3EJ4 LAB FIVE

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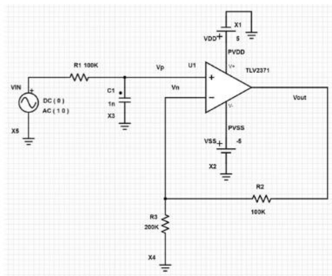
Questions for Part 1:

Q1.

(1)



Yichen Xu 400247938 July 19

(1) $T_{(s)} = \frac{V_o(s)}{V_{in}(s)}$

$\because V_+ = V_- \quad \therefore V_- = V_{o(s)} \frac{R_3}{R_2+R_3} = V_+$

$\therefore \frac{V_{i(s)} - V_+}{R_1} = \frac{V_+}{\frac{1}{C_1 s}} \quad \therefore \frac{V_{i(s)} - V_{o(s)}\frac{R_3}{R_2+R_3}}{R_1} = V_{o(s)} \frac{R_3}{R_2+R_3} C_1 s$

$\therefore \frac{V_o(s)}{V_{in}(s)} = \frac{R_3+R_2}{R_3(R_1 s C_1 + 1)}$

$\therefore T(s) = \frac{R_3+R_2}{R_3(R_1 s C_1 + 1)}$ (The transfer function)

* Low frequency gain

(when $S=0$, we can get low-frequency gain)
($R_2 = 100k$, $R_3 = 200k$, $R_1 = 100k$)

$T_{(s=0)} = A_v = 1 + \frac{R_2}{R_3} = 1 + 0.5 = 1.5 V/V \approx 3.52\,dB$ Page 4

* the -3dB frequency f_c ($f_{3dB} = \frac{1}{2\pi RC}$ = cut off frequency)

$f_c = \frac{1}{2\pi R_1 C_1} = 1591\,Hz = 1.591\,kHz$

(2)

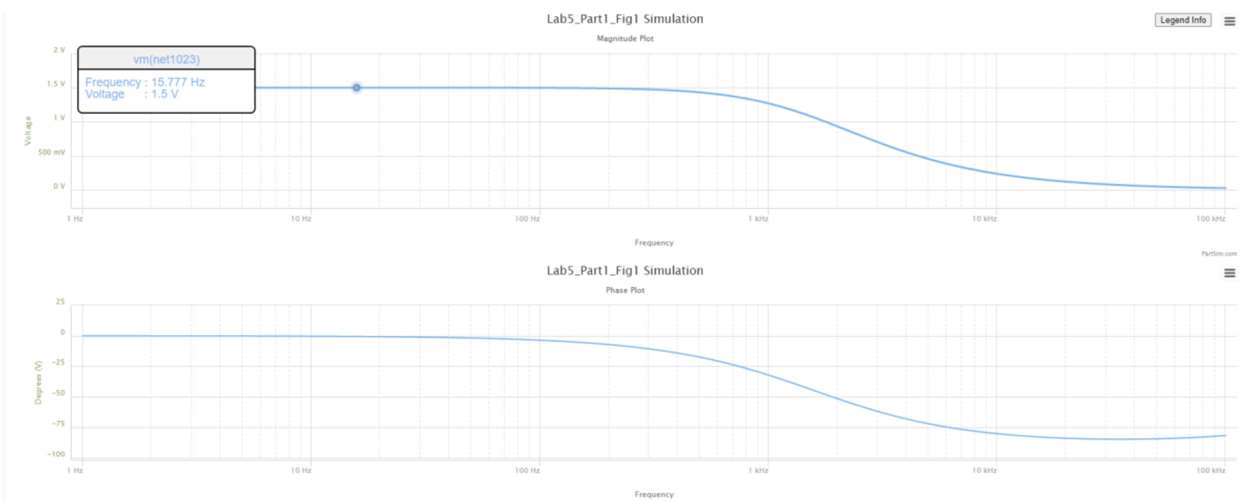


Figure 1(Transfer Function of the First orderrr low pass filter)

