MGSC 662 Group Project Report

Patrolling Policemen

Group 18: Diwei Zhu, Yichen Wang, Danyal Hamid, Hugo Garcia, Alice Liu

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1 Introduction

Police power, as a capacity of states to enforce the law, maintain orders, and protect public welfare, has the origin in the law of necessity (Prentice, 2015). Being one of the police services to reinforce the police power against the occurrence of crime, patrolling is of dispute for a long time for whether it has a statistically significant effect on street crime. Sherman and Weisburd (1995), with their study, argued that substantial increases in police patrol presence can indeed cause modest reductions in crime and more impressive reductions in disorder within high crime locations. The conclusion is supported by works from other researchers in terms of studies based on different cities (Ratcliffe et al., 2011; Andresen & Lau, 2013). However, the activities of state-sponsored agencies like police systems are of a high vulnerability to fiscal distress, especially during economic shocks and when the political system becomes less supportive of police agencies.

In this project, we set an eye on Toronto, a city that has demonstrated 78 homicides, 373 shootings, and 5,268 breaks and enters in 2021 till December 2$^{\text{nd}}$ (Toronto Crime Incidences, 2021), highlighting the necessity of police patrol for crime prevention and improving citizens' feeling of safety. However, during the pandemic, the Canadian police system is experiencing pressures from the increased spending on healthcare and unemployment compensation (Chudik et al., 2021). Because of the limitations in fiscal support, time and police resources (police equipment and officers) counterplays the need to reinforce patrolling powers in crime hot spots, there is a necessity to optimize patrolling plans to ensure the best crime prevention performance under those limitations. Responding to the need, based on a simplified simulation of the Toronto police system and public safety conditions, we built a model to minimize the budget spent on police patrolling among divisions in downtown Toronto via optimizing the patrolling routes and determining the number of police cars.

2 Problem Description and Formulation

2.1 Model Structure

We first imagined a complete police patrol route as a closed loop that can be broken into different line segments connected together by checkpoints. This means, a police car will travel from the starting point to point 1, then to point 2,3, 4, etc. When all points are visited, the police car will return to the starting point (see fig 2.1). According to the map of Command and Divisional Boundaries, the downtown area (in yellow)is divided into 8 divisions. with Division 53 being the "Central Field" (see fig 2.2).

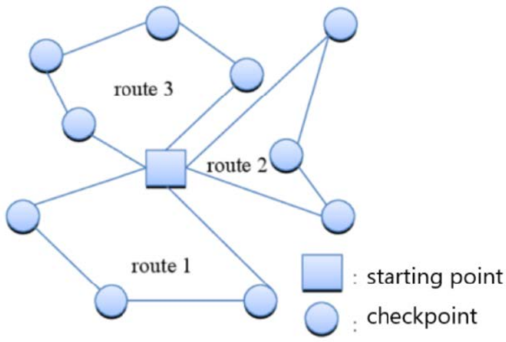

Fig 2.1 - Police Patrol Route Basic Model

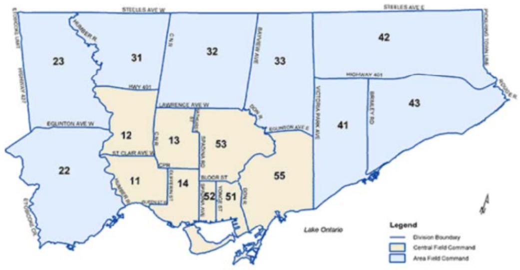

Fig 2.2 - Toronto Police Service: Command and Divisional Boundaries

Toronto Police is facing a limited budget from the government. Therefore, we also need to take the consideration that, while ensuring the quality of patrolling, we limit the patrolling cost as much as possible. We form our total patrolling cost function based on the logic of total patrolling cost = the sum of patrolling time for each police car $\times$ patrolling cost per minute.

2.2 Sets, Indices and Parameters

$i, j \in$ Divisions $\equiv D = \{0, 1..(n-1)\}$: Set of divisions where 0 is the index for Central Field, and n is the number of divisions.

$k \in$ Police cars $\equiv P = \{0..K-1\}$: Index and set of police cars, where K is the number of police cars.

$S_k \in S$: Tour of police car k, i.e. subset of divisions visited by the police car.

