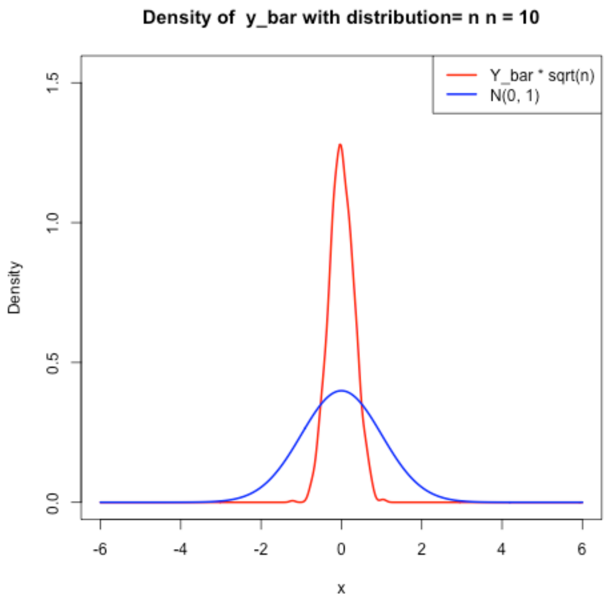
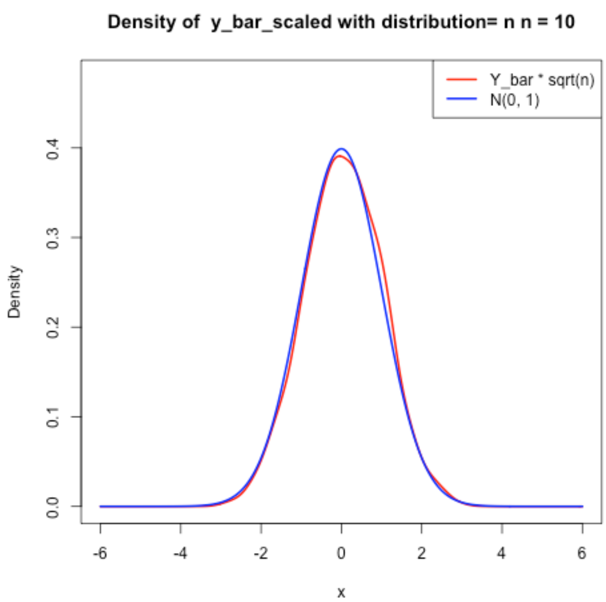
Problem 1.Outputs of the simulations

