Boosting the Equity Momentum Factor in Credit: Is it still significant in Convertible Bonds?

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I. INTRODUCTION

The intricate dance between equity and debt markets has long been a subject of fascination for financial scholars and practitioners alike. One of the most compelling phenomena in this realm is the equity momentum factor, a well-documented anomaly wherein stocks that have performed well in the recent past tend to continue outperforming, and vice versa. Recent research, notably by Kaufmann et al. [1], has expanded this concept to the corporate bond market, revealing that the momentum observed in equities can be harnessed to predict the movement of corporate bonds with significant accuracy.

However, a segment of the market that bridges the gap between pure equities and pure bonds remains relatively unexplored in this context: convertible bonds. These hybrid instruments, with their unique blend of bond-like safety and equity-like upside potential, present a compelling case for investigation.

Conventional valuation processes for convertible bonds are in some sense always one step behind the true market value. In order to validate the price of the convertible bond in either way, the trader has to gather enough information about both the firm and the market. That means instead of predicting the value of the convertible bonds in the near future, conventional traders can only validate the value of convertible bonds in the path and seek for arbitrage opportunities. The finding by Kaufmann et al. [1] has introduced an opportunity in which using the past momentum traders can predict the future value of a convertible bond and actively create arbitrage opportunities.

II. UNIQUE NATURE OF CONVERTIBLE BONDS

Convertible bonds distinctly differ from traditional bonds. Unlike their conventional counterparts which are primarily influenced by macroeconomic shifts, convertible bonds show a heightened sensitivity to company-specific news. This makes them less reactive to broader economic trends but more attuned to the dynamics that influence equity prices. Furthermore, their typical structure offers lower coupon rates, compensating for the conversion feature. It's pivotal to note that post-conversion, the principal isn't repaid as these bonds metamorphose into equity shares.

When diving into the issuers, we find two predominant types: large corporations and growth-centric companies. Large corporations, having diverse financing avenues, often issue fewer convertible bonds. These usually come with lower interest rates and conversion factors. On the other hand, growth-focused entities, constrained in their financial access but aiming for rapid expansion, frequently release convertible bonds with elevated rates and conversion terms.

Typically, the valuation process for convertible bonds involves two primary approaches: the structural approach (also known as the firm-value approach) and the reduced-form approach (or stock-value approach). Their fundamental distinction lies in the input variables used. The structural approach relies on company-specific information, while the reduced-form approach uses market information (Kao, 2000). The reduced-form approach highly relies on credit risk as the primary market information, contingent on the estimation of default risk and recovery rate. Nevertheless, it disregards stock momentum which this exploration aims to clarify as distinctive market information concerning convertible valuation.

III. Hypothesis

Given the distinct characteristics of convertible bonds and their closer alignment with equities, this research seeks to validate the relationship between equity momentum and convertible bond price trends. Traditional convertible bond pricing models perceive them as a bond amalgamated with a call option, a foundation laid by the seminal work of Black and Scholes [2]. This dual nature implies that the equity momentum's influence on convertible bonds might be even more pronounced than on regular corporate bonds.

Convertible bonds, by design, benefit from the rise in equity prices, all while safeguarding with the bond's underlying assets. This blend of equity sensitivity and bond security might magnify the impact of equity momentum on their price movements. Consequently, our central hypothesis revolves around the idea that the relationship between equity momentum and convertible bond price trends should be more significant, given the bond's intrinsic equity-like characteristics. This research will test this hypothesis, aiming to elucidate the intricate dynamics between equity momentum and convertible bond behaviors.

IV. Methodology

In this section, we will briefly cover the methodology that we are going to employ in the research and the reason for choosing those methods.

The primary purpose of employing boosted regression trees and the associated methodology was to prove the relationship between equity momentum and the convertible bond market. The research aimed to determine whether information from past equity returns could predict future convertible bond returns. By using non-linear models such as Boosted Regression Tree instead of linear regression model, the study sought to capture complex relationships in the data that might be missed by traditional linear models.

The prediction of the trained model will be used to prove the correlation between equity momentum and convertible bond price. We will manually separate part of the data to be the testing set and test the performance of the model on that set of data. If such positive correlations exist then the model should react before the convertible bond price changes.

A. Boosted Regression Tree

Boosted regression trees are an ensemble learning technique that combines the predictions of several decision tree models to improve accuracy and reduce the likelihood of overfitting. The approach is called 'boosting' because it involves sequentially adding trees to the model; each new tree is created to correct the errors made by the previous ones. This sequential process continues until no significant improvements can be made or a predetermined number of trees is reached. Each tree in the ensemble is a weak learner—meaning it may only be slightly better than random guessing—but when combined, they form a strong learner that can make highly accurate predictions. The final prediction is typically made by taking the weighted average of the predictions from all the trees.