$t_{i,j} \in \mathbb{R}^+$: Travel time from division i to division j.

2.3 Decision variables

For the path between each division, we should specify which police car travels this path from which division to which division: $x_{i,j,k} \in \{0, 1\}$: This binary variable is equal 1, if police car k visits and goes directly from division i to division j, and zero otherwise.

For each division, we should decide which police car will visit this point: $y_{i,k} \in \{0, 1\}$: This binary variable is equal to 1, if police car k visits division i, and zero otherwise.

Our model should include a variable that determines how many police cars we need to traverse through all divisions: $z_k \in \{0, 1\}$: This binary variable is equal 1, if police car $k \in \{1, 2..K\}$ is used, and zero otherwise.

With route variables determined, we then need a variable that represents the individual travelling time of each police car: $t_k \in \mathbb{R}^+$: This continuous variable denotes the travel time for police car $k \in \{1, 2..K\}$.

To determine the total cost and total time of the whole patrolling process, we need to find the sum of total time for each police car: $st \in \mathbb{R}^+$: This continuous variable is the sum of travel time for police car $k \in \{1, 2..K\}$.

We also need the time length beginning from when all police cars depart at the starting point to when the last police car finishes the patrolling duty: $s \in \mathbb{R}^+$: This continuous variable denotes $max(t_k)\ \forall$ police car $k \in \{1, 2..K\}$.

Notice that we don't include the part where police cars return from the last checkpoint to the starting point in our fee calculation, because police cars don't need to perform patrolling duty on the way back to the starting point.

2.4 Objective Functions

We recognize three aspects that can affect total cost and rank their influence accordingly. The most important aspect is the sum of patrolling hours of all police cars. This is because the increase of work hours directly increases the total cost linearly. The second important aspect is the longest time a police car travels, because total cost will rise if too much time is used. Lastly, the third aspect is how many police cars to employ, since increasing the number of police cars induces work hours to accumulate, causing escalation of total cost.

First priority: Minimize the total travel time, which is the sum of travel time for each police car.

$$\text{Minimize} \quad \sum_{k=1}^{K} t_k \tag{1}$$

Second priority: Minimize the maximum travel time, which is the time for all police cars to complete patrol.

$$\text{Minimize} \quad max(t_k) \tag{2}$$

Third priority: Minimize the number of police cars used.

$$\text{Minimize} \quad \sum_{k=1}^{K} z_k \tag{3}$$

2.5 Constraints

We enriched our model by adding constraints to enable logically correct route planning as well as to comply with real-world scenarios.

Police car utilization: For all divisions different from the Central Field, i.e. $i > 0$, if the division is visited by a police car k, then it is used.

$$y_{i,k} \leq z_k \quad \forall i \in D,\ i > 0,\ k \in P \tag{4}$$

Travel time: No police car travels for more than 60 min. Note that we do not consider the travel time to return to Central Field.

$$\sum_{i \in D} \sum_{j \in D, j > 0} t_{i,j} \cdot x_{i,j,k} \leq 60 \quad \forall k \in P \tag{5}$$

Visit all divisions: Each division is visited by exactly one police car.

$$\sum_{k \in P} y_{i,k} = 1 \quad \forall i \in D,\ i > 0 \tag{6}$$

Starting point: Central Field is visited by every police car used.

$$\sum_{k \in P} y_{1,k} \geq \sum_{k \in P} z_k \tag{7}$$

Arriving at a division: If division j is visited by police car k, then the police car is coming from another division i.

$$\sum_{i \in D} x_{i,j,k} = y_{j,k} \quad \forall j \in D,\ k \in P \tag{8}$$

Leaving a division: If a police car k leaves division j, then the police car is going to another division i.

$$\sum_{i \in D} x_{j,i,k} = y_{j,k} \quad \forall j \in D,\ k \in P \tag{9}$$

Breaking symmetry: all routes that visit the same divisions are only considered as one route

$$\sum_{i \in D} y_{i,k} \geq \sum_{i \in D} y_{i,k+1} \quad \forall k \in \{0..K-1\} \tag{10}$$

Subtour elimination: These constraints ensure that for each police car route, there is no cycle.

$$\sum_{(i,j) \in S_k} x_{i,j,k} \leq |S_k| - 1 \quad \forall k \in K,\ S_k \subseteq D \tag{11}$$

3 Numerical implementation and results

After getting an understanding of the problem at hand and formulating it mathematically we set about acquiring relevant data sets and pre-processing them for our problem.