Case1: $Y_i \sim N(0,1)$ & n = 10

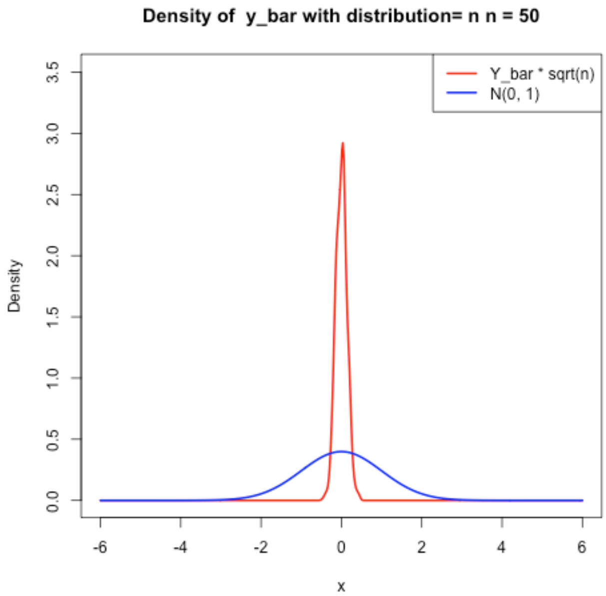
$\overline{Y}$



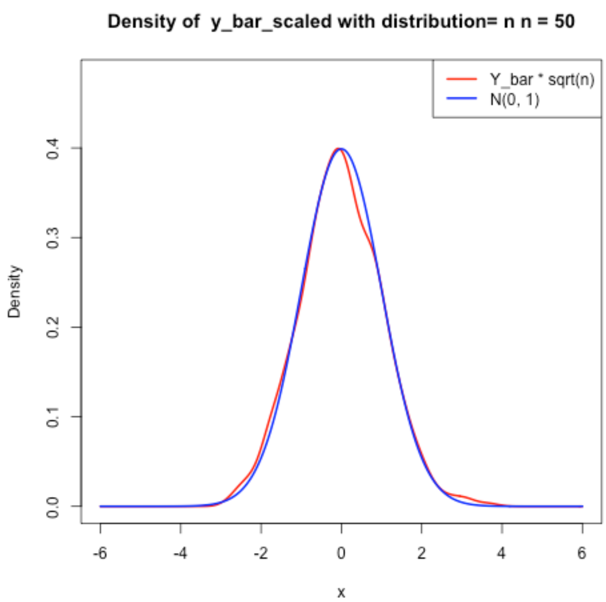
$n^{1/2}\overline{Y}$



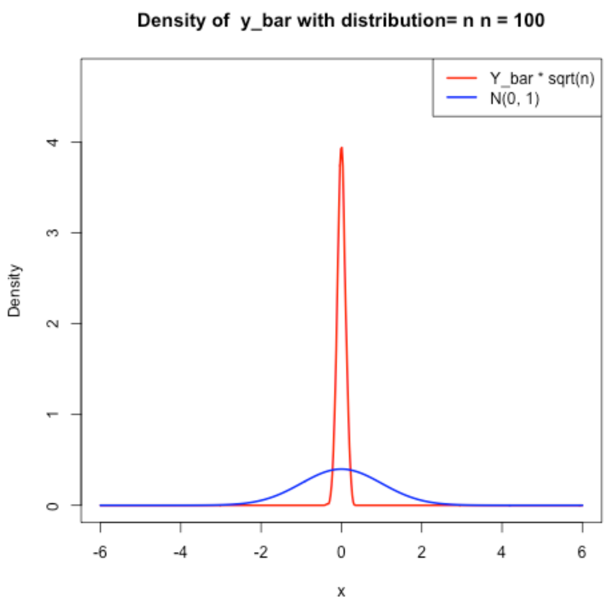
Case2: $Y_i \sim N(0,1)$ & n = 50
$\overline{Y}$