3	1	1.500000172	-0.036069274
4	1.02305973	1.500000158	-0.036901021
5	1.046651211	1.500000143	-0.037751948
6	1.070786705	1.500000128	-0.038622498
7	1.095478757	1.500000112	-0.039513122
8	1.120740201	1.500000095	-0.040424284
9	1.146584168	1.500000078	-0.041356456
10	1.173024089	1.50000006	-0.042310125
11	1.200073707	1.500000041	-0.043285784
12	1.227747083	1.50000002	-0.044283942
13	1.256058599	1.5	-0.045305118

(Data for Av in Step 1.3)

3	1	0.150733	-0.04738
4	1.023293	0.150697	-0.03871
5	1.047129	0.150725	-0.05035
6	1.071519	0.150721	-0.05976
7	1.096478	0.15074	-0.04839
8	1.122018	0.150721	-0.05523
9	1.148154	0.150721	-0.05791
10	1.174898	0.150733	-0.05753
11	1.202264	0.150766	-0.0554
12	1.230269	0.150736	-0.08636
13	1.258925	0.150673	-0.05251

(Data for Av in Step 1.8)

321	1407.715367	1.121597902	-41.50095622
322	1440.176903	1.110238218	-42.14788529
323	1473.386993	1.098710673	-42.79617597
324	1507.362899	1.087021837	-43.44549366
325	1542.122281	1.075178653	-44.09550093
326	1577.683204	1.06318842	-44.74585836
327	1614.064152	1.051058766	-45.39622546
328	1651.284035	1.038797632	-46.04626151
329	1689.362199	1.02641324	-46.69562647
330	1728.318435	1.013914073	-47.34398184
331	1768.172991	1.001308839	-47.99099149
332	1808.946583	0.988606451	-48.63632259
333	1850.660402	0.975815989	-49.27964637
334	1893.336131	0.962946674	-49.92063896
335	1936.995951	0.950007836	-50.55898215

(Data for fc in Step 1.3)

307	1096.478	0.11406	-40.4487
308	1122.018	0.112948	-41.064
309	1148.154	0.111945	-41.7639
310	1174.898	0.11062	-42.4198
311	1202.264	0.109444	-43.0177
312	1230.269	0.108339	-43.7244
313	1258.925	0.107066	-44.4059
314	1288.25	0.105939	-44.9776
315	1318.257	0.104512	-45.7147
316	1348.963	0.103408	-46.2817
317	1380.384	0.102189	-47.025
318	1412.538	0.100867	-47.5719
319	1445.44	0.099675	-48.2746
320	1479.108	0.098333	-48.9631
321	1513.561	0.097076	-49.5829

(Data for fc in Step 1.8)

For the low-frequency gain, we can observe that Vout is 1.500000172V with Vin = 1V, computing, the low-frequency gain Av=1.5/1 = 1.5 V/V from its simulated data Step1.3. In terms of the measurement data, we can observe that Vout is 0.150733V with Vin = 100mV, computing, the low-frequency gain Av=0.150733V /100mV = 1.5 V/V from its measured data Step1.8. These two A values are identical to the ones we estimated in Part 1 calculation.

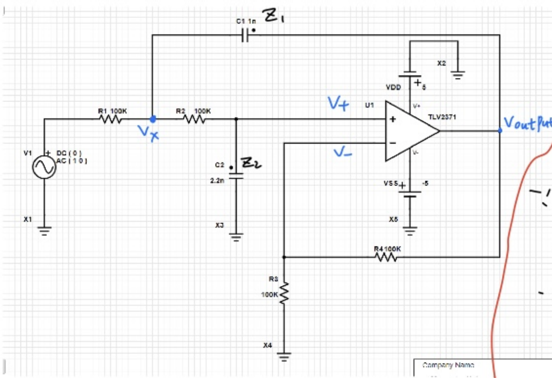
For the -3dB frquency fc, we can see that in Steps 1.3 and 1.8 that the simulated and measured values of fc are 1614Hz and 1288.25Hz respectively given the data above,. The simulated value is a better match. This measurement mistake could be attributable to the amplifier's physical qualities.