3.1 Data Collection

For the scope of our project, we limited ourselves to the 8 police divisions that together form the downtown Toronto area. For these 8 divisions, we created a route time matrix (Fig 3.1) with the help of google maps (containing the time taken to get from one division to every other division).

Figure 1: Fig 3.1 - Route time matrix

		2054 Davenport	200 Threthewey Dr.	1435 Elington Av. W.	350 Dovercourt Rd.	51 Parliament St.	255 Dundas St. W.	75 Elington Av. W.	101 Coxwell Av.
		11	12	13	14	51	52	53	55
2054 Davenport Rd.	11	0	11	15	13	26	23	27	40
200 Threthewey Dr.	12	11	0	13	23	32	31	25	38
1435 Elington Av. W.	13	15	13	0	16	27	21	15	35
350 Dovercourt Rd.	14	13	23	16	0	20	13	28	28
51 Parliament St.	51	26	32	27	20	0	10	19	10
255 Dundas St. W.	52	23	31	21	13	10	0	21	18
75 Elington Av. W.	53	27	25	15	28	19	21	0	26
101 Coxwell Av.	55	40	38	35	28	10	18	26	0

We also acquired data on the per-hour utilization cost of a police patrolling vehicle. Which we found to be 37.38 CAD/hour for Toronto.

3.2 Gurobi formulation

Having acquired the data needed we then chose python as our modeling language (in accordance with the MGSC662 course) and the Gorobi add-on as our solver and started to formulate the problem. Model implementation followed the following key steps :

Import relevant libraries

Gurobipy, Pandas, Math, Sys, Itertools

Load the datasets

1. Route time matrix from excel using the pandas read_excel() functionality.
2. Define the number of available police vans.
3. Define bounds.

Declare the model

Using Gurobipy.model("Model name") functionality

Create all the decision variables

Using model.addVars() implement all the decision variables mentioned in the previous section.

Add constraints

Using model.addConstr() implement all the constraints mentioned in the previous section. We had to deal with the subtour constraints a little differently when adding them to python because there is an exponential number of these constraints, we don't want to add them all to the model. Instead, we use a callback function to find violated subtour constraints and add them to the model as lazy constraints.

Define the objective functions

Using model.setObjectiveN() to set multiple objective functions and assign each of them a priority.

Solve for the objective value

Solve to minimize the objective functions. using model.optimize()

3.3 Solution and analysis

When we solve our model the solution takes the following form. It informs us on each of the criteria set in our objective function. It gives us the number of cars used for patrolling, their individual routes and the time taken for them to complete those routes as well as the total cost of travel for the advised routes.

Solution:

```
Police Division index:
1: Central Field, 75 Eglinton Av. W.
2: 11 Division, 2054 Davenport Rd.
3: 12 Division, 200 Trethewey Dr.
4: 13 Division, 1435 Eglinton Av. W.
5: 14 Division, 350 Dovercourt Rd.
6: 51 Division, 51 Parliament St.
7: 52 Division, 255 Dundas St. W.
8: 55 Division, 101 Coxwell Av.
```

```
Optimization Results:
Route for police car 1: 1 -> 4 -> 3 -> 2 -> 5 -> 1.
Travel time: 52.0 min
Route for police car 2: 1 -> 7 -> 6 -> 8 -> 1.
Travel time: 41.0 min
```

```
Total travel cost: $ 57.939
```

The solution is in line with our understanding of the problem. We were essentially trying to solve a variation of the travelling salesman problem in which the locations to be visited are divided between **K** number of **agents** or in our case **cars**. The solution follows our requirements, all locations are visited between the cars, no location beside the home base is visited twice (there are no subtours), none of the cars individually go over 60 mins of usage time.

Lastly, if we visualize the routes we can see that our solution gives a viable path for the patrol cars to follow that is directly implementable and takes the least amount of time to traverse (Fig 3.2).

