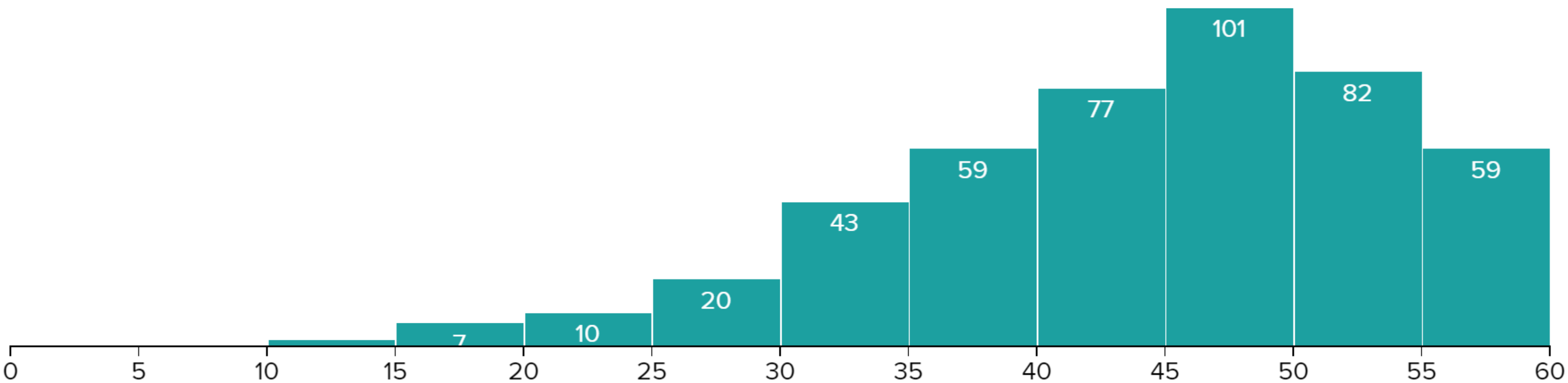
Lec 10.



MINIMUM	MEDIAN	MAXIMUM	MEAN	STD DEV
10.0	45.0	60.0	43.81	9.75

Warmup

a) $AB = AC \Rightarrow B = C$? No

b) $AB = 0 \Rightarrow A = 0$ or $B = 0$? No.

a) $\underbrace{A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}}$ $C = \begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix}$

b) $\downarrow$

Subspace.

<u>Def</u> A subspace H of $\mathbb{R}^n$ is a subset of vectors in $\mathbb{R}^n$.

(1) $\vec{0} \in H$ $\longrightarrow$ ruling out ϕ.

(2) $\vec{u}, \vec{v} \in H$, then $\vec{u}+\vec{v} \in H$

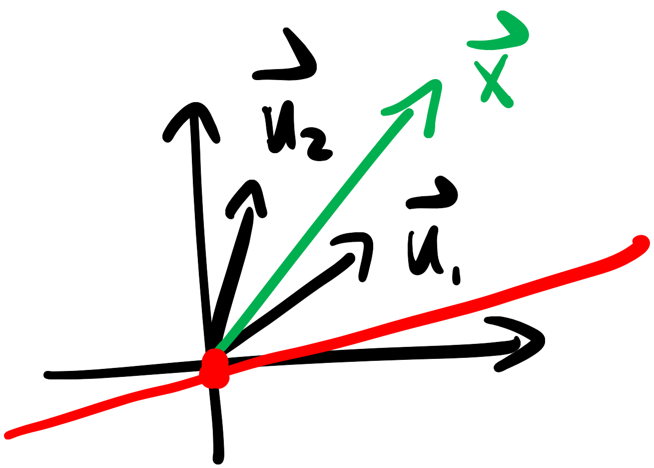
(3) $\vec{u} \in H$, $c \in \mathbb{R}$, then $c\vec{u} \in H$

Ex. $\mathbb{R}$. possible subspaces?

$\{\vec{0}\}$, $\mathbb{R}$

2 subspaces.

Ex. $\mathbb{R}^2$ possible subspaces?