Questions for Part 2:

Q2.

2.6, respectively. Yichen Lu 400247938 Jul 19

Q2.



Transfer function:

$$T(s) = \frac{V_o(s)}{V_i(s)}$$

KCL at Node V_x:

$$\therefore \frac{V_x - V_{in}}{R_1} + \frac{V_x}{R_2 + Z_2} + \frac{V_x - V_{out}}{Z_1} = 0 \quad ①$$

Voltage division:

$$\because V_- = \frac{R_3}{R_3 + R_4} V_{out} = \frac{1}{2} V_{out} \quad ②$$

Ideal op amp:

$$\because V_+ = V_- \quad ③$$

$$\therefore 2V_+ = 2V_- = V_{out} \quad ④$$

$\because$ voltage division

$$\therefore V_x = \frac{R_2 + Z_2}{Z_2} (V_+) \quad ⑤$$

$\because$ ④ & ⑤

$$\therefore V_x = \frac{R_2 + Z_2}{Z_2} \cdot \frac{V_{out}}{2} = \frac{R_2 + Z_2}{2Z_2} (V_{out})$$

Substitute V_x in ①

$$\therefore \frac{V_{in}}{R} = \left(\frac{R + Z_2}{2Z_2 R} + \frac{1}{2Z_2} + \frac{R - Z_2}{2Z_2 Z_1} \right) V_{out}$$

$$\therefore \frac{V_{out}}{V_{in}} = T(s) = \frac{2 Z_1 Z_2}{R^2 + R(2Z_1 - Z_2) + Z_1 Z_2}$$

$$\therefore T(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{2 \cdot \frac{1}{s \cdot 1n} \cdot \frac{1}{s \cdot (2.2n)}}{(100k)^2 + 100k \cdot \left(\frac{2}{s \cdot 1n} - \frac{1}{s \cdot (2.2n)} \right) + \frac{1}{s \cdot 1n} \cdot \frac{1}{s \cdot (2.2n)}}$$

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$$\therefore T(s) = \frac{\frac{10}{11} \times 10^8}{s^2 + \frac{17}{11} \times 10^4 \cdot s + \frac{5}{11} \times 10^8}$$

* Low-frequency Gain (when $s=0$)

$$A_{low\text{-}f} = \frac{\frac{10}{11} \times 10^8}{0 + 0 + \frac{5}{11} \times 10^8} = 2\ V/V = 20 \log 2\ dB \approx 6.02\ dB$$

2	Hz	dBV	deg
3	1	6.020577025	-0.122644031
4	1.02305973	6.020576448	-0.125472165
5	1.046651211	6.020575844	-0.128365515
6	1.070786705	6.020575212	-0.131325585
7	1.095478757	6.020574551	-0.134353913
8	1.120740201	6.020573859	-0.137452073
9	1.146584168	6.020573134	-0.140621675
10	1.173024089	6.020572376	-0.143864367
11	1.200073707	6.020571583	-0.147181834

3	1	6.045335537	-0.11051686
4	1.023292992	6.045384875	-0.11227988
5	1.047128548	6.045768201	-0.10623526
6	1.071519305	6.045115884	-0.1097998
7	1.096478196	6.045379051	-0.11671016
8	1.122018454	6.044316838	-0.11553917
9	1.148153621	6.044502496	-0.11861858
10	1.174897555	6.044317209	-0.11439544
11	1.202264435	6.044304848	-0.11798997

(Data for Av in Step 2.2) (Data for Av in Step 2.6)

The low frequency gain should be around 2 V/V, or roughly 6.02dB, according to the calculation in (1). With Vin = 1V = 0 dB, the output for simulation in step 2.2 is 6.020577025 dB. As a result, the low frequency gain is 6.020577025 dB, which is nearly identical to our calculation. The gain is 6.045335537 dB for step 2.6, which is fairly close to the value we intended (6.02 dB).

Q3.

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Q3.

Q3. (20 Points) Calculate (1) the pole frequency f_o, (2) the cut-off frequency (or -3dB frequency) f_c, (3) the pole quality factor Q, (4) the peak value of the magnitude of the transfer function, and (5) the frequency f_{max} where the peak value of the magnitude of the transfer function happens. Verify the calculated results using the simulated data obtained in Step 2.2 and the measured data obtained in Step 2.6, respectively. Yichen Lu 400247938 luy191

(1) ∵ $s^2 + \frac{17}{11}\times 10^4 s + \frac{5}{11}\times 10^8 = 0$

∵ $s^2 + \frac{\omega_0}{Q}s + \omega_0^2 = 0$

∴ $\omega_0^2 = \frac{5}{11}\times 10^8$

$f_0 = \frac{\omega_0}{2\pi} = \frac{\sqrt{\frac{5}{11}\times 10^8}}{2\pi} = 1.073\text{kHz}$

∴ $Q = 0.33$

(2) $|T(j\omega_c)| = \frac{\sqrt{2}}{2}|T(j\omega_c)|$; $\omega_L = 0$

$\left|\frac{a_0}{-\omega_c^2 + \frac{\omega_0\omega_c}{Q}j + \omega_0^2}\right| = \frac{1}{\sqrt{2}}\cdot\frac{a_0}{\omega_0^2}$

∴ $|-\omega_c^2 + 4.545\times10^7 + 2.045\times10^4\omega_c j|$

$= \sqrt{2}\times 4.545\times10^7$

∴ $(\omega_c^2 - 4.545\times10^7)^2 + (2.045\times10^4)^2\omega_c^2 = 2\times(4.545\times10^7)^2$

∴ $\omega_c^2 \approx 6.19407\times10^6$

$\omega_c = 2488.79$ rad/s

∴ $f_c = \frac{\omega_c}{2\pi} = \frac{2488.79}{2\pi} = 396.1\text{Hz}$

(3) $Q = 0.33$

(4) ∵ $|T(s)|$ to be maximum

∴ denominator to be minimum

∴ $\sqrt{\omega^2 + 3.273\times10^8\omega + 2.066\times10^{15}}$ to be minimum

∵ $\omega \geq 0$ ∴ $\omega = 0 = \omega_{max}$ ∴ $|T s| = |T s|_{max}$

∴ $|T(s)|_{max} = 2\text{ V/V} = 6.02\text{dB}$

(5) $f_{max} = \frac{\omega_{max}}{2\pi} = \frac{0}{2\pi} = 0\text{Hz}$

(1) fo =1.073kHz

(2) fc = 396.1Hz

(3) Q = 0.33

(4) |T(s)|max = 2V/V = 6.02dB

(5) fmax = 0Hz

DATA for q3	Calculated	Simulated	Measured
fo	1.073kHz	1070.78670498641Hz	954.992586021435Hz
fc	396.1Hz	401.754172617276Hz	436.515832240166Hz
Q	0.33	/	/
|T(s)|max	6.02dB	6.020577025 dB	6.045335537 dB
fmax	0Hz	1	1

In conclusion, the calculated data is almost the same with the data from the simulated and measured data.

Questions for Part 3:

Q4.

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∵ Ideal op-amp

∴ $V_- = V_+ = 0V$ (GND)

∵ KCL at V_x

$$\frac{V_x - V_{in}}{R} + \frac{V_x - V_{out}}{Z_C} + \frac{V_x - V_{out}}{Z_C + R} = 0 \quad ①$$

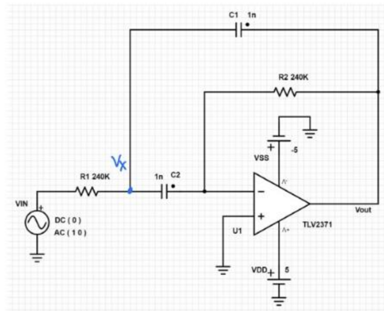
∵ voltage division

$$\therefore \frac{V_x - 0}{Z_C} = \frac{0 - V_{out}}{R} \quad \therefore V_x = -\frac{Z_C}{R} V_{out} \quad ②$$

