Classical Logic – Exercises

[Thanks to Woodcock/Davies and Wooldridge]

1. Using these propositions:

p	It is raining
q	I have an umbrella
r	I get wet

   formulate the following expressions in words:

   (a) $(p \wedge q)$

   (b) $(p \wedge \neg q) \wedge r$

   (c) $\neg p \wedge \neg r$

   (d) $p \wedge (q \vee r)$

   (e) $\neg p \vee r$

   (f) $\neg(p \vee r)$

题目中要求expression in previous question
就只是表示，表达之前问题的真值表就行了

2. Construct truth tables for the expressions in the previous question.

3. Using these propositions:

p	Logic is easy
q	There is a logic question
r	I pass the exam

   symbolise the following statements:

   (a) If logic is easy then I pass the exam.

   (b) If I pass the exam then logic is easy.

   (c) I fail the exam if logic is difficult.

   (d) If there is no logic question, then I pass the exam.

   (e) Logic is difficult. I pass the exam if there is no logic question.

   (f) Either logic is easy, and I pass the exam, or logic is hard, and I fail.

   (g) If logic is easy, then if there is a logic question, then I pass the exam.

   (h) If logic is difficult then I pass the exam if there is no logic question.

   (i) If I fail the exam, then there is a logic question and logic is difficult.

4. Define a predicate $p(x)$ such that $\forall x : \mathbb{N} \bullet p(x)$ is false, but $\exists x : \mathbb{N} \bullet p(x)$ is true. (Note that, $\mathbb{N}$ is the set of Natural Numbers, i.e. 0, 1, 2, 3, ...)

1. Sol:
a) p (It is raining) and q(I have an Umbrella);
b) p (It is raining) and -q(I do not have an Umbrella) and r(I get wet).
c) -p (It is not raining) and -q(I do not have an Umbrella)
d) p (It is raining), either q(I do an Umbrella) or I get wet.
e) -p (It is not raining) or r(I get wet)
f) - (it is false that) p(It is raining) or r(I get wet)

2. Sol:

Value	Input			Output					
	p	q	r	$(p \land q)$	$(p \land \neg q) \land r$	$\neg p \land \neg r$	$p \land (q \lor r)$	$\neg p \lor r$	$\neg(p \lor r)$
1	T	T	T	T	F	F	T	T	F
2	T	T	F	T	F	F	T	F	F
3	T	F	T	F	T	F	T	T	F
4	T	F	F	F	F	F	F	F	F
5	F	T	T	F	F	F	F	T	F
6	F	T	F	F	F	T	F	T	T
7	F	F	T	F	F	F	F	T	F
8	F	F	F	F	F	T	F	T	T

3. Sol:
a) p -> r
b) r -> p
c) -r <- -p
d) -q -> r
e) -p ∧ (r <- -q)
f) (p ∧ r) v (-p ∧ -r)
g) p -> (q -> r)
h) -p -> (r <- -q)
i) r -> (q ∧ -p)

4. Sol
p(x): x == 4

5. Which of the following statements is TRUE; argue informally in each case to justify your answer. (Note that, $\mathbb{Z}$ is the set of Integers.)

   (a) $\forall i : \mathbb{N} \bullet i = i$
   (b) $\exists i : \mathbb{N} \bullet i \neq i$
   (c) $\forall i, j : \mathbb{N} \bullet i \neq j$
   (d) $\forall x : \mathbb{N} \bullet \exists y : \mathbb{N} \bullet (y < x)$
   (e) $\forall x : \mathbb{Z} \bullet \exists y : \mathbb{Z} \bullet (y < x)$
   (f) $\forall i, j : \mathbb{N} \bullet (i \leq j) \Rightarrow \exists k : \mathbb{N} \bullet ((i + k) = j)$

6. The predicate $p(x, y)$ is defined as

$$(x + y) > x^2$$

   Which of the following are true?

   (a) $\forall x : \mathbb{N} \bullet \exists y : \mathbb{N} \bullet p(x, y)$
   (b) $\exists y : \mathbb{N} \bullet \forall x : \mathbb{N} \bullet p(x, y)$
   (c) $(\forall x : \mathbb{N} \bullet \exists y : \mathbb{N} \bullet p(x, y)) \;\Rightarrow\; (\exists y : \mathbb{N} \bullet \forall x : \mathbb{N} \bullet p(x, y))$

7. Express the fact that there is not a largest Integer using first-order logic.

5. Sol:

a) True: because, no matter what i always equal i;

b) False: as previous;

c) False: Because, it is possible that i and j is same number;

d) True: Because, it equals

((y1 < x1) v (y2 < x1) v.... v (yn < x1)) ∧
((y1 < x2) v (y2 < x2) v.... v (yn < x2)) ∧
....
((y1 < xn) v (y2 < xn) v.... v (yn < xn)) ∧
it will be always true;

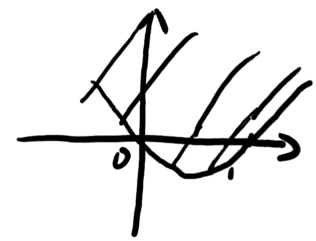
X $x=0$

y cannot less

more focus on domain.

$x^2-x < y$

$x(x-1) < y$



e) True: similar;

f) True: Because, if number a >b, the a-b =k

6. Sol:

a) it is true;

b) it is false;

c) It is false; T => F

7. Sol:

-∃x∈ Z · ∀y∈ Z · x $\geq$ > y 包括自己 所以用≥.