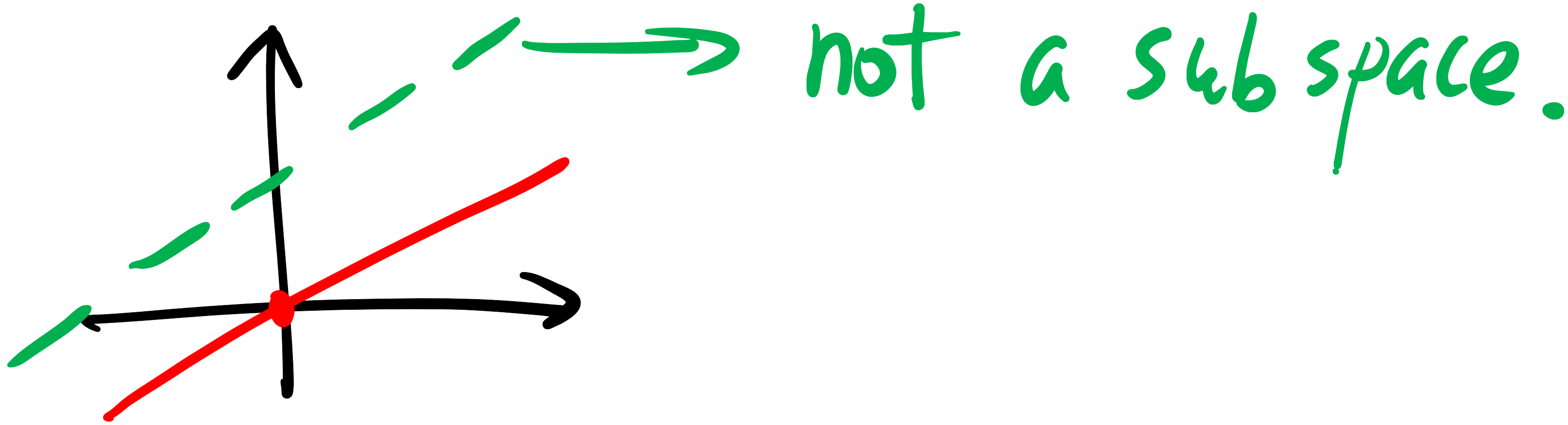


$\{\vec{0}\}$

$\text{span}\{\vec{u}\}$, $\vec{u} \in \mathbb{R}^2$, $\vec{u} \neq \vec{0}$

$\mathbb{R}^2$

infinite number of subspaces.



Thm. $\{\vec{v}_1, \cdots, \vec{v}_k\}$ in $\mathbb{R}^n$, $k \geq 1$.

Then $\text{span}\{\vec{v}_1, \cdots, \vec{v}_k\}$ is a subspace of $\mathbb{R}^n$

This is the smallest subspace containing

$\{\vec{v}_1, \dots, \vec{v}_k\}$.

Two important examples of subspaces.

$A \in \mathbb{R}^{m \times n}$

1) Column space

$\mathrm{col}(A) :=$ span of column vectors of A

2) Null space

$\mathrm{Null}(A) :=$ sol. set of $A\vec{x} = \vec{0}$

$\mathrm{Col}(A)$ is a subspace of $\mathbb{R}^m$

$\mathrm{Null}(A)$ " " " " $\mathbb{R}^n$

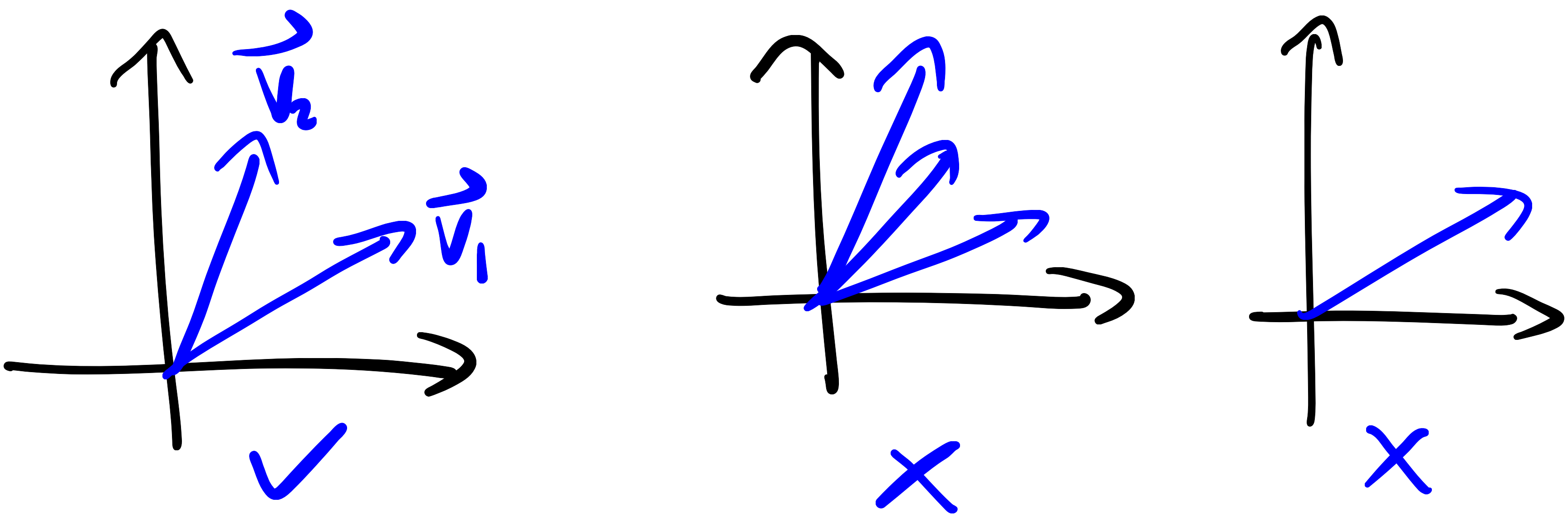
Def. A basis for a subspace H of $\mathbb{R}^n$ is an ordered set of vectors

