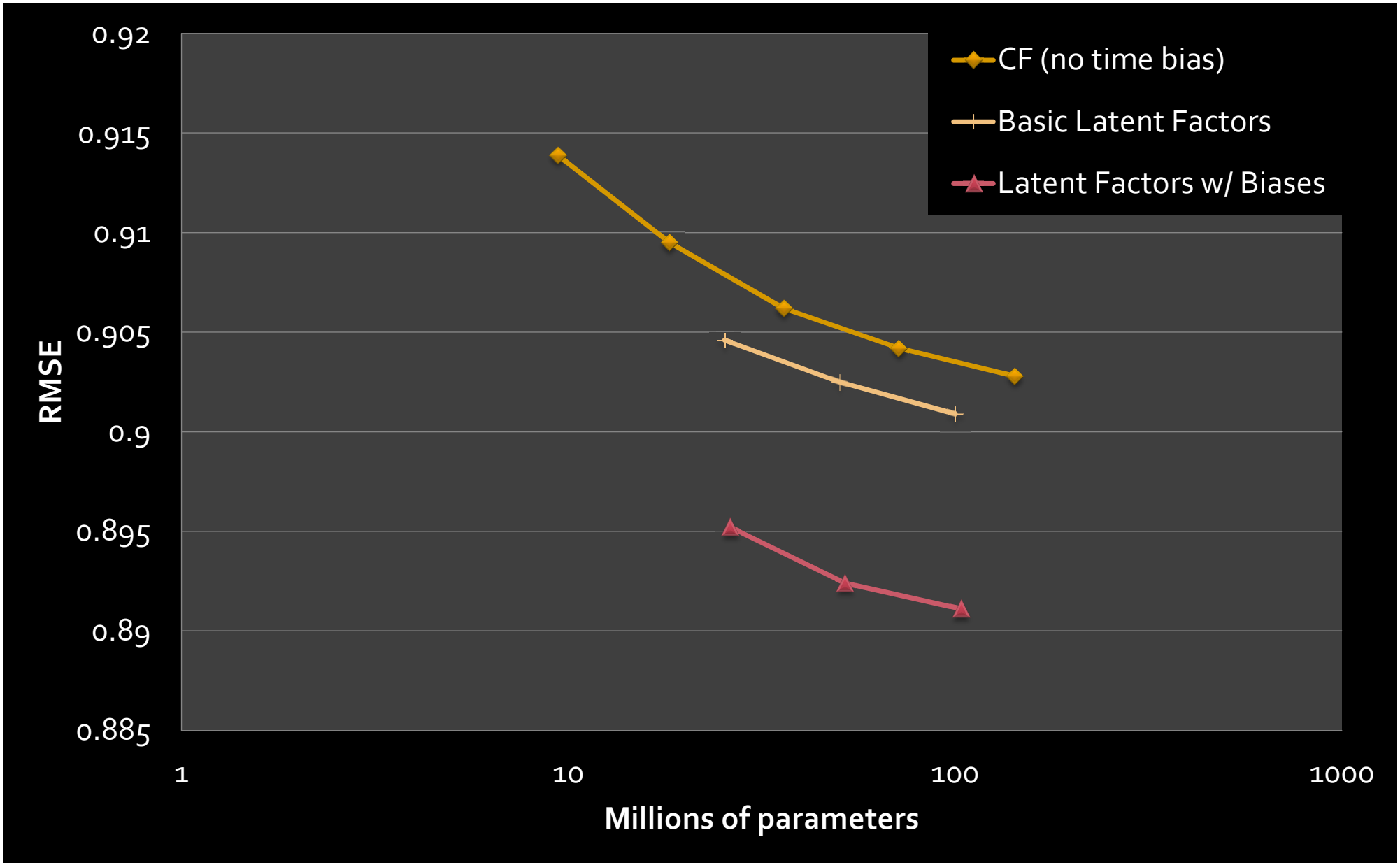
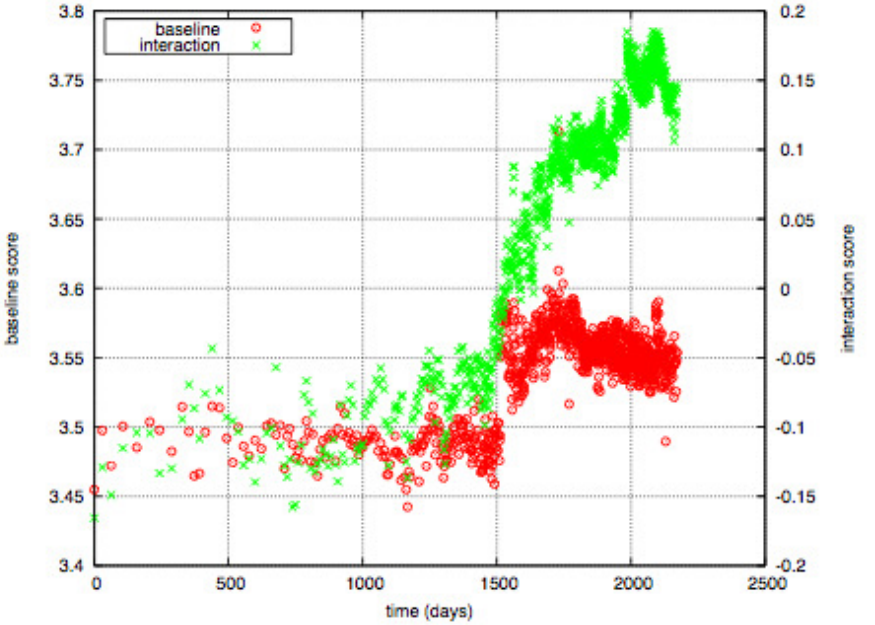