Following this introduction, the choice of boosted regression trees for the research is driven by their multifaceted strengths in handling financial data. The flexibility of boosted regression trees allows them to capture intricate patterns in the data, making them suitable for financial datasets that often exhibit non-linear relationships and interactions between variables. The resilience of boosted regression trees to outliers ensures that the model's performance is not unduly swayed by anomalous data points, which are commonplace in financial datasets. The implicit feature selection capability of the method is invaluable in dealing with the high-dimensional nature of financial data, where numerous potential predictors can overwhelm traditional analysis techniques. The built-in mechanisms to prevent overfitting, such as controlling the number of trees and their depth, are essential for constructing a model that generalizes well to unseen data. The proven track record of boosted regression trees in predictive accuracy, particularly in the domain of finance, underscores their suitability for the task at hand—unraveling the predictive relationship between equity momentum and bond prices. While they offer a moderate level of interpretability, the insights into feature importance they provide are crucial for understanding the driving factors behind the model's predictions.

Our research plans to use boosted regression trees to model the relationship between various factors (like past equity returns, size, and liquidity, potential corporate bond price and return) and the equity momentum factor in the convertible bond market. The trees were trained on historical data using an "expanding window" approach, where all available data up to a certain point in time was used to train the model, which then predicted the subsequent period's returns.

B. *Model Hyperparameters*

In this research project, the choice and calibration of the model class, as well as the restriction of complexity, were carefully considered to mitigate the risk of overfitting, which is particularly pertinent in financial applications where the signal-to-noise ratio is low. Boosted trees were chosen as the model class due to their moderate complexity compared to other machine learning models, which is suitable for applications with low signal-to-noise ratios. The complexity of the model was controlled by selecting optimal hyperparameters, which define the model's capacity to learn from data without capturing too much noise.

The hyperparameters of the model play a crucial role in determining its structure and complexity. The boosted regression trees, which form the core of the methodology, are defined by several hyperparameters. Among these, the number of trees was set at 1,000, ensuring a comprehensive ensemble for accurate predictions. The decay rate, which influences the learning rate of the model, was experimented with values of 0.01, 0.05, and 0.1. Additionally, the depth of the individual trees, which dictates how many splits they can have, was tested with values of 5, 10, and 20. These hyperparameters were meticulously chosen to ensure the model is sufficiently intricate to capture the underlying patterns in the data, yet not overly complex to the point of overfitting to the training data. The right balance of these hyperparameters is vital for the model's success in predicting bond returns based on equity momentum.

A fivefold cross-validation was applied to ensure robustness, where the training set was divided into five subsets. The models for each hyperparameter combination were estimated five times, each time leaving out a different subset for calibration and using the omitted data to evaluate the model. This method allowed all the training data to be used for both training and validation purposes, ensuring that the model's complexity was appropriately restricted to balance the trade-off between learning from the data and avoiding overfitting to the training data set.

C. *Features*

Features chosen for this research are based on their potential influence on convertible bond returns. The idea was to capture the momentum in equities and see if it had predictive power for convertible bond returns.

Features include past equity returns(which will be used to calculate momentum in equity), size (defined by bond market value and equity market capitalization), and liquidity (from both bond and equity perspectives). Past returns were taken as instrumental variables, with the last 24 one-month equity returns included. Past returns will be used to calculate the momentum of equity. The momentum of equity was calculated based on the two most prominent studies by Jegadeesh and Titman (1993) and Carhart (1997).[3][4] Jegadeesh and Titman explained momentum strategies for equity, and Carhart showed that most of the equity mutual fund persistence is attributable to exposure to momentum.

The definition of the one-month momentum factor (1×0) is to divide the return index at date t (RI_t) by the return index one month ago (RI_{t-1}). Yet, due to the pronounced impact of microstructural disturbances on momentum factors, we determine the one-month momentum factor using a daily average centered around the relevant dates:

$$1\times0 = \frac{\frac{1}{7}\sum_{i=0}^{6} RI_{t-i}}{\frac{1}{7}\sum_{i=-3}^{3} RI_{(t-1)-i}}$$

At time t, the 1×0 momentum factor, along with the 1×0 momentum factors from the previous 23 months, are directly incorporated into the model as features, resulting in a total of 24 one-month returns. In this study, we also reference the 0.33-, 0.67-, and 3- month momentum factors (0.33×0, 0.67×0, and 3×0) for comparative purposes. These momentums have also been provided as lagged signals over time resulting in one to three-month lagged signals (1x1, 1x2, and 1x3 for one month momentum feature). These factors follow the same calculation principle. The only variation is that an 11-day average of the Return Index (RI) is used due to the extended time frame of the original signal.

