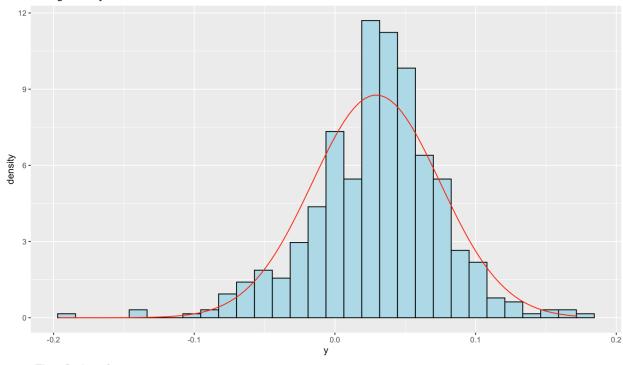
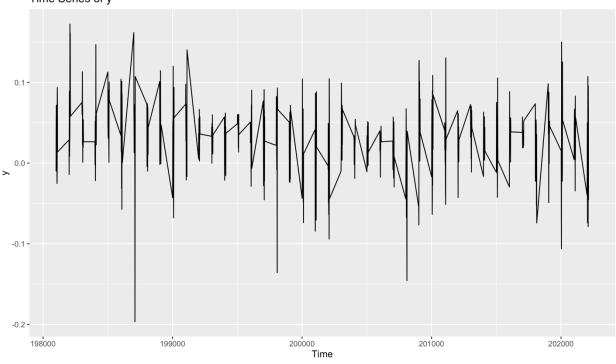
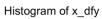
Q1. Data Visualization Histogram of y