Figure 2: Fig 3.2 - Route time matrix

4 Problem Extensions

Thinking beyond our base model, we now consider that in cities, certain parts experience less crime than others, and Toronto is no exception. As such, we can introduce another dimension which represents crime rates, and increase police vehicle patrols in those areas where more crime is occurring. There are many different types of crimes to consider, however with limited time and data on all crimes in Toronto, we chose to use homicides in the city between 2004 to 2021 as a representation of crime rates. Grouping total homicides by division (see Appendix Fig 4.1), we created a heatmap to see how each division compared in homicide rates (see Appendix Fig 4.2).

We observed that division 12, 14 and 51 handle the most homicide cases. This information can be used to inform budgetary decisions for each division. Indeed we would want to deploy additional patrolling resources to patrol: D12, D14, D51.

Using this information, we updated our base model so that police patrols have to visit the high concentration crime areas twice. It is hard adding new decision variables to achieve this since we have functions eliminating subtours, so the way we implemented it is to modify the original dataset. The three high concentration crime areas are treated as new divisions and added to the dataset so we could solve it in the same way as we did in the base model, where the police must visit all of the areas. The only difference is that the travelling time between the same division must be set to a large number (1000000 in our case) to prevent the car patrolling in the same division twice without leaving. What's more, to eliminate forward-backward problem (a police car moves from one division A to a dangerous division), and moves back to the division A, i.e. A-B-A), we add new constraints that xi,j,k+xj,i,k ==1 (Appendix Fig 4.7). The modified result is shown below:

The solution changes as follows (see Appendix Fig 4.3):

Where: 6 = Division 51, 5 is Division 14, and 3 is Division 12

Base model: 99 minutes, $61

Extended model: 146 minutes, $91

Δ: 47 minutes, $30, one extra vehicle

However, there is still a problem: Our base model's initial objective of minimizing patrol time and vehicle costs is not necessarily a good one, rather it should be a two-tier objective of minimizing crime (the general goal of the police system) and minimizing costs (the goal of our base model). With this, segmenting homicides based on division (and thus one central latitude & longitude coordinates) does not help to inform crime rates, and thus a meaningful patrolling schedule.

Going down this path adds complexity and changes our understanding of the problem because we must account for other factors and variables in the police force. For example, patrols by vehicle are not the only type of patrolling method. There are also foot patrols, horse patrols, motorbike patrols, and bicycle patrols, where the use of these different types of patrol will vary based on factors such as:

geography/topography, time of day (e.g. rush hour), climate and seasons, resource availability, and large events. Additionally, different patrol types come with their own sets of constraints and costs. For example, a horse patrol requires a trained human police officer, horse riding gear, a transportation vehicle to the patrol site, and horse food/water. Moving to a more granular view of homicides in Toronto, without grouping by division, we see the following concentrations of crime on this heatmap (see Appendix Fig 4.4):

Notice the blue markers overlayed on the heatmap which represent police division stations. Using our extended problem's two-tier objective of minimizing crime and minimizing costs, we explored one specific station, division 52, and area (University of Toronto area) to focus on. This area is located in the city and would be best patrolled by either foot, bicycle (in summer), or vehicle. Research has found that "targeted foot patrols in violent crime hotspots can significantly reduce violent crime levels" (J.Ratcliffe et al., 2011), so in this case we use foot patrols to illustrate the extension of our base model. By observing crime spots on the heat map (see Appendix Fig 4.5) and setting Division 52's Station as the starting and ending point, we can trace a foot patrol path to visit those areas which may be more prone to crime (see Appendix Fig 4.5). Doing so turns our problem into a much more contained model which looks only at Division 52 by creating a new tour with nodes at 2,3,4, and 5 before returning to 1. Given that walking patrol is the cheapest form of patrolling, this becomes a more realistic patrol that minimizes both costs and crime rates.

A limitation and constraint of foot patrolling is the area which can be covered. To effectively patrol a large area, we should utilize various patrol types and account for their capacity to cover an area over time. To do this, we segment a large area based on a feasible radius for each patrol type which is set as their constraints, centered around a division station. Using our example above, to effectively patrol the D52 University Area, we can assign three types of patrol, segmented by a given radius which extends outwards: foot patrol for the innermost area, bicycle patrol for the mid-area, and vehicle patrol for the outermost area (see Appendix Fig 4.6). In this way we cover the entire area while playing to the strengths of each patrol type.