Not all the features of the dataset have been passed into our machine-learning model. Unlike the original research where the researcher tries to predict the return in the bond market by capturing the non-linear relationship between equity momentum and bond return, our research lies on a derivative that is highly correlated with equity. So, if we provide the return of the equity over the same time period, it is fairly easy for the model to learn from the information of the equity market and utilize those to predict the convertible bond market. During our research, we have tried to feed the return of the equity market as a feature into our model but quickly realized that this is an excellent parameter that can be used to do the regression. Then, suddenly the model will only take that as the parameter that is important. Based on our observation, the return of the equity market usually has 10 times or more importance than any other features we have fed into the model. As a result, we decided to remove some of the features that can give the model future information and information that is too highly related which can cause the model to overfit. Features removed are Equity returns, dates, and spreads of different markets.

V. DATASETS AND RESOURCES

The data used in the original research is derived from monthly constituent data spanning from January 2020 to February 2023. This data comes from two indexes of the Intercontinental Exchange, Inc. (ICE): the Global Corp IG Index (G0BC, referred to in the research as IG) and the Global HY Index (HW00, referred to as HY). For both indexes, only bonds denominated in US dollars were considered. Every month-end, ICE provides various bond characteristics, including credit spread (option-adjusted spread, OAS), credit rating, time to maturity (TTM), total return, excess return over US Treasuries, and sector. The research focused on bonds from the sectors "Financial," "Utility," and "Industrials."

For our research, we retrieve data from the Bloomberg terminal. In this study, a set of representative convertible bonds are chosen for analysis, along with their corresponding equity data.

In our cross-asset momentum analysis, we concentrated on convertible bonds tied to listed companies, emphasizing those with direct connections to publicly traded entities. Given our keen interest in understanding how equity momentum factors can predict convertible bond returns, it's pertinent to note that certain companies hold both convertible and common bonds in their portfolio. Recognizing this, it becomes imperative to consider both types of bonds comprehensively to assess the impact of equity momentum factors on the return dynamics of convertible securities. To identify the convertible bonds for consideration, we consulted the primary convertible bond ETF and selected the top 15 most traded convertible bonds. Subsequently, we matched them to their corresponding equity and common bonds. However, it's worth noting that some companies exclusively issued convertible bonds without common bonds. Consequently, in the later stages of our model testing, we segregated the convertible bonds into two groups: one comprising companies issuing both convertible and common bonds, and the other consisting of companies exclusively issuing convertible bonds.

Our main focus was on creating momentum factors and testing their capability to predict convertible bond returns. To achieve this, we only considered features that played a role in the speed of information flow from the stock market to the bond market and in assessing performance quality. These features encompass factors like market liquidity and the condition of the bond market. Another possible approach would be to introduce additional features, such as bond spreads. We believe that these features notably affect the expected returns of bonds. The specific data fields selected are: Security Price Series, Security Yield Series, Convertible Bond Prices, Convertible Bond Yields, Convertible Bond Spreads and Convertible Bond Trading Volumes. As we explore the data that we collected, some companies exclusively issue convertible bonds, while others also issue common bonds. Consequently, we can select representatives from each group and visualize the data correlation for a comparative analysis.

Within the subset of companies that have issued common bonds, we take Ford's stock, convertible bond, and common bond as examples, the price trend and the correlation between their time series are plotted in Fig 1 and Fig 2:

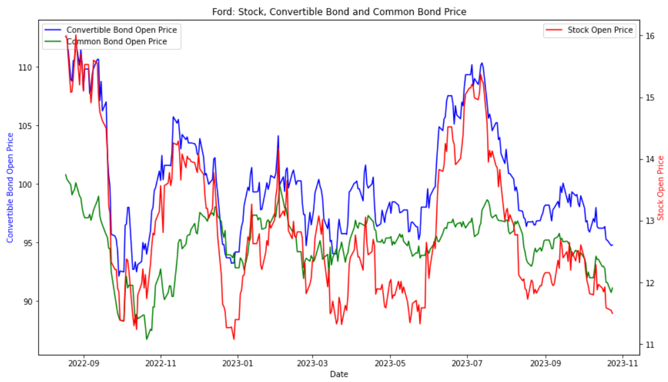

Fig 1: Ford Stock & Bonds Price

Fig 1 shows the observed similarities in the trends of Ford’s convertible bonds, common bonds, and their corresponding stocks suggest a potential interconnectedness in their movements. Also, we can observe synchronization in these three trends, which implies that the momentum in stock prices may have a concurrent impact on both convertible bonds and common bonds.