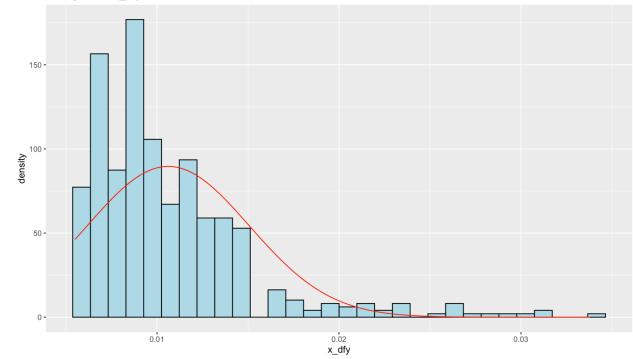




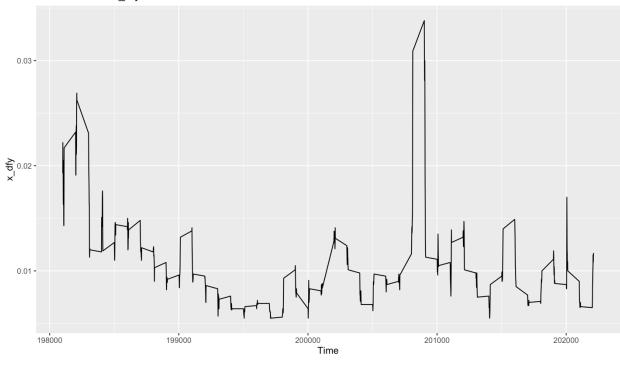


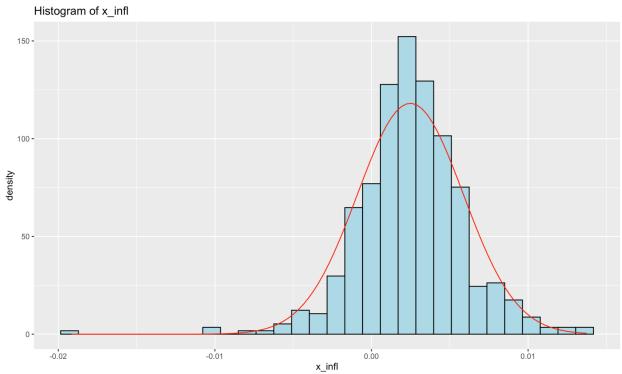


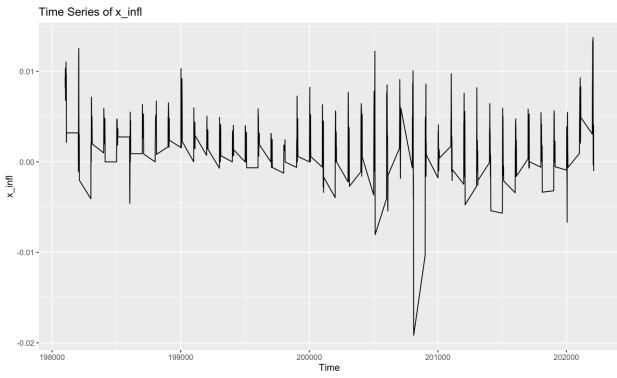


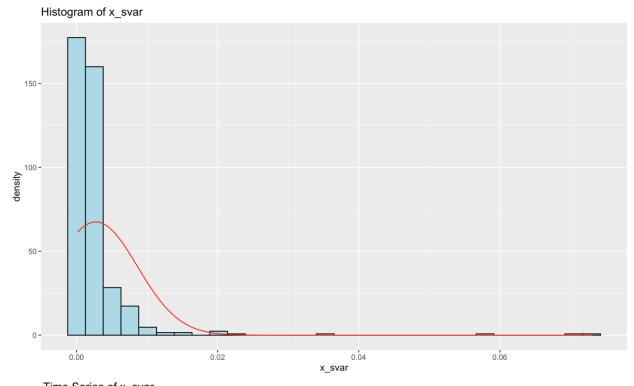


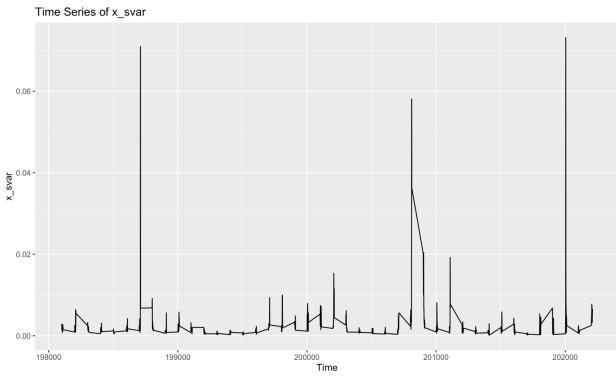
Time Series of x_dfy

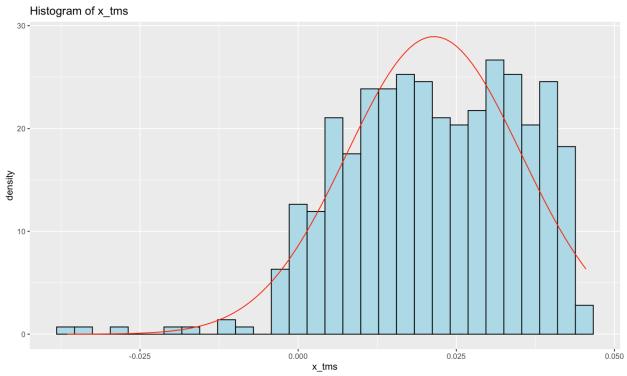


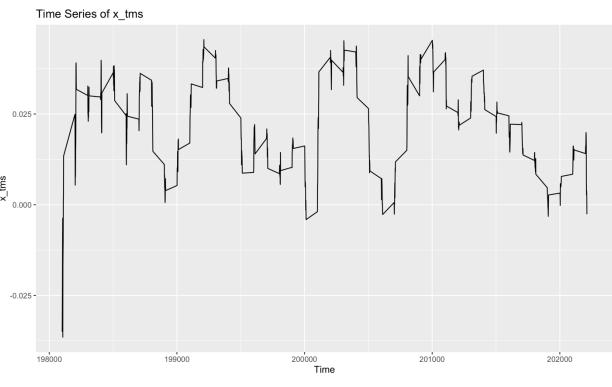


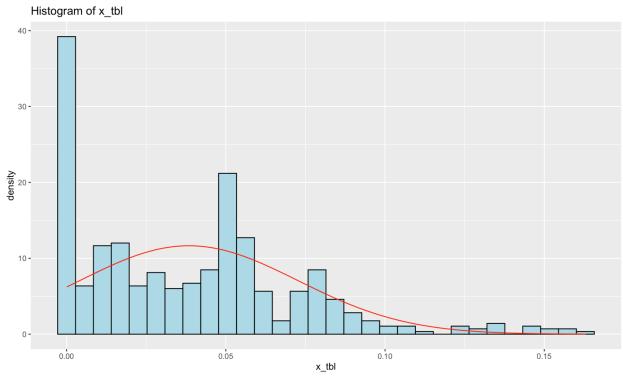


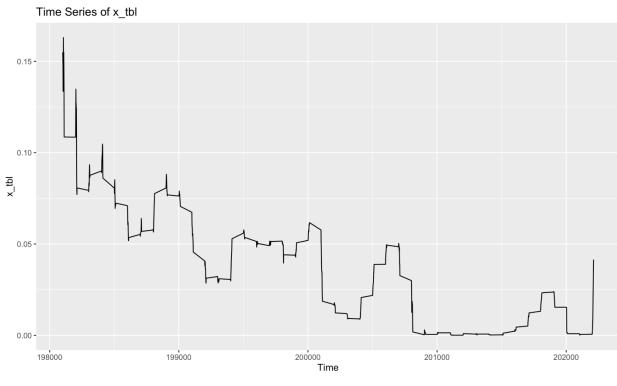


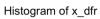