$\{\vec{v}_1, \dots, \vec{v}_k\}$ s.t.

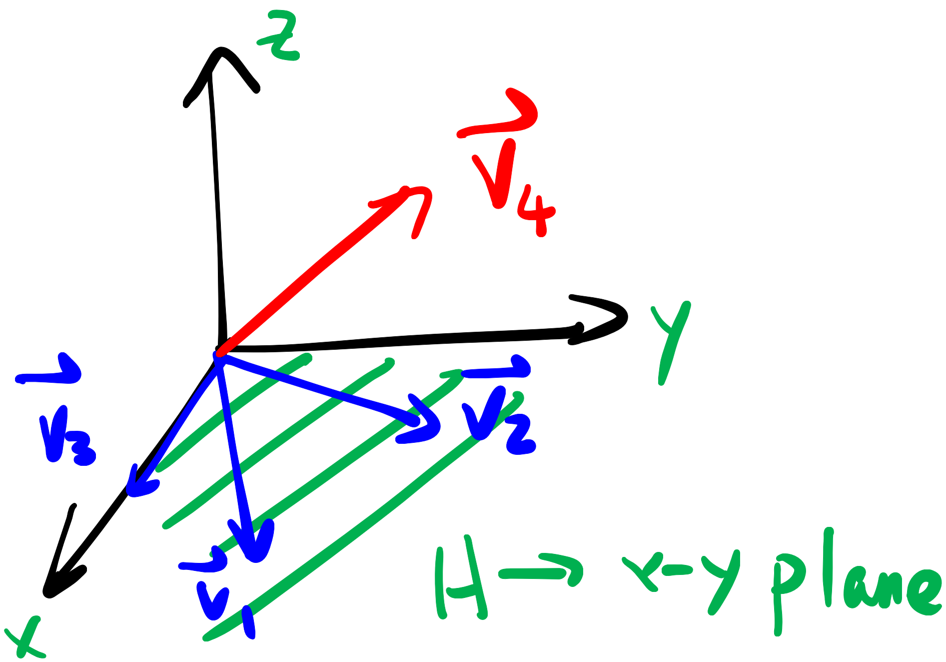
1) $\{\vec{v}_1, \dots, \vec{v}_k\}$ spans H (big enough)

2) $\{\vec{v}_1, \dots, \vec{v}_k\}$ lin. indep. (small enough)

Ex. $\mathbb{R}^2 = H$



Ex. $\mathbb{R}^3 \supset H$



$\vec{v}_i \in \mathbb{R}^3$

$\{\vec{v}_1, \vec{v}_2\}$

Ex. Which set of vectors is a basis of $\mathbb{R}^3$?

1) $\left\{\begin{bmatrix}1\\2\\3\end{bmatrix}, \begin{bmatrix}2\\3\\4\end{bmatrix}\right\}$ ✗

2) $\left\{\begin{bmatrix}0\\1\\0\end{bmatrix}, \begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}\right\}$ ✓

3) $\left\{\begin{bmatrix}1\\1\\0\end{bmatrix}, \begin{bmatrix}1\\1\\1\end{bmatrix}, \begin{bmatrix}0\\1\\1\end{bmatrix}, \begin{bmatrix}1\\0\\1\end{bmatrix}\right\}$ ✗

4) $\left\{\begin{bmatrix}1\\1\\0\end{bmatrix}, \begin{bmatrix}1\\1\\1\end{bmatrix}, \begin{bmatrix}1\\0\\1\end{bmatrix}\right\}.$ ✓ (exer)

Def. H is a subspace of $\mathbb{R}^n$

the dimension of H

$\dim(H) :=$ size of any basis of H.

Fact (exer) size of basis is independent of the particular choice of basis.

Def. $A \in \mathbb{R}^{m \times n}$.

$$\mathrm{rank}(A) := \dim \mathrm{col}(A) = \text{"\# pivots"}$$

Thm. (Rank thm).

$$n = \mathrm{rank}(A) + \dim \mathrm{Null}(A).$$

$$n = \text{\# pivots} + \text{\# free variable}$$