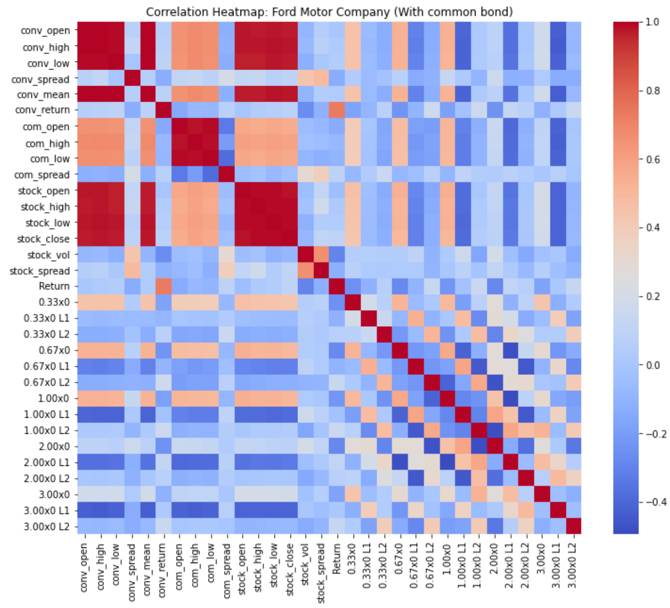

Fig 2: Ford Correlation Matrix of Original Data

The correlation matrix plot above illustrates the relationships between variables, indicating a robust correlation between the prices and volumes of both stocks and common bonds with convertible bond price data. The calculated momentum factors exhibit a systematic correlation with the return of convertible bonds. This consistent relationship suggests a discernible connection between the derived momentum factors and the performance of convertible bonds. However, given that our chosen model is the Boosting algorithm, which inherently possesses the capability to filter and iterate through variables, there might be no need for explicit feature selection. The algorithm's inherent ability to adapt and select variables during iterations allows for a dynamic approach, potentially eliminating the need for manual variable curation.

Within the subset of companies that have not issued common bonds, we take Airbnb's stock, and convertible bond as examples, the price trend and the correlation between their time series is plotted in Fig 3 and Fig 4:

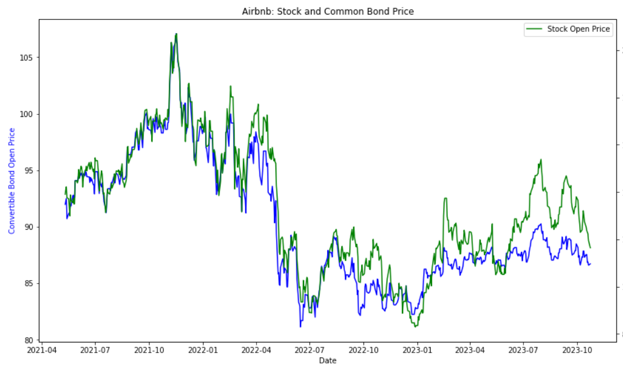

Fig 3: Airbnb Stock & Bonds Price

In the plot of the prices of Airbnb's convertible bonds and stocks, a striking similarity in their trends is evident. This provides a promising foundation for exploring momentum in our research.

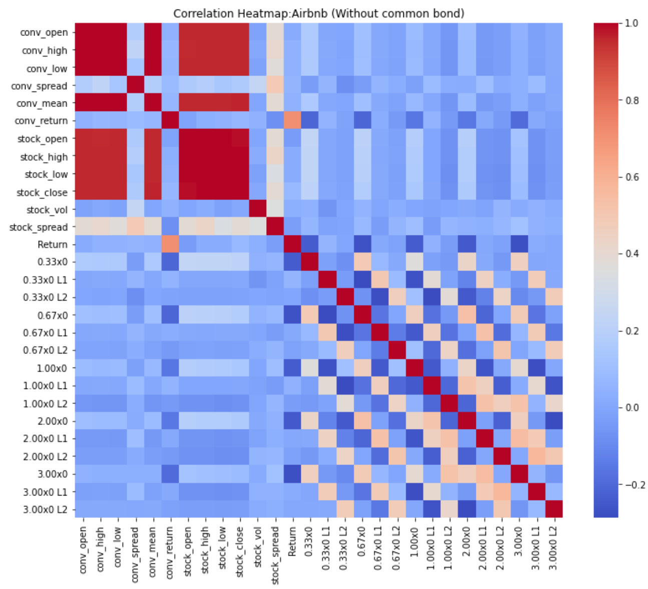

Fig 4: Airbnb Correlation Matrix of Original Data

In the correlation matrix plot for Airbnb, a noticeable increase in correlation is observed between convertible bond returns and momentum factors. This indicates that in the absence of common bond issuance, there is a stronger correlation between convertible bond returns and stock price momentum.

In conclusion, we can see that the convertible bond price and the stock price are highly correlated, irrespective of whether the company has issued common bonds or not. Simultaneously, it can be observed that the correlation between the momentum factor and convertible bond returns strengthens when the company does not issue common bonds. Meanwhile, other factors still need to be adjusted and this provides a great opportunity for the Boosting algorithms to thrive.

The choice of these data sets enables us to delve into the dynamics and interplay between securities and convertible bond prices. Additionally, it serves as our primary data source for the construction of momentum factors, which will be subject to testing within the realm of convertible bonds.