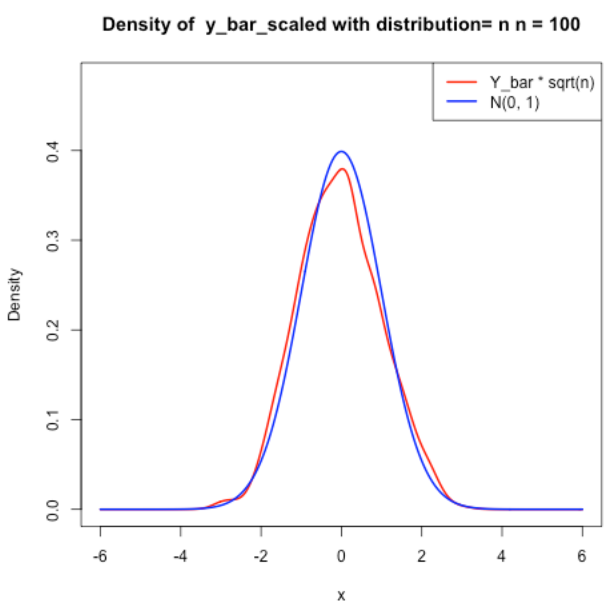
$n^{1/2}\overline{Y}$



Case3: $Y_i \sim N(0, 1)$ & n = 100
$\overline{Y}$

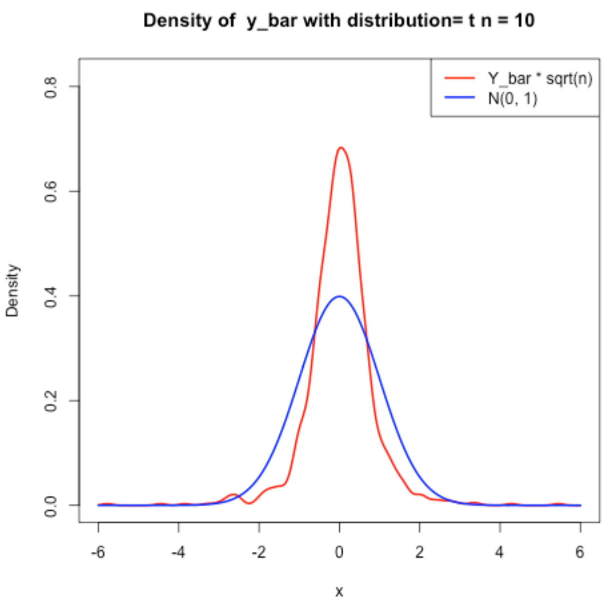


$n^{1/2}\overline{Y}$

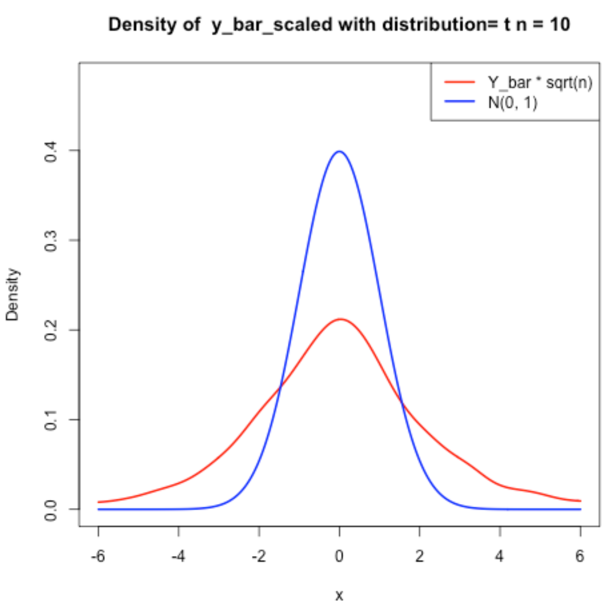


Case4: $Y_i \sim t(2)$ & n = 10

$\overline{Y}$

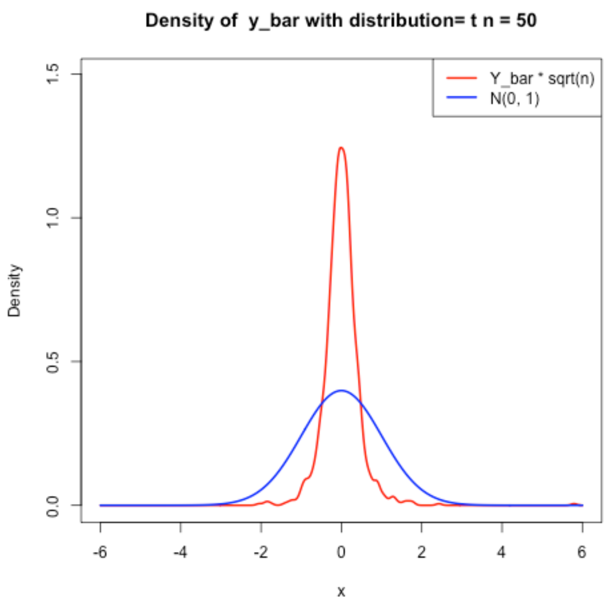


$n^{1/2}\overline{Y}$

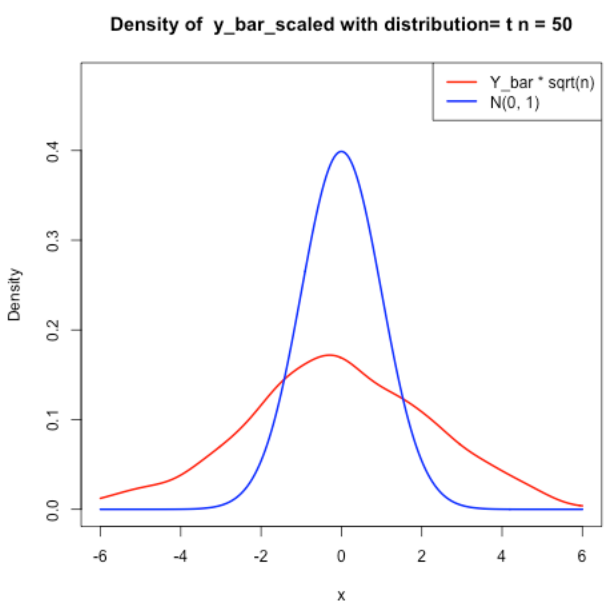


Case5: $Y_i \sim t(2)$ & n = 50

