2、

$$y[n] = y_1[n] + y_2[n], \quad y_1[n] = x[n] * 0.8^n u[n], \quad y_2[n] = x[n] * 0.5^n [n]$$

$$\Rightarrow y_1[n] \xrightarrow{z} Y_1(z) = X(z) \cdot \frac{1}{1-0.8z^{-1}} \qquad y_2[n] \xrightarrow{z} Y_2(z) = X(z) \cdot \frac{1}{1-0.5z^{-1}}$$

$$Y(z) = X(z) \cdot \left( \frac{1}{1-0.8z^{-1}} + \frac{1}{1-0.5z^{-1}} \right)$$

$$= X(z) \cdot \frac{2-1.3z^{-1}}{(1-0.8z^{-1})(1-0.5z^{-1})}$$

$$Y(z) \cdot (1 - 1.3z^{-1} + 0.4z^{-2}) = X(z) \cdot (2-1.3z^{-1})$$

$$\Rightarrow y[n] = 1.3y[n-1] - 0.4y[n-2] + 2x[n] - 1.3x[n-1] \ \#$$

3、

Advantages

① It simplifies convolution to multiplication, which saves computing resource and time significantly.

② We can do spectrum analysis by Fourier Transform.

Problems

① not real operation ( more calculation )

② irrational number multiplication

4、

suppose $X[n]$ start with $n=0$ & end with $n=1999$

$\Rightarrow X[m]$ start with $m=0$ & end with $m=1999$

$FT \Rightarrow X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi f t} dt$ for finite length $x(t) \Rightarrow FS: X[m] = \frac{1}{T}\int_T x(t) e^{-j2\pi m \Delta f t} dt$

Discretize FS $\Rightarrow$ $t = n\Delta t$, $dt = \Delta t$, $T = N \cdot \Delta t$

$$\Rightarrow X[m] = \frac{1}{N \cdot \Delta t} \cdot \sum_{n=0}^{N-1} \cdot x(n\Delta t) \cdot e^{-j2\pi m \Delta f \cdot n\Delta t} \Delta t = \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-j2\pi n m \Delta t \Delta f}$$

for FFT, $X[m]$ has N points, too

$\Rightarrow e^{-j2\pi n m \Delta t \Delta f}$ should repeats every N points for $n, m$ $\Rightarrow \Delta f \cdot \Delta t = \frac{1}{N}$ should be satisfied.

$$X[m] = \frac{1}{N} \cdot \sum_{n=0}^{N-1} x[n] \cdot e^{-j\frac{2\pi m n}{N}}, \quad m = 0, 1 \cdots N-1 \quad N = 2000$$

$\Rightarrow$ it is DFT with a scaling $N$ on amplitude ( DFT: $X[m] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N} \cdot n \cdot m}$ $m = 0, 1, 2 \cdots N-1$ )

$\Rightarrow \Delta t \cdot \Delta f = \frac{1}{N}$ $\Rightarrow \Delta f = \frac{1}{\Delta t \cdot N} = \frac{1}{0.002 \cdot 2000} = 0.25$ Hz $\Rightarrow$ $X[200]$, $X[1600]$ are correspond to

$f = 50$Hz, $400$Hz, respectively.

# other sol.:

$X[m]$ is equivalent to $X(f)$ sample with $\Delta f$ within $\left[-\frac{f_s}{2}, \frac{f_s}{2}\right]$ N points, $f_s = \frac{1}{\Delta t}$ $\Rightarrow \Delta f = \frac{\frac{f_s}{2} - (-\frac{f_s}{2})}{N} = \frac{1}{N \Delta t} = 0.25$

$\Rightarrow m = 160 \Rightarrow m \cdot \Delta f = 200 \cdot 0.25 = 50$Hz $\quad m = 1600 \Rightarrow m \cdot \Delta f = 1600 \cdot 0.25 = 400$Hz #

5.(a) Step invariance samples signal from the unit step response

$\Rightarrow$ in freq-domain, $H_u(f) = \frac{H(f)}{j2\pi f}$ $\Rightarrow$ Decrease the high-freq. part

$\Rightarrow$ reduce the aliasing effect since high-freq. part is much smaller

(b)

Bilinear Transform do Laplace Transform on an analog response $h_a(t)$, which is $H_a(s)$, then design a digital filter by transforming $H_a(s)$ to $H(z)$ by conformal mapping $s = c \cdot \frac{1 - z^{-1}}{1 + z^{-1}}$.