VI. Model Results

The difficulty in using machine learning models often stems from concerns about their explainability. To address this, our approach goes beyond merely examining the overarching model structure. We delve deeply into the results, placing a strong emphasis on the importance of each feature. This granular analysis is pivotal in understanding not just the model's predictions, but also the underlying data patterns and the relative influence of each factor. Furthermore, we rigorously test the boosting factor to ensure the reliability of our findings. By doing so, we gain valuable insights into the model's behavior, enabling us to simplify and refine it for better performance and generalization. This process also aids in identifying key features for effective domain-specific applications and in ensuring that the model's reliance on certain features does not introduce bias, thereby aligning with ethical and regulatory standards. In essence, this detailed examination enhances our comprehension of the entire narrative the data tells, ensuring that our machine-learning models are not just powerful, but also transparent and trustworthy.

A. *Feature Importance*

The importance of variables was determined by assessing their relative influence, considering whether each variable was chosen for splitting during the tree-building process and the degree to which it contributed to the improvement in squared error across all trees. When the algorithm split a node based on a particular feature, it led to an enhancement in forecasting accuracy and a decrease in squared error. The extent of this reduction in squared error was indicative of the importance of the selected feature. Additionally, a higher reliance on a specific feature for predictions corresponded to a greater variable importance in the model.

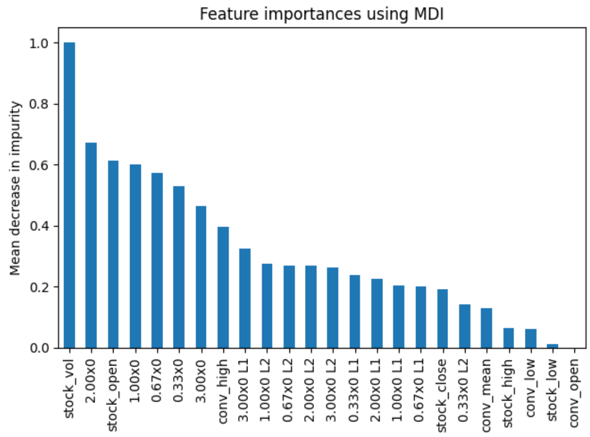

Fig 5: Feature importance of dataset without common bond

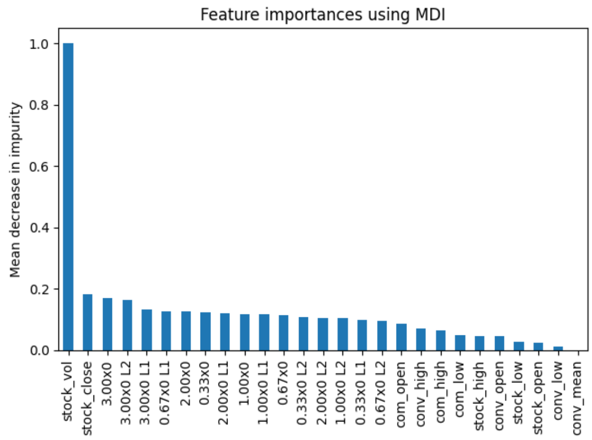

Fig 6: Feature importance of dataset with common bond

Fig 5 and 6 indicates that, for both groups, irrespective of whether common bonds are issued, the volume of stock emerges as the most influential feature affecting the return on convertible bonds. Meanwhile, we can see that other features measuring size and liquidity are among the most important features.

It is worth noting that momentum factors constructed based on equity also possess high feature importance. Moreover, in the group without common bonds, the feature importance of equity momentum factors is even higher. This could be because, in the group without common bonds, equity momentum factors are more prominent, possibly due to the greater impact of stock market trends on convertible bond returns in such scenarios. This may reflect the significance of stock market movements in predicting returns on convertible bonds, especially in the absence of common bonds.

Another interesting phenomenon has also been observed in the importance graph, which is that for companies which issued only convertible bonds without common bonds, the short-term equity momentum factor has a more significant influence on the model. On the contrary, for companies that issued both convertible bonds and common bonds, longer-term momentum factors such as three-month momentum and three-month momentum with a lag of 1 to 2 months are more important.

B. *Feature Importance Over Time*

As Fig 5 and Fig 6 have illustrated before, equity market volume is always the most important feature in the regression model. Nevertheless, a noteworthy observation is that periodically some momentum factors emerge as significant features in the dataset without common bonds. From the figure, we can see that for most of the features, its importance has a periodic movement. This suggests that the prediction of convertible bond returns issued by companies without common bonds relies more on equity momentum.