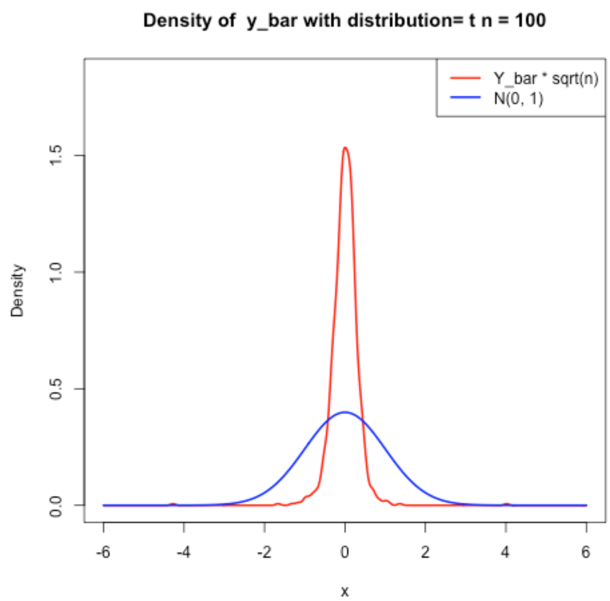
$\overline{Y}$



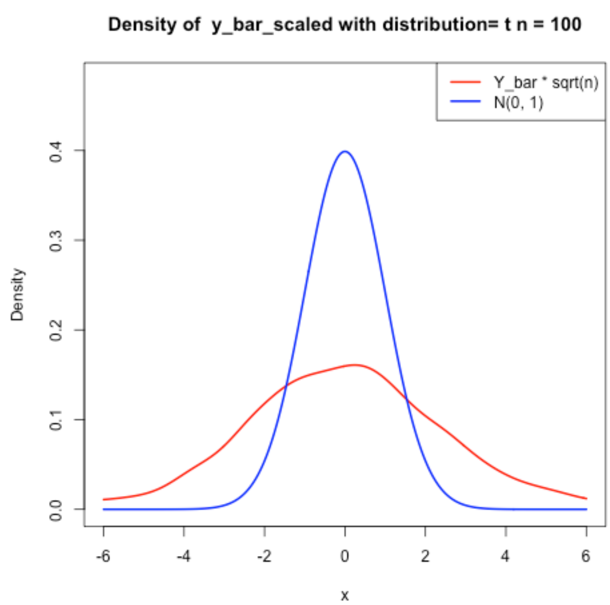
$n^{1/2}\overline{Y}$



Case6: $Y_i \sim t(2)$ & n = 100
$\bar{Y}$



$n^{1/2}\bar{Y}$



Results observation and Analysis

By the results above, we can easily find out that $\bar{Y}$ estimation of two types of random variables both converge to the expectation value of the Y_i as n becomes larger, which is just matching the **Large Sample Theorem**.

