

## Sets, Functions and Schemas – Exercises

[ Thanks to Wooldridge ]

1. [Sets] In the following expressions, replace the question mark so as to make a true statement about sets.

- (a)  $A \cup A =? A$
- (b)  $A \cap \emptyset =? \emptyset$
- (c)  $(A \subseteq B) \Leftrightarrow (A \setminus B =?) \emptyset$
- (d)  $A \cap A =? A$
- (e)  $A \cup \emptyset =? A$
- (f)  $(A \subseteq B) \wedge (B \subseteq C) \Rightarrow (A ? C)$
- (g)  $\emptyset ? A \quad \emptyset \subseteq A$

2. List the elements of each of the following sets:

- (a)  $\{n \mid (n \in \mathbb{N}) \wedge (n < 6) \wedge (n > 10)\} \quad \emptyset$
- (b)  $\{n \mid (n \in \mathbb{N}) \wedge (n > 7) \wedge (n \leq 8)\} \quad \{8\}$

3. Consider the following sets:  $A == \{1, 9\}$ ,  $B == \{1, 3\}$ , and  $C == \{1, 5, 9\}$

- (a) What is  $\mathbb{P}C \quad \{\emptyset, \{1\}, \{9\}, \{1, 9\}, \{1, \emptyset\}, \{1, 9, \emptyset\}\}$
- (b) What is  $(\mathbb{P}C \setminus A) \cap \mathbb{P}B \quad \{\emptyset, \{1\}\}$
- (c) What is  $A \times B \quad \{(1, 1), (1, 3), (9, 1), (9, 3)\}$
- (d) What is  $\#((A \times B) \cap (A \times C)) = 2$

4. [Functions] Let

$$\begin{aligned} A &== \{a, b, c\} \\ B &== \{x, y, z\} \end{aligned} \quad .$$

and

$$\begin{aligned} f_1 &== \{a \mapsto y, c \mapsto x\} \quad i, iii, \\ f_2 &== \{a \mapsto y, b \mapsto z, c \mapsto x, c \mapsto z\} \times \\ f_3 &== \{a \mapsto y, b \mapsto y, c \mapsto z\} \quad i, ii, \\ f_4 &== \{a \mapsto z, b \mapsto y, c \mapsto x\} \quad i, ii, iii, iv, v \end{aligned}$$

- (a) For each  $f_i \in \{f_1, \dots, f_4\}$ , say whether  $f_i$  has the property of being:
- i. a partial function from  $A$  to  $B$ ;
  - ii. a total function from  $A$  to  $B$ ;
  - iii. an injection from  $A$  to  $B$ ;
  - iv. a surjection from  $A$  to  $B$ ;
  - v. a bijection from  $A$  to  $B$ .

If in any case your answer is negative, then justify it.

- (b) For each  $f_i$  that is a function, give
  - i.  $\text{dom } f_1 = \{a, c\}$ ,  $\text{dom } f_3 = \{a, b, c\}$ ,  $\text{dom } f_4 = \{a, b, c\}$
  - ii.  $\text{ran } f_1 = \{x, y\}$ ,  $\text{ran } f_3 = \{y, z\}$ ,  $\text{ran } f_4 = \{x, y, z\}$
- (c) For each of the following, say whether it is a legal expression, and if it is legal, evaluate it.
  - i.  $f_1 \cup f_2$  Yes, but we can express functions as a set of ordered pairs.
  - ii.  $(f_1 \cup f_4)(a)$  No, because there is no any operator between the set and element 'a'
  - iii.  $f_1 \oplus \{a \mapsto z\}$  Yes, the expression updates the previous function.
  - iv.  $(f_1 \oplus \{a \mapsto z\}) \cup \{b \mapsto y\}$  Yes.
  - v.  $((f_1 \oplus \{a \mapsto z\}) \setminus \{a \mapsto z\})$  Yes.

## 5. [Schema Calculus]

Refer to the phone book example given in the first lecture on schemas.

- (a) Specify an operation to delete an entry from the book.
  - (b) Specify an operation to change a telephone number.
  - (c) Write a ‘robust form’ of the deletion operation above (i.e. specify what happens when deletion is attempted on an entry that doesn’t occur in the book).  $\text{RobustDelete} \triangleq ((\text{Delete} \wedge \text{Success}) \vee \text{Failure})$
6. Specify a phone book together with its operations. However, this time, ensure that the phone book will hold at most 100 entries.

$\begin{array}{l} \text{PhoneBook} \\ \text{---} \\ \text{name! : Name} \end{array}$