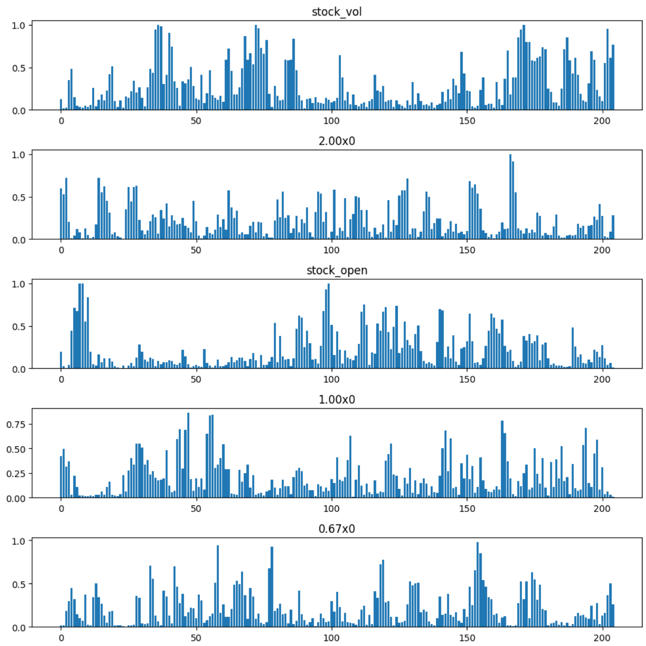

Fig 7. Feature importance over Time of the dataset without common bond

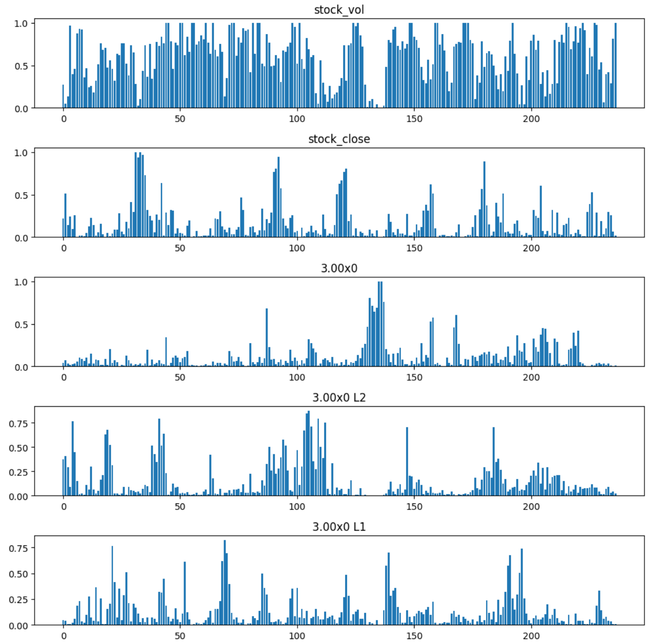

Fig 8. Feature importance over time of the dataset with common bond

C. *Partial Dependence*

Variable importance serves as the most prominent method to indicate the relevance of variables for our model. However, it does not elucidate the functional form of the relationship between predictors and the outcome variable. This challenge can be addressed by utilizing partial-dependence plots (PDPs).

Partial Dependence Plots (PDP) are a crucial analytical tool in machine learning that provide insights into the influence of a single feature on the predictions made by a model. These plots offer a visual representation of the relationship between a specific variable and the outcome predicted by the model, while averaging out the effects of other variables. The core purpose of PDPs is to decipher and communicate the impact of features on model predictions, aiding in understanding and interpreting complex models.