Considering this problem extension, we start to understand how quickly this problem can grow based on the complexity of the factors outlined above: multiple division stations, multiple patrol types, patrol type constraints, multiple types of crime, multiple potential crime sources, with many changing contextual and environmental factors; all of which changes the nature of our base model and problem drastically.

5 Recommendations and conclusions

5.1 Recommendation

According to our results generated from the model, there are several recommendations for Toronto police if they want to implement the patrol plan, where the top priority is lowering the cost, the second priority is to minimize the maximum individual patrolling time and the third priority is to minimize the number of police cars used.

For normal patrol without revisiting certain high crime-rate districts, the entire patrol mission time could be controlled under 52 minutes using 2 cars, with the cost minimized at around $58. However, if a second re-visit is required to high crime-rate districts, then an additional car will be utilized, with a marginal cost of $30 and an increased patrol time of 13 minutes for a single patrol mission. These solutions could provide Toronto police with good information on how to balance between neighbourhood safety and operation cost.

5.2 Reflection

In this project we have learnt how to solve a real-life problem by using concepts from Travelling Salesman Problem, incorporated with other techniques such as categorizing data by using the heatmap package from python to highlight high crime-rate regions. We have also implemented a multi-objective model, with different weights assigned to each objective function. This technique really helps our model as we need to consider many different goals to help the Toronto police design the best patrol route.

When talking about the bottlenecks, the time data for travelling between two different districts is the one. Since we need to consider the situation in reality such as traffic jams and complicated road maps, instead of calculating time using Euclidean distance, we manually gathered the travelling time using Google Map by putting starting point and destination coordinates in the app and recording the recommended travel time. This step is time-consuming and repetitive and restricts our dataset to only eight districts at the center of Toronto. In the future for further improvements, we could incorporate related packages to automatically generate a time matrix between all the districts we preferred, which would definitely improve the applicability of our model.

6 Reference

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7 Appendix

	Division	Total Homicides
0	D11	41
1	D12	78
2	D13	38
3	D14	80
4	D51	109
5	D52	38
6	D53	35
7	D55	44

Figure 3: Fig 4.1 - Toronto Homicides 2004-202 Grouped by Police Division where Homicide Occurred

Figure 4: Fig 4.2 - Toronto Homicides 2004 - 2020 Heatmap Grouped by Police Division where Homicide Occured

```
Route for police car 1: 1 -> 6 -> 7 -> 5 -> 3 -> 1.
Travel time: 65.0 min
Route for police car 2: 1 -> 4 -> 3 -> 2 -> 5 -> 1.
Travel time: 52.0 min
Route for police car 3: 1 -> 6 -> 8 -> 1.
Travel time: 29.0 min
Total travel cost: $ 90.958
Max travel time: 65.0 min
```

Figure 5: Fig 4.3 - Extended Base Model where Patrol Vehicles visit high crime zones twice

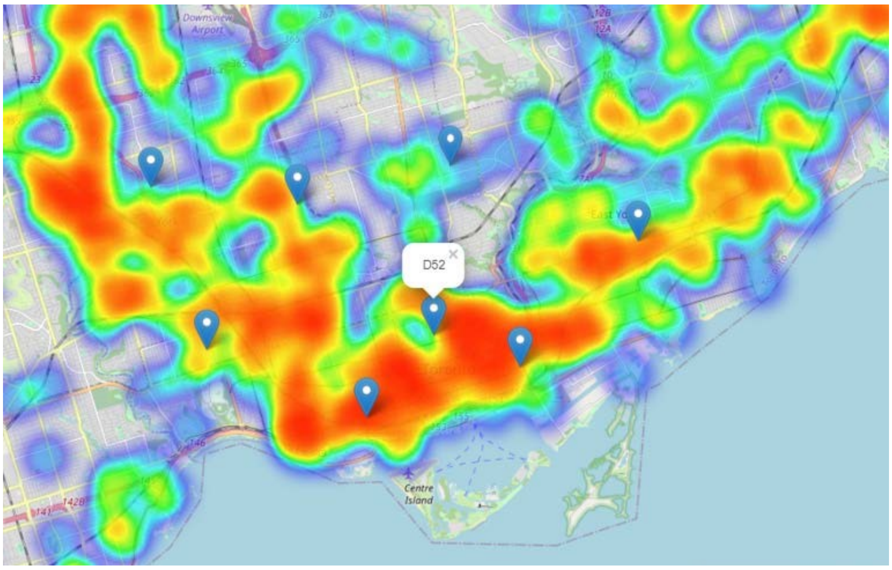

Figure 6: Fig 4.4 - Toronto Homicides 2004 - 2020 Heatmap

Figure 7: Fig 4.5 - Toronto Homicides 2004 - 2020 Heatmap, Zoomed in on Division 52 Station (University of Toronto Area) Foot Patrol tour traced with nodes based on visible crime distribution