By doing so, we maps $H_a(jf_{old}) \to H_a(e^{-j2\pi \frac{f_{new}}{f_s}})$, $f_{new} = \frac{f_s}{\pi} \cdot \tan^{-1}(\frac{2\pi}{c} f_{new})$

since $\tan^{-1} x \in (-\frac{\pi}{2}, \frac{\pi}{2})$ $\Rightarrow f_{new} \in (-\frac{f_s}{2}, \frac{f_s}{2})$ $\Rightarrow$ aliasing effect will be fully avoided.

6.

(a)

(i) even

(ii) even

(iii) odd

(iv) even

(v) even

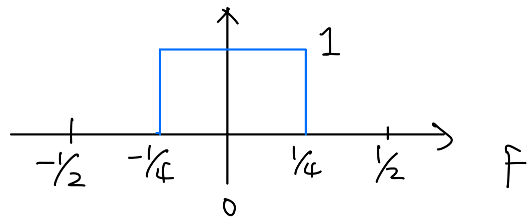
(vi) odd

(vii) even

2.

Desired Frequency Response $H_d(F)$

$H_d(F)$




$r[n] = h[n+k]$

$\Rightarrow R(F) = e^{-j2\pi kF} H(F)$

$$S[n] = \begin{cases} r[n] & n=0 \\ 2r[n] & 0<n\le k \end{cases}$$

$N=7 \Rightarrow k=3$

$$MSE = \frac{1}{f_s}\int_{-f_s/2}^{f_s/2} \left| R(f) - H_d(F) \right|^2 df = \int_{-\frac{1}{2}}^{\frac{1}{2}} \left| \sum_{n=0}^{k} S[n]\cos(2\pi nF) - H_d(F) \right|^2 dF$$

$$\frac{\partial MSE}{\partial S[n]} = 0 = 2\sum_{\tau=0}^{k} S[\tau]\int_{-\frac{1}{2}}^{\frac{1}{2}} \cos(2\pi nF)\cos(2\pi\tau F)\,dF - 2\int_{-\frac{1}{2}}^{\frac{1}{2}} H_d(F)\cos(2\pi nF)\,dF$$

$$\Rightarrow \sum_{\tau=0}^{3} S[\tau]\int_{-\frac{1}{2}}^{\frac{1}{2}} \cos(2\pi nF)\cdot\cos(2\pi\tau F)\,dF = \int_{-\frac{1}{4}}^{\frac{1}{4}} \cos(2\pi nF)\,dF$$

for $n=0 \Rightarrow S[0] = \int_{-\frac{1}{4}}^{\frac{1}{4}} dF = \frac{1}{2} = r[0]$

$n=1 \Rightarrow S[1]\cdot\frac{1}{2} = \int_{-\frac{1}{4}}^{\frac{1}{4}} \cos(2\pi F)\,dF = \frac{1}{\pi} = r[1]$

$S[2]\cdot\frac{1}{2} = \int_{-\frac{1}{4}}^{\frac{1}{4}} \cos(4\pi F)\,dF = 0 = r[2]$

$S[3]\cdot\frac{1}{2} = \int_{-\frac{1}{4}}^{\frac{1}{4}} \cos(6\pi F)\,dF = \frac{-1}{\pi} = r[3]$

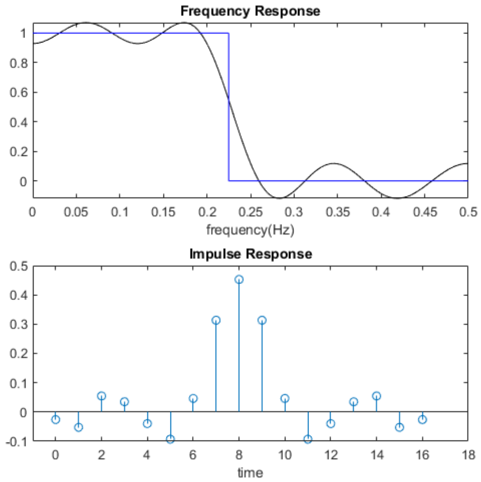
$r[0:3] = \left[\frac{1}{2}, \frac{1}{\pi}, 0, -\frac{1}{\pi}\right] \Rightarrow h[0:6] = \left[-\frac{1}{\pi}, 0, \frac{1}{\pi}, \frac{1}{2}, \frac{1}{\pi}, 0, -\frac{1}{\pi}\right]$ #

extra (3, 8)

$f_s = 40\,kHz \Rightarrow F = 0.2 \sim 0.3$ be abandoned

$\Rightarrow f = 8 \sim 12\,kHz$ are wasted.

1.




error of each iteration

1: 0.170681

2: 0.104844

3: 0.072793

4: 0.070772

5: 0.070768