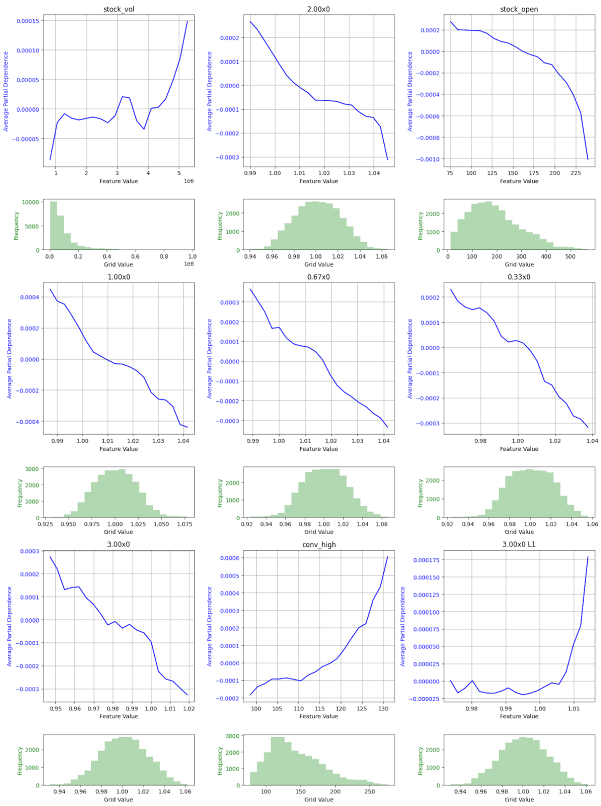

Fig 9. Partial dependence of the dataset with common bond

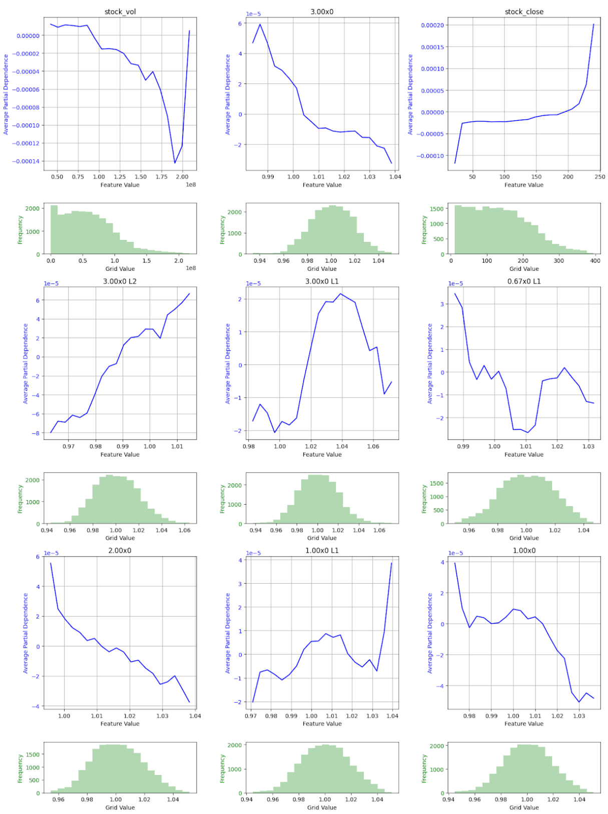

Fig 10. Partial dependence of the dataset with common bond

PDPs are particularly valuable because they help in identifying the nature of the relationship between the feature of interest and the predicted outcome – whether it is linear, non-linear, or more intricate. By isolating and examining each feature individually, PDPs simplify the understanding of the model's behavior, making them an indispensable tool in areas where model interpretability is as crucial as model accuracy.

Results from both Fig 7 and Fig 8 show that higher equity momentum has a distinct effect on these two different groups. For the group where common bonds are not issued, the PDPs for different momentum factors demonstrated a variety of relationships. Meanwhile, for the group with the issuance of common bonds, the consistent pattern in the PDPs for equity momentum factors was observed. Such inconsistency between the action of momentum factor in two different groups could be caused by the existence of a common bond or due to the limitation of PDPs.

While Partial Dependence Plots are a valuable tool in interpreting machine learning models, they come with certain limitations that must be acknowledged. Firstly, PDPs assume that the feature being analyzed is independent of other features, which is often not the case in real-world datasets. This assumption can lead to misleading interpretations, especially in the presence of strong correlations between features. In our research it is very hard to say that all momentums are independent from each other. Some relationships might have evolved after common bonds were introduced.

D. R-Squared

R-squared is a statistical measure that represents the proportion of the variance in the dependent variable that is predictable from the independent variables in a regression model.

The in-sample R-squared over the training data window provided in Fig 11 and Fig 12 suggests that the chosen independent variables predict the dependent variable better in the model with convertible bonds than in the model with common bonds. A reasonable assumption emerged based on the underlying truth that convertible bond equals to an option plus a bond. When there does not exist a common bond, convertible bond return is only influenced by equity return. While on the other hand, when there exists a common bond, the return of the convertible bond has also included the movement in the common bond market. Thus, since we haven't provided enough information related to the bond market and only feeding momentum from the equity market, it is making sense that our model has a better interpretation when there is no common bond exists. Another possibility is that companies that have not issued common bonds are predominantly in a growth stage, and their convertible bonds tend to closely mirror their stock performance.

