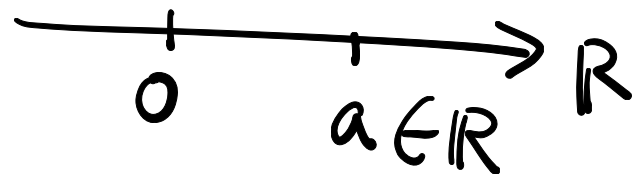


# Lec 1.

## Notation

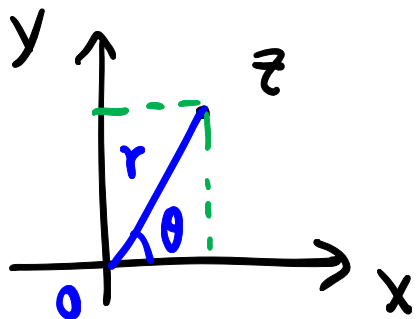
$\mathbb{R}$  = real numbers



$\mathbb{C}$  = complex numbers

$$z = x + iy \in \mathbb{C}$$

$$x, y \in \mathbb{R}, i = \sqrt{-1}$$



polar form  $z = x + iy = r e^{i\theta}$

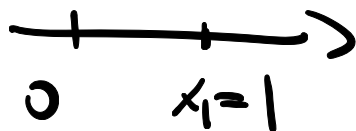
$$e^{i\theta} = \cos \theta + i \sin \theta \quad \begin{cases} x = r \cos \theta \\ y = r \sin \theta \end{cases}$$

$a, b, c \dots$  numbers / constants

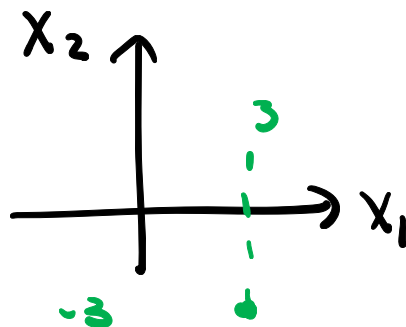
$x, y, z, \dots$  variables / unknowns.

## Systems of linear equations

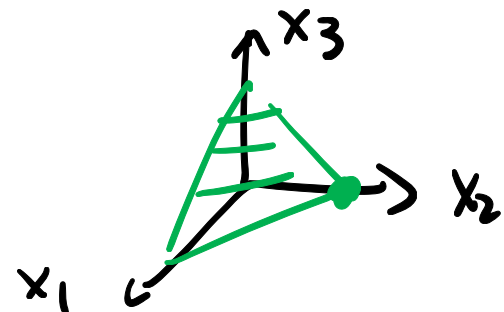
$$x_1 = 1$$



$$\begin{cases} x_1 + x_2 = 0 \\ x_1 = 3 \end{cases}$$



$$\begin{cases} x_1 + x_2 + x_3 = 1 \\ x_1 = 0 \\ x_2 = 1 \end{cases}$$



Def A linear eqn in  $n$  variables is  
an eqn. written in the form

$$a_1 x_1 + a_2 x_2 + \dots + a_n x_n = b$$

A system of linear eqns / linear system  
in  $n$  variables is a finite collection  
of lin. eqns

$$\left\{ \begin{array}{l} a_{11}x_1 + \dots + a_{1n}x_n = b_1 \\ a_{21}x_1 + \dots + a_{2n}x_n = b_2 \\ \vdots \\ a_{m1}x_1 + \dots + a_{mn}x_n = b_m \end{array} \right.$$

Def A solution set of a lin. sys.

is the set of tuples  $(s_1, \dots, s_n)$

that solve all eqns. in the lin. sys.

Def Two lin. sys. are equivalent  
if they have the same sol. set.

Ex. Find sol sets for.

$$1) \begin{cases} 3x_1 - x_2 = 0 \\ 2x_1 = 6 \end{cases}$$

sol set  $\{(3, 9)\}$

$$2) \begin{cases} 3x_1 + x_2 = 1 \\ -6x_1 - 2x_2 = 0 \end{cases}$$

sol set  $\emptyset$

$$3) \begin{cases} x_1 - x_2 + x_3 = 0 \\ x_2 + x_3 = 0 \end{cases} \quad \text{Sol set } \{(-2s, -s, s) \mid s \in \mathbb{R}\}$$

Three possible outcomes

1) no sol.  $\rightarrow$  inconsistent

2) unique sol

3) Infinitely many sols.

}  $\rightarrow$  consistent.

Ex. For what value of  $c$  are the following lin. sys. equivalent?

$$\textcircled{1} \begin{cases} x_1 - c x_2 = 0 \\ x_1 + x_3 = 0 \end{cases}$$

$$\textcircled{2} \begin{cases} 2x_1 - x_2 + x_3 = 0 \\ x_2 + x_3 = 0. \end{cases}$$

sol set

$$\{ (cs, s, -cs) \mid s \in \mathbb{R} \}$$

sol set

$$\{ (-t, -t, t) \mid t \in \mathbb{R} \}$$

$$\Leftrightarrow c = 1.$$

Two sets  $A, B,$

$$A \neq B$$

$$\forall x \in A \Rightarrow x \in B$$

$$\forall y \in B \Rightarrow y \in A$$