In addition, observing the scaled distribution $\sqrt{n} * \bar{Y}$, the case of $Y_i \sim N(0,1)$ successfully converge to normal distribution, just as central limit theorem expected. On the other hand, the case of $Y_i \sim t(2)$ just become flatter and flatter as n becomes larger, which is not matching to the **Central Limit Theorem**. In my inspection, It is due to the fact that the variance of the t-distribution is not defined when the degrees of freedom are 2 (according to the variance formula $\frac{q}{q-2}$, which yields an infinite result). Therefore, this distribution does not meet the conditions of the Central Limit Theorem, and hence the simulation results do not converge to a normal distribution.

Problem 2

1.By the code below, we can conduct the Wald's test on each $\hat{\beta}_j$ with null hypothesis $H_0: \hat{\beta}_j = 0$

```
x <- cbind(ones, x_dfy, x_infl, x_svar, x_tms, x_tbl, x_dfr, x_dp, x_ltr, x_ep, x_bmr, x_ntis)

#Do the Wald's test for each beta_hat_j with null hypothesis beta_hat_j = 0

fit <- lm(y ~ x - 1)
beta_hat <- coef(fit)
vcov_matrix <- vcov(fit)

wald_test_results <- data.frame(
  beta = beta_hat,
  wald_statistic = beta_hat^2 / diag(vcov_matrix),
  p_value = 1 - pchisq(beta_hat^2 / diag(vcov_matrix), df = 1)
)

#reject the null hypothesis if p_value < alpha
alpha <- 0.05
wald_test_results$reject_null <- wald_test_results$p_value < alpha

print(wald_test_results)
```

Results:

```
                beta wald_statistic      p_value reject_null
xones    0.215519353    12.09970254 0.0005042987        TRUE
xx_dfy  -1.167618067     1.58644273 0.2078351056       FALSE
xx_infl -0.379379508     0.34824283 0.5551095120       FALSE
xx_svar -0.101604035     0.06654763 0.7964313957       FALSE
xx_tms  -0.329207402     2.54984370 0.1103051673       FALSE
xx_tbl  -0.317573893     7.89488681 0.0049574740        TRUE
xx_dfr   0.275242786     3.43280454 0.0639134796       FALSE
xx_dp    0.045320259    13.44255962 0.0002459798        TRUE
xx_ltr   0.126357857     2.91990190 0.0874931830       FALSE
xx_ep   -0.002077709     0.05652437 0.8120761352       FALSE
xx_bmr   0.028790417     0.79661273 0.3721080411       FALSE
xx_ntis  0.070079631     0.30858752 0.5785482365       FALSE
```

Result shows that we reject H_0 only for intercept, tbl and dp.

2. Joint Wald's test on H_0: $\hat{\beta}_1 = 0$ & $\hat{\beta}_2 + \hat{\beta}_3 = 0$

```r
cat("\n====================================================\n")
#conducted joint wald's test with null hypothesis beta_hat_1 = 0 and beta_hat_2 + beta_hat_3 = 0
# Calculate the Wald statistic
R <- matrix(c(1, 0, 0, 0, 1, 1), nrow = 2, ncol = 3)
r <- c(0, 0)
R_beta <- R %*% beta_hat[c(1, 2, 3)]
R_vcov <- R %*% vcov_matrix[c(1, 2, 3), c(1, 2, 3)] %*% t(R)
wald_statistic <- t(R_beta) %*% solve(R_vcov) %*% (R_beta - r)

p_value = 1 - pchisq(wald_statistic, df = 2)

cat(paste("Wald statistic:", wald_statistic, "\n"))
cat(paste("P-value:", p_value, "\n"))
cat(paste("Reject null hypothesis if p_value < alpha: ", p_value < alpha, "\n"))
cat("====================================================\n")
```

Results:

```
====================================================
Wald statistic: 12.8672361246544
P-value: 0.00160662742794437
Reject null hypothesis if p_value < alpha:  TRUE
====================================================
```

Result shows that we reject the null hypothesis.