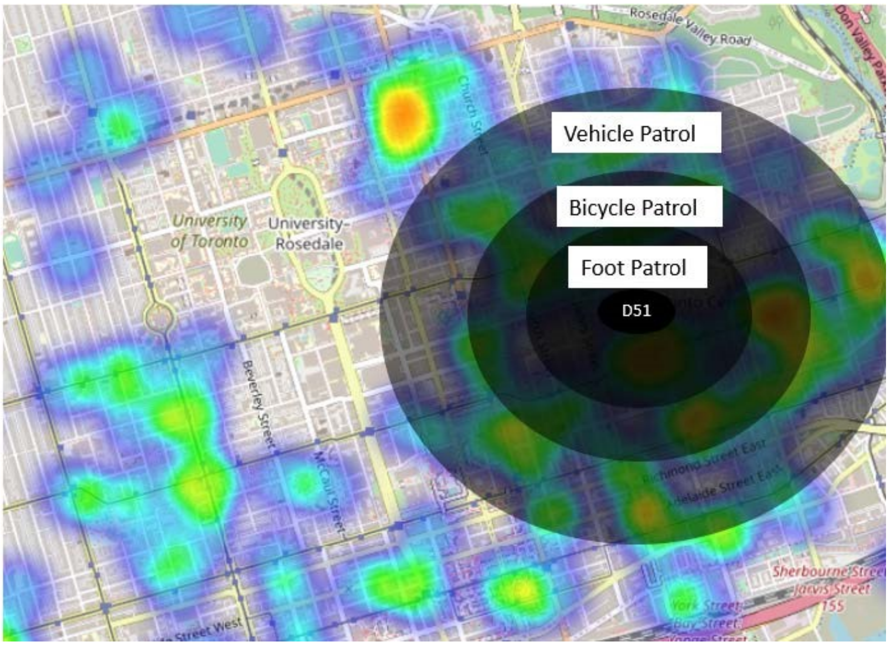

Figure 8: Fig 4.6 - Toronto Homicides 2004 - 2020 Heatmap: Overlayed with spheres denoting radius for patrol type to cover

New Set and Indices

$v \in \{\text{Division 12, Division 14, Division 51}\} \subseteq D = \{2, 4, 5\}$: Set of divisions that need to patrolled twice,
2 are the indexes for Division 12, 4 are the indexes for Division 14, and 5 are the indexes for Division 51.

$w \in \{\text{Division 12, Division 14, Division 51}\} \equiv ExtraD = \{8, 9, 10\}$: Set of divisions that need to patrolled twice,
8 are the indexes for Division 12, 9 are the indexes for Division 14, and 10 are the indexes for Division 51.

$d \in D$: a general denotion of all divisions.

New Constraints

Forward-Backward Prevention: prevent police car from first patrol a high crime rate area, then patrol another area, then goes back to the same high crime rate area.

$\forall\, d \in D, d \notin \{v, w\}, d > 0, k \in K$:

$$
\begin{aligned}
x_{v,d,k} + x_{d,w,k} &\leq 1,\ v = 2, w = 8 \\
x_{v,d,k} + x_{d,w,k} &\leq 1,\ v = 4, w = 9 \\
x_{v,d,k} + x_{d,w,k} &\leq 1,\ v = 5, w = 10 \\
x_{w,d,k} + x_{d,v,k} &\leq 1,\ v = 2, w = 8 \\
x_{w,d,k} + x_{d,v,k} &\leq 1,\ v = 4, w = 9 \\
x_{w,d,k} + x_{d,v,k} &\leq 1,\ v = 5, w = 10
\end{aligned}
$$

Figure 9: Fig 4.7 - Extension constraint