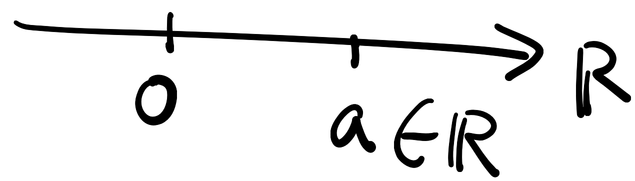
Lec 1.

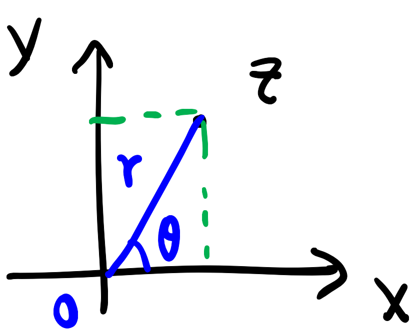
Notation

$\mathbb{R}$ = real numbers



$\mathbb{C}$ = complex numbers $\quad z = x + iy \in \mathbb{C}$

$x, y \in \mathbb{R}, \; i = \sqrt{-1}$



Polar form $\quad z = x + iy = re^{i\theta}$

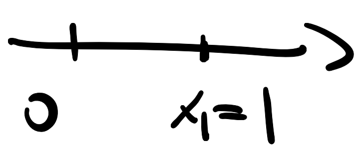
$$e^{i\theta} = \cos\theta + i\sin\theta \qquad \begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$$

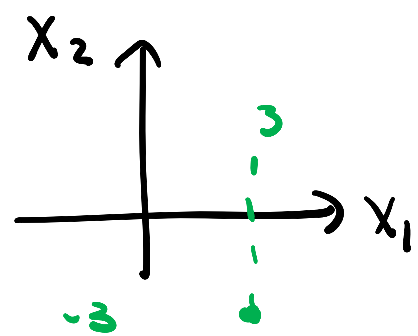
$a, b, c, \ldots$ numbers / constants

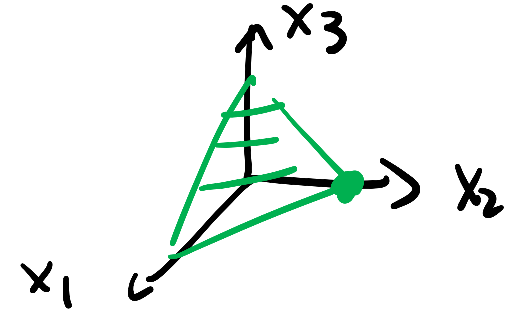
$x, y, z, \ldots$ variables / unknowns.

Systems of linear equations

$$x_1 = 1 \qquad \begin{cases} x_1 + x_2 = 0 \\ x_1 = 3 \end{cases} \qquad \begin{cases} x_1 + x_2 + x_3 = 1 \\ x_1 = 0 \\ x_2 = 1 \end{cases}$$







Def A linear eqn in n variables is an eqn. written in the form

$$a_1 x_1 + a_2 x_2 + \cdots + a_n x_n = b$$

A system of linear eqns / linear system in n variables is a finite collection of lin. eqns

$$
\begin{cases}
a_{11}x_1 + \cdots + a_{1n}x_n = b_1 \\
a_{21}x_1 + \cdots + a_{2n}x_n = b_2 \\
\vdots \\
a_{m1}x_1 + \cdots + a_{mn}x_n = b_m
\end{cases}
$$

<u>Def</u> A solution set of a lin. sys. is the set of tuples $(s_1, \cdots, s_n)$ that solve all eqns. in the lin. sys.

<u>Def</u> Two lin. sys. are equivalent if they have the same <u>sol. set</u>.

<u>Ex</u>. Find sol sets for.

1) $\begin{cases} 3x_1 - x_2 = 0 \\ 2x_1 = 6 \end{cases}$ sol set $\{(3, 9)\}$

2) $\begin{cases} 3x_1 + x_2 = 1 \\ -6x_1 - 2x_2 = 0 \end{cases}$ sol set ϕ

3) $\begin{cases} x_1 - x_2 + x_3 = 0 \\ x_2 + x_3 = 0 \end{cases}$ Sol set $\{(-2s, -s, s) \mid s \in \mathbb{R}\}$

Three possible outcomes

1) no sol. → inconsistent

2) unique sol

3) Infinitely many sols.

$\left.\begin{array}{l} \text{2) unique sol} \\ \text{3) Infinitely many sols.} \end{array}\right\}$ → consistent.

Ex. For what value of c are the following lin. sys. equivalent?

① $\begin{cases} x_1 - c\,x_2 = 0 \\ x_1 + x_3 = 0 \end{cases}$ ② $\begin{cases} 2x_1 - x_2 + x_3 = 0 \\ x_2 + x_3 = 0. \end{cases}$

sol set sol set

$\underbrace{\{ (cs, s, -cs) \mid s \in \mathbb{R} \}}_{A}$ $\underbrace{\{ (-t, -t, t) \mid t \in \mathbb{R} \}}_{B}$

$\Longleftrightarrow \quad c = 1.$

Two sets A, B,

$$A \overset{?}{=} B$$

$$\forall x \in A \Rightarrow x \in B$$

$$\forall y \in B \Rightarrow y \in A$$

Pick $t=1$

$(-1,-1,1) \in A$

$s=-1$

$(-c,-1,+c) = (-1,-1,1)$

$\Rightarrow$ A necessary condition is $c=1$.

When $c=1$,

$A = \{(s,s,-s) \mid s \in \mathbb{R}\}$

$= B$.