Substitute ② in ①

$$\therefore \frac{V_{out}}{V_{in}} = -\frac{1}{\frac{Z_C}{R} + \frac{Z_C + R}{Z_C} + 1} = -\frac{1}{\frac{Z_C}{R} + \frac{R}{Z_C} + 2}$$

$$\therefore T(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{-\frac{5}{12} \cdot 10^4 s}{(s + \frac{5}{12} \cdot 10^4)^2}$$



$$|T(s)| = \left| \frac{-\frac{5}{12} \cdot 10^4 s}{(s + \frac{5}{12} \cdot 10^4)^2} \right| = \left| \frac{\frac{5}{12}(10^4) j\omega}{(j\omega + \frac{5}{12}(10^4))^2} \right|$$

Let $a = \frac{5}{12} \cdot 10^4$

$$\therefore |T(s)| = \left| \frac{aj\omega}{(j\omega + a)^2} \right| = \left| \frac{a\omega j}{-\omega^2 + a^2 + 2a\omega j} \right| = \frac{a\omega}{\sqrt{(a^2 - \omega^2)^2 + (2a\omega)^2}}$$

$$= \frac{a\omega}{a^2 + \omega^2} \le \frac{a}{\frac{a^2}{\omega} + \omega} = \frac{a}{2a} = \frac{1}{2} \text{ (when } \omega = a)$$

∴ Center frequency gain $A_C = \frac{1}{2}$ V/V $= -6.0206$ dB

The center frequency gain is -6.02dB. From the calculation, we know. that ω0 = 41667 rad/s, then we can get f0 = ω0 / 2π= 663.15 Hz.

We can observe that the center frequency gain is 6.02058966 dB with frequency of 663.4102514 H from its simulated data Step3.2. In terms of the measurement data, we can observe that the center frequency gain is -5.56073747 dB with frequency of 501.1872336 Hz from its measured data Step3.6 These two values are identical to the ones we estimated in calculation.

Q5.

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from Q4 we know

$$T(s) = \frac{4166.7s}{s^2 + 8333.3s + 4166.7^2}$$

when $\omega = \frac{1}{RC} = 4166.7$ rad/s

when $\omega_0 = 4166.7$ rad/s, $|T(s)|$ reaches it max

$$\therefore \left(s + \frac{1}{RC}\right)^2 = s^2 + \frac{2}{RC}s + \frac{1}{R^2C^2}$$

$$= s^2 + \frac{\omega_0}{Q}s + \omega_0^2$$

$$\therefore \begin{cases} \omega_0 = \frac{1}{RC} = 4166.7 \text{ rad/s} \\ Q = \frac{1}{2} \end{cases}$$

$$\because p = -\frac{\omega_0}{2Q} \pm j\omega_0\sqrt{1-\frac{1}{4Q^2}}$$

$$\therefore p_1 = p_2 = -\frac{\omega_0}{2Q} = -4166.7$$

$$\therefore \omega_{p_1} = \omega_{p_2} = 4166.7 \text{ rad/s}$$

$$BW = \omega_{p_2} - \omega_{p_1} = 0$$

(1) center frequency ω0 = 4166.7 rad/s

(2)pole frequency factor Q = 0.5

(3)ωp1= ωp2= 4166.7 rad/s

(4)3-dB bandwidth BW = 0

Data for Q5	Calculated	Simulated	Measured
ω0	4166.7rad/s	663.4102514*2π=4168.3 rad/s	501.1872336*2π=3149.1 rad/s
Q	0.5	/	/

ωp1	4166.7 rad/s	663.4102514*2π=4168.3 rad/s	501.1872336*2π=3149.1 rad/s
ωp2	4166.7 rad/s	663.4102514*2π=4168.3 rad/s	501.1872336*2π=3149.1 rad/s
BW	0	0	0

In conclusion, the calculated data is almost the same with the data from the simulated and measured data.