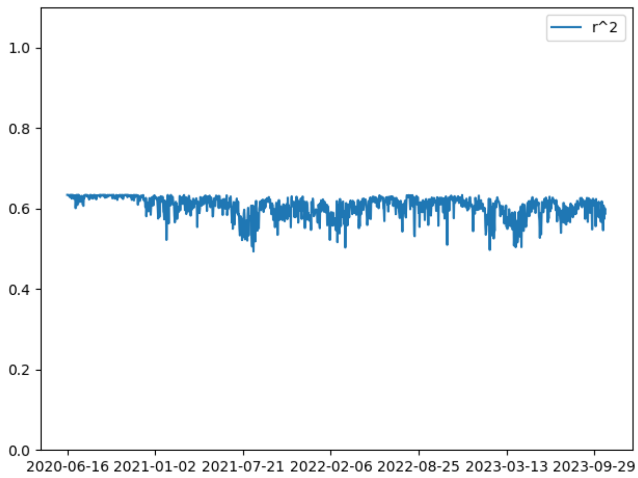

Fig 11. R-squared of the dataset with common bond

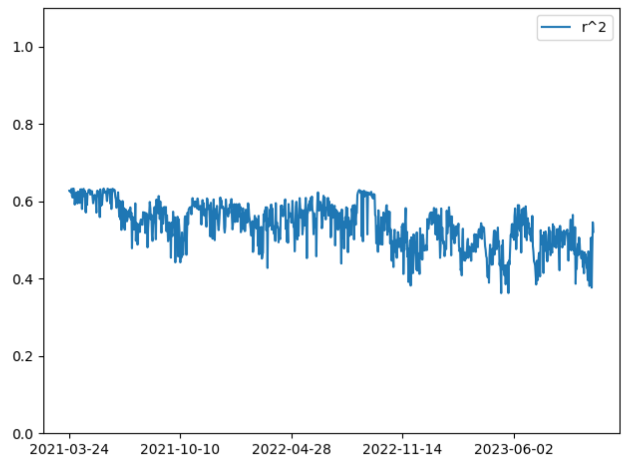

Fig 12. R-squared of the dataset with common bonds

E. Performance on Boosting Factor

To ensure result accuracy and assess the model's robustness, we employed a rolling time window to evaluate the predictive accuracy over time. It is evident that the performance is more robust in the group without common bonds, with a relatively stable prediction error. In the group with common bonds, there is greater fluctuation in predictive accuracy.

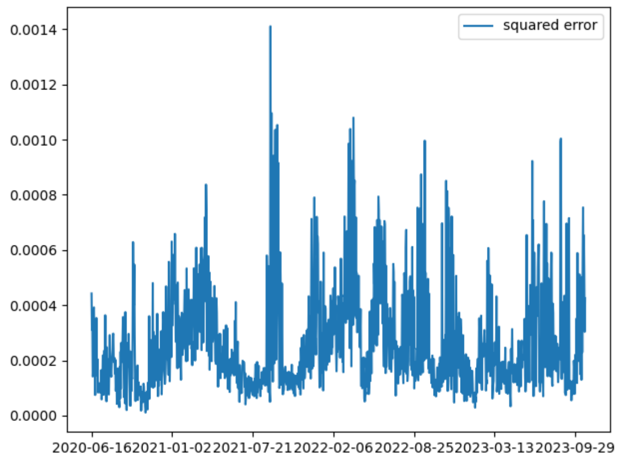

Fig 13. The squared error of the dataset with common bond

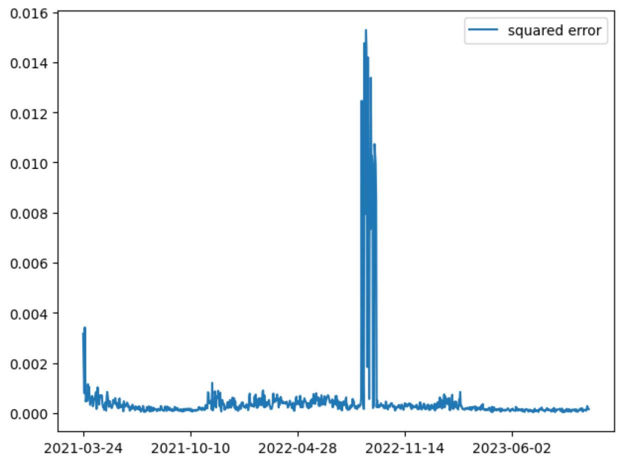

Fig 14. The squared error of the dataset with common bond

VII. Conclusion

Regarding predictive effectiveness, feature importance, and partial dependence, the presence or absence of common bond issuance introduces variations in outcomes. The speculated rationale is that when a company exclusively issues convertible bonds without common bonds, all information embedded in the equity price sequence can be harnessed to predict convertible bond returns.

Conversely, if a company issues both common and convertible bonds, it is reasonable to assume that convertible bond returns are influenced by both the equity sequence and common bonds. Given that companies issuing both types of bonds are primarily large corporations utilizing convertible bonds for debt financing or stock repurchasing, the pricing of convertible bonds becomes more intricately linked to the relationship between their debt and equity position.

This study focuses on the predictive utility of the equity market momentum for convertible bonds. Consequently, when predicting the returns of these convertible bonds, the input information is relatively limited, leading to a comparatively lower accuracy in prediction.

VIII. Group member's contribution

The main research process of this paper includes literature search and method conception, data collection and organization, model construction, and result collection and reflection.

To be more specific, Yichen Liu is responsible for data collection and organization, as well as some coding work for model construction. Zaichuan You will be mainly responsible for replicating methodology that has been mentioned in H.Kaufmann's research and creation of boosted regression tree. Liching Tseng will be mainly responsible for data processing and statistical analysis.

IX. Reference

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