Sequences and Bags – Exercises

[Thanks to Wooldridge]

1. **[Sequences]**

   Assuming that $s, t, u : \mathrm{seq}\, Char$ are defined as follows:

   $$s == \langle a, c, k \rangle$$
   $$t == \langle b, l \rangle$$
   $$u == \langle j \rangle$$

   evaluate the following expressions:

   (a) $(t \frown s) \frown (u \frown s)$ <b, l, a, c, k> ⁀ <j, a, c, k> = <b, l, a, c, k, j, a, c, k>

   (b) $\mathrm{head}(\mathrm{front}(t \frown s))$ b

   (c) $\mathrm{last}(\mathrm{front}(t \frown s))$ c

   (d) $\mathrm{tail}(\mathrm{front}(t \frown s))$ <l, a, c>

   (e) $s \frown t$ <a, c, k, b, l>

   (f) $\mathrm{rev}(s \frown t)$ <l, b, k, c, a>

   (g) $\mathrm{dom}(s \frown t)$ {1..5}

   (h) $\mathrm{ran}(s \frown t)$ {a, b, c, k, l}

   (i) $(s \frown t) \rhd \{a, e, i, o, u\}$ $\{1 \mapsto a\}$

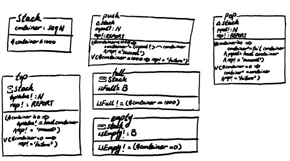
   (j) $(t \frown s) \oplus \{3 \mapsto o\}$ <b, l, o, c, k>

   HINT: Remember that a sequence

   $$\langle a, b, c \rangle$$

   is really just a function

   $$\{1 \mapsto a, 2 \mapsto b, 3 \mapsto c\}.$$



2. Using sequences, develop a Z specification for a *stack of natural numbers*. The stack should be of limited size (say 1000 elements), and should have the following operations defined:

   - *push* — push an item onto the top of the stack;
   - *pop* — remove the top item from the stack;
   - *top* — obtain the top item from the stack, without changing the stack;
   - *full* — returns a variable *isfull*! which contains *true* if the stack is full, *false* otherwise;
   - *empty* — as for *full*, but for finding out whether the stack is empty.

Your specification should be *robust*, in that it should capture error situations (such as trying to push an item onto a full stack).

3. **[Bags]**

Assuming that $B, C : \text{bag}\, Char$ are defined as follows:

$$B == [\![k, c, a, j, k, c, a, l, b]\!]$$
$$C == [\![a]\!]$$

evaluate and give the type of the following expressions:

(a) $B \sqsubseteq C$ false

(b) $a \text{ in } B$ True.

(c) $a \text{ in } B \wedge a \text{ in } C$ True.

(d) $B \# a$ = 2

(e) $(B \# a) + (C \# a) - 1$ = 2

(f) $B \oplus \{c \mapsto 5\}$ {k↦2, c↦5 a↦2, j↦1, l↦1, b↦1}

(g) $B \oplus \{c \mapsto 5\}) \setminus k \mapsto 1$ {k↦2....}