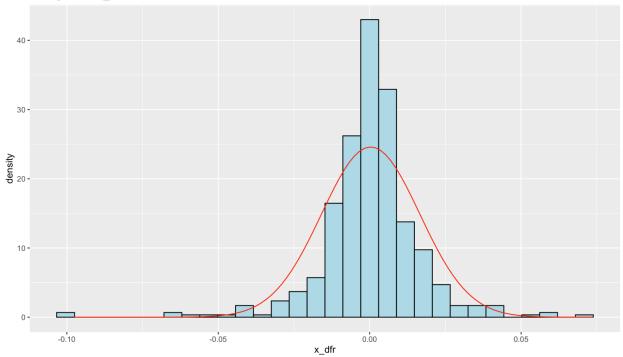




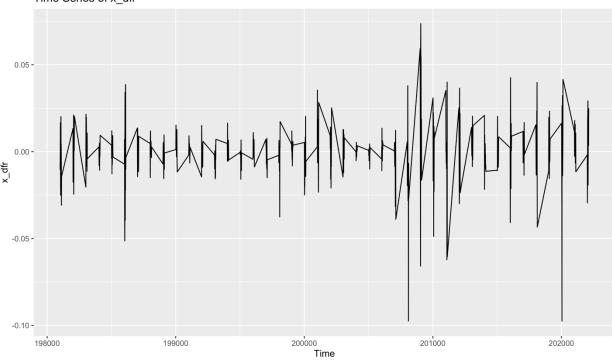




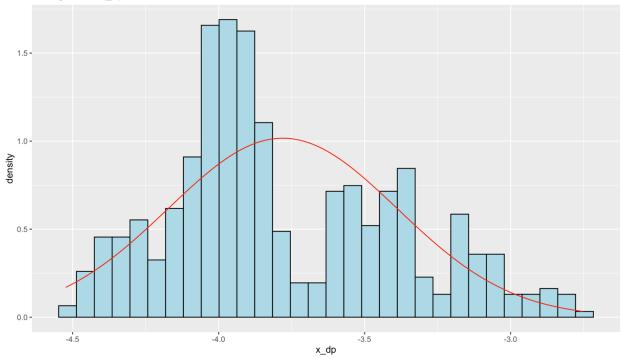




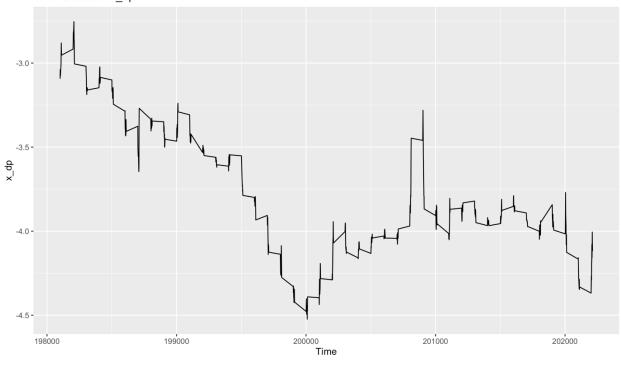
Time Series of x_dfr



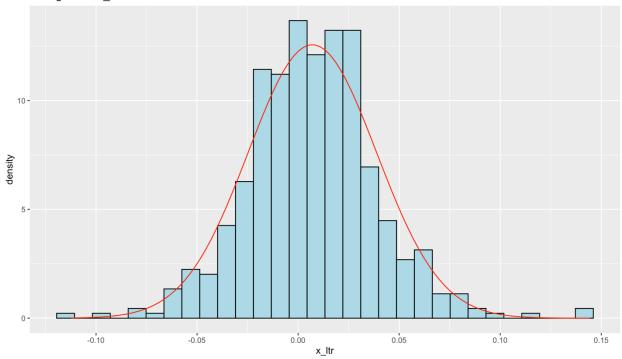




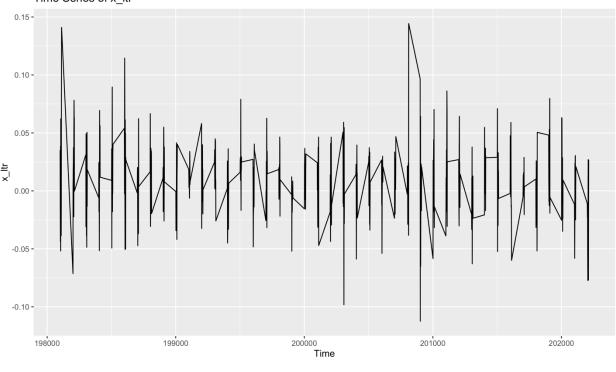
Time Series of x_dp

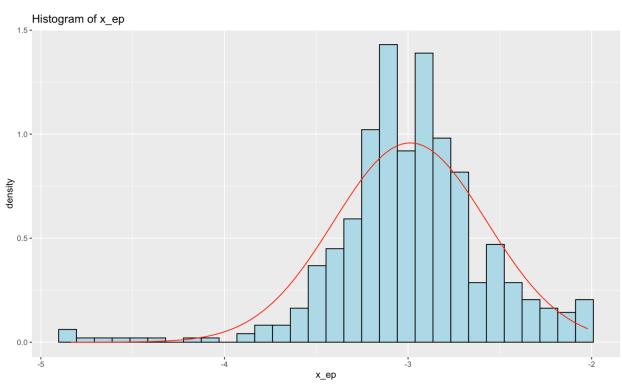


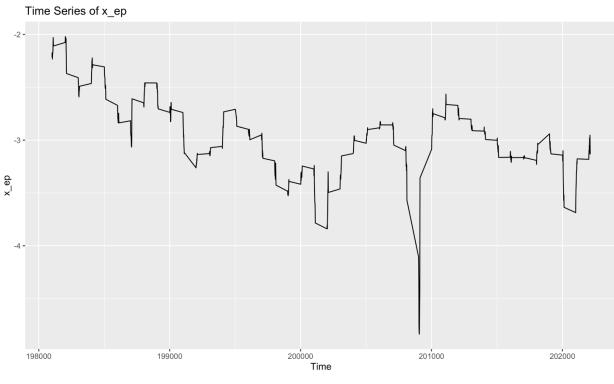


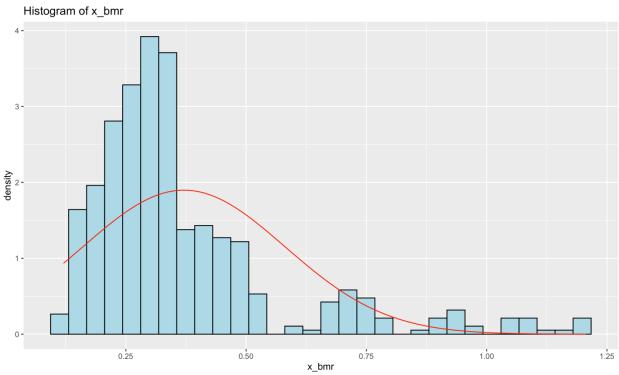


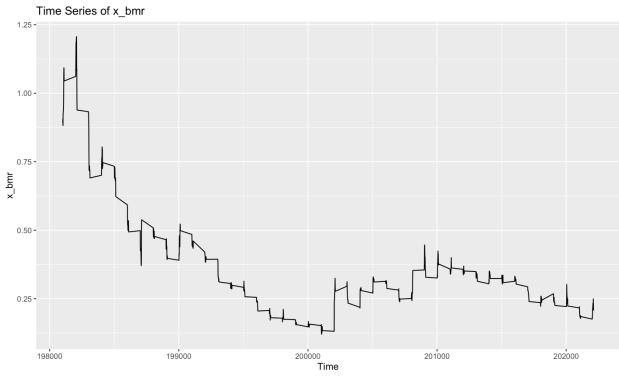












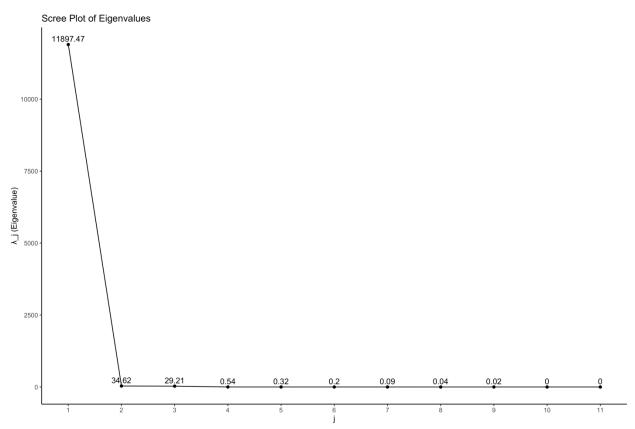
