
鼠
新年快乐
Happy Chinese new year
2020

Solving lin. sys.

a simple system. $\longrightarrow$ back substitution

$$\begin{cases} x_1 - 2x_2 + x_3 = 0 \\ \quad\quad\; x_2 - 4x_3 = 4 \\ \quad\quad\quad\quad\;\; x_3 = 3 \end{cases}$$

$\longrightarrow$ $x_1 = 2\times 16 - 3 = 29$

$\longrightarrow$ $x_2 = 4 + 4\cdot 3 = 16$

$\longrightarrow$ $x_3 = 3$

Ex. $\begin{cases} x_1 - 2x_2 + x_3 = 0 & (1) \\ 2x_2 - 8x_3 = 8 & (2) \\ -4x_1 + 5x_2 + 9x_3 = -9 & (3) \end{cases}$

$\frac{1}{2} \times (2)$:

$$x_2 - 4x_3 = 4 \qquad (2')$$

$4 \times (1) + (3)$:

$$-3x_2 + 13x_3 = -9 \qquad (3')$$

$3 \times (2') + (3')$:

$$x_3 = 3 \qquad (3'')$$

$(1)\,(2)\,(3) \Longleftrightarrow \underbrace{(1)\,(2')\,(3'')}_{\text{triangular}}$.

General strategies: change to simpler & equivalent lin. sys.

Elementary row operation.

(R1) Add a multiple of any row to any other row

(R2) Exchange 2 rows

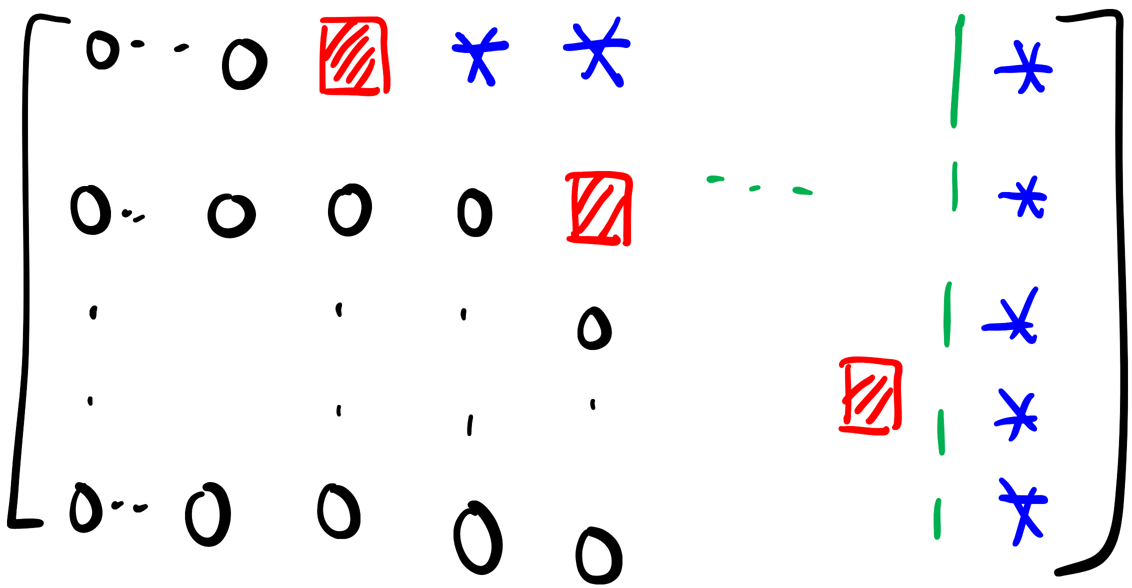
(R3) Scale any row by a nonzero number.

Ex. $\begin{cases} 2x_1 - 3x_2 \qquad\qquad - x_4 = 7 \\ x_1 + x_2 - 3x_3 + 2x_4 = 0. \end{cases}$

augmented matrix of lin sys.

$$\left[\begin{array}{cccc|c} 2 & -3 & 0 & -1 & 7 \\ 1 & 1 & -3 & 2 & 0 \end{array}\right]$$

Goal: Row echelon form (REF)



▨ $\neq 0$ pivot / leading entries

$*$ any number.

Ex.

$$\left[\begin{array}{cccc|c} \boxed{2} & 0 & 1 & -1 & 5 \\ 0 & \boxed{3} & 0 & 0 & c \\ 0 & d & \boxed{-2} & 0 & 1 \end{array}\right]$$

For what values of c,d is this REF

$d=0$, $c \in \mathbb{R} \Rightarrow$ REF.

Even better Reduced row echelon form (RREF).

$$\left[\begin{array}{cccccccc|c}
0 & \cdots & 0 & 1 & * & 0 & & 0 & * \\
0 & \cdots & 0 & 0 & 0 & 1 & \cdots & & * \\
\vdots & & & \vdots & \vdots & 0 & & 0 & * \\
\vdots & & & \vdots & \vdots & \vdots & & 1 & * \\
0 & \cdots & 0 & 0 & 0 & 0 & & 0 & *
\end{array}\right]$$

Thm. Given any augmented matrix. We can find a RREF equivalent to original aug. matrix by elementary row op. (Alg.)

Think: RREF is unique.

(Appendix A1)