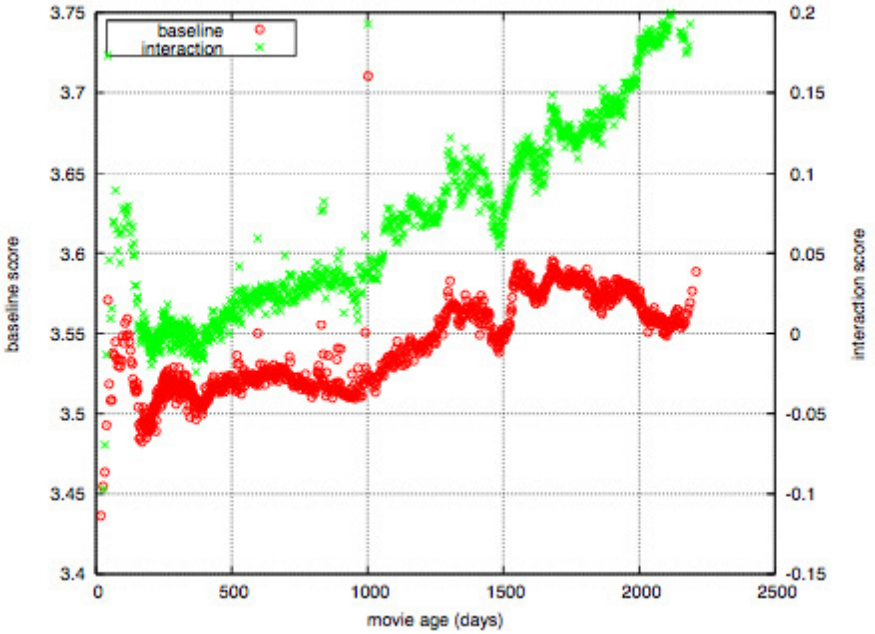

0.92
0.915
0.91
0.905
0.9
0.895
0.89
0.885
RMSE
1
10
100
1000
Millions of parameters
CF (no time bias)
Basic Latent Factors
Latent Factors w/ Biases

Temporal Biases of Users

- **Sudden rise in the average movie rating** (early 2004)
  - Improvements in Netflix
  - GUI improvements
  - Meaning of rating changed
- **Movie age**
  - Users prefer new movies without any reasons
  - Older movies are just inherently better than newer ones





Y. Koren, Collaborative filtering with temporal dynamics, KDD '09

Temporal Bias and Users

- **Original model:**

$$r_{xi} = \mu + b_x + b_i + q_i \cdot p_x$$

- **Add time dependence to biases:**

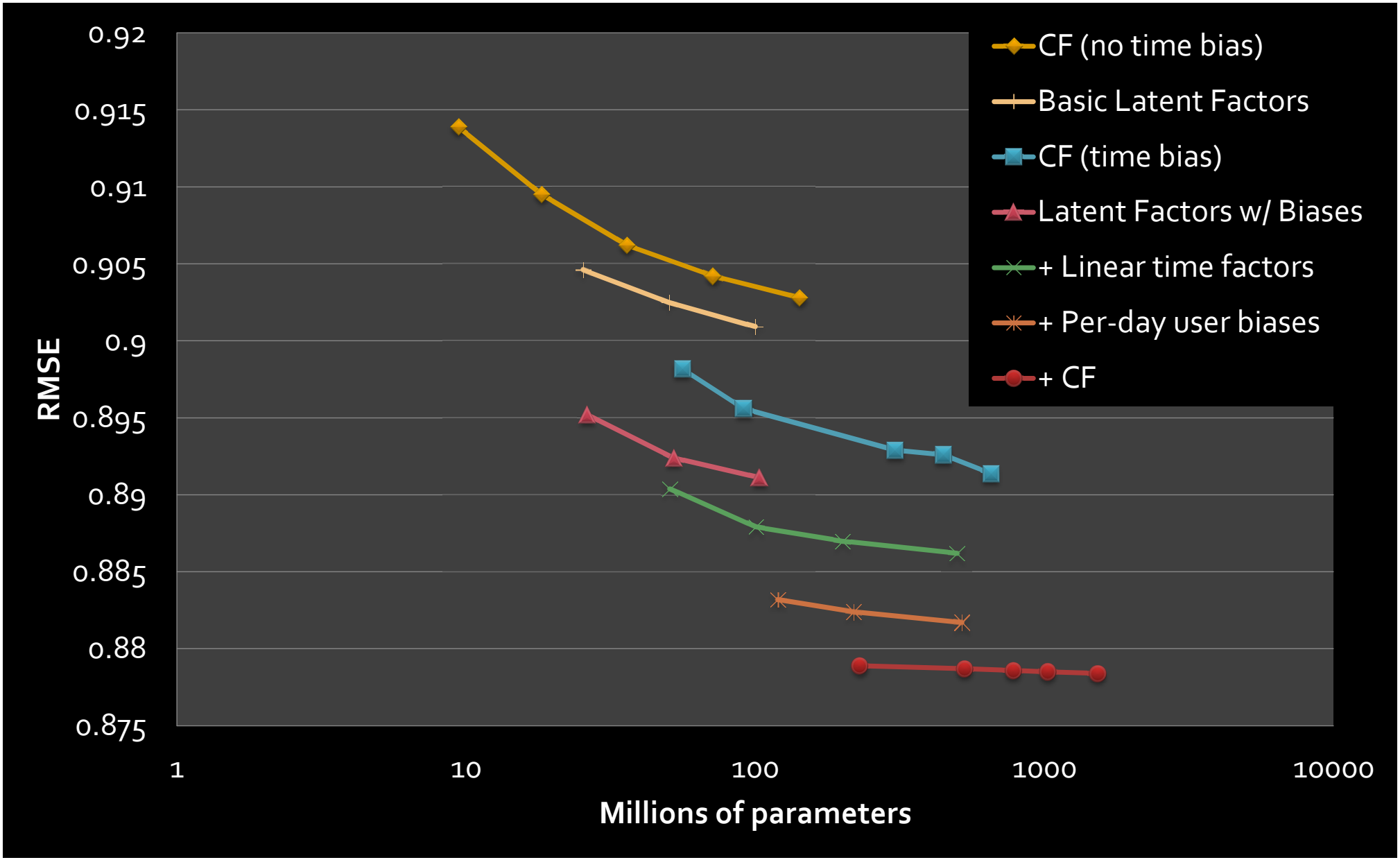
$$r_{xi} = \mu + b_x(t) + b_i(t) + q_i \cdot p_x$$

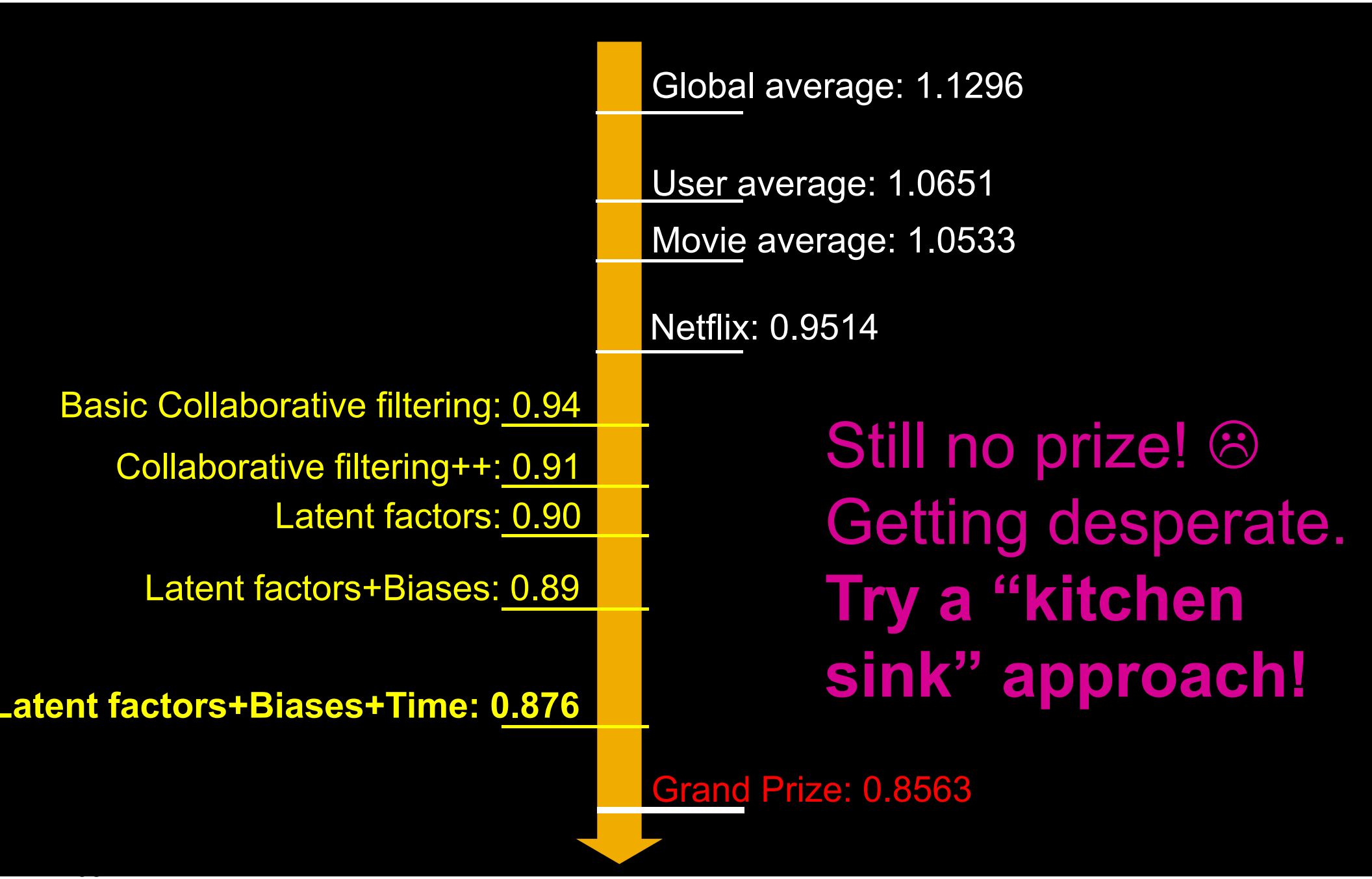
  - Make parameters b_x and b_i to depend on time
  - **(1)** Parameterize time-dependence by linear trends
    (2) Each bin corresponds to 10 consecutive weeks

$$b_i(t) = b_i + b_{i,\mathrm{Bin}(t)}$$

- **Add temporal dependence to factors**
  - $p_x(t)$... user preference vector on day t

Y. Koren, Collaborative filtering with temporal dynamics, KDD '09


0.92
0.915
0.91
0.905
0.9
0.895
0.89
0.885
0.88
0.875
RMSE
1
10
100
1000
10000
Millions of parameters
CF (no time bias)
Basic Latent Factors
CF (time bias)
Latent Factors w/ Biases
+ Linear time factors
+ Per-day user biases
+ CF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Latent factors: 0.90
Latent factors+Biases: 0.89
Latent factors+Biases+Time: 0.876
Grand Prize: 0.8563
Still no prize! ☹
Getting desperate.
Try a "kitchen sink" approach!