Q2.

Q2.1(calculate by R)
$$trace(X(X'X)^{-1}X') = 11$$
 Q2.2(calculate by R)

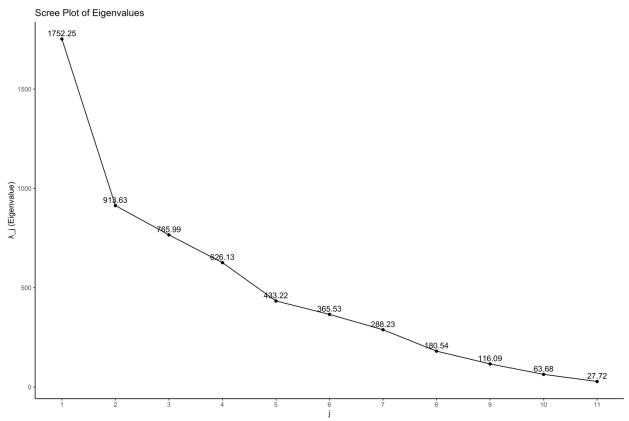
$$trace(I_n - X(X'X)^{-1}X') = 493$$

Q2-1: The result of
$$trace(X(X'X)^-1X')$$
 is: 11 Q2-2: The result of $trace(I_n - X(X'X)^-1X')$ is: 493

Q3.



Note: Eigenvalues are sorted in advance.



Note: Eigenvalues are sorted in advance.

Q5. (calculate by R)

Apply spectral decomposition:

$$H = \tilde{X}' \cdot \tilde{X}$$
$$A = H\Lambda^{-1}H'$$

Then we can verify the equation in the problem statement by R. The result is:

Q5. Verify that AA^{-1} equals to I_k by calculating the maximum element-wise difference: Maximum element-wise difference: 7.771561e-15. => The result is effectively the identity matrix within the specified tolerance 10^-12.

Q6. (calculate by R)

The best fitted b is the projection part of y in the row space of \tilde{X} , which is calculated as follow equation:

$$b = (\tilde{X}' \cdot \tilde{X})^{-1} \cdot \tilde{X}' \cdot y$$

Then we got the result of vector b by R: