

Temporal Logic – Exercises

1. We wish to say that

表示3next is. $\square\square(x=0 \wedge y>1)$
 “in three moments in time, the variable x will have value 0 and, from that moment on, the variable y will always be greater than 1.” *表示3next is*

How might we represent this in temporal logic?

2. Give a semantic definition for a “sometime in the past” operator in the style of semantics you saw in the lectures. $\langle M, i \rangle \models \Diamond\varphi \Leftrightarrow \exists j \in \mathbb{N}. (j \leq i) \wedge \langle M, j \rangle \models \varphi$.
3. As we have seen,

$$\varphi W\psi \Leftrightarrow (\varphi U\psi \vee \square\varphi)$$

Fill in the missing formulae below using $\varphi W\psi$ as part of the right-hand side:

$$\varphi U\psi \Leftrightarrow \varphi W\psi \wedge \Diamond\varphi$$

4. The formula $\square\Diamond\varphi$ is commonly termed “infinitely often φ ”. Describe what property the formula $\Diamond\square\psi$ characterises? *$\square\varphi$ means φ will eventually be true.
 $\square\varphi$ means φ is always true.
 Therefore, it means that at sometime in future,
 φ will always be true (forced as above always)*
5. We can conjoin together next-formulae such as

$$p \wedge \square p \wedge \square\square p \wedge \square\square\square p \wedge \dots$$

How many such formulae would we have to conjoin together to give the same behaviour as $\square p$? *infinite.*

6. Is the following true?

$$\varphi U(\varphi U\psi) \Rightarrow \varphi U\psi$$

Justify your answer, either informally or by appealing to the semantics given. (Hint: if we assume that $\varphi U(\varphi U\psi)$ is satisfied in a model, does it follow that $\varphi U\psi$ must also be satisfied?)

7. The formula $\square(\varphi \Rightarrow \Diamond\psi)$ says that the formula $\varphi \Rightarrow \Diamond\psi$ is always true.

If also we know that $\Diamond\varphi$, then how many times will ψ be forced to occur?

8. Again, given $\square(\varphi \Rightarrow \Diamond\psi)$, then if we also know that $\square\varphi$ is true, how many times will ψ be forced to occur?

*$\square\varphi \Rightarrow \Diamond\psi$
 The formula means that at anytime in future,
 if φ is true, ψ will eventually be true.*

7. $\Diamond\psi$ means that ψ will be eventually true, sometimes in future.

So, ψ will similarly be true at sometime in future.

Therefore, there will be at least one time when the ψ will be true.

8. $\square\varphi$ means that φ will keep being true in future.

So $\Diamond\psi$ will be true at anytime in future.

Therefore, ψ will be infinitely often. 1

$$\diamond \varphi U(\varphi U\psi)$$

*means φ will always be true
 until $\varphi U\psi$ occurs
 In addition, φ means that
 φ will always be true and ψ occurs.
 Therefore, φ keeps being true until
 ψ keeps being true and ψ occurs.
 It means that φ keeps being true
 until $\varphi U\psi$ occurs.*