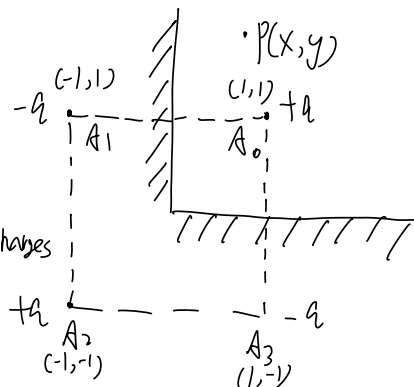


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$P(x, y)$ is in the first quadrant

$A_0(1, 1)$ is the point charge

$A_1(-1, 1)$, $A_2(-1, -1)$, $A_3(1, -1)$ are the image charges



$$\therefore V = \frac{q}{4\pi\epsilon_0} \cdot \frac{1}{|r|}$$

$$\therefore V_P = V_{P0} + V_{P1} + V_{P2} + V_{P3}$$

$$V_{P0} = \frac{q}{4\pi\epsilon_0} \cdot \frac{1}{\sqrt{(x-1)^2 + (y-1)^2}} \quad V_{P1} = \frac{-q}{4\pi\epsilon_0} \cdot \frac{1}{\sqrt{(x+1)^2 + (y-1)^2}}$$

$$V_{P2} = \frac{q}{4\pi\epsilon_0} \cdot \frac{1}{\sqrt{(x+1)^2 + (y+1)^2}} \quad V_{P3} = \frac{-q}{4\pi\epsilon_0} \cdot \frac{1}{\sqrt{(x-1)^2 + (y+1)^2}}$$

$$\therefore V_P = \frac{q}{4\pi\epsilon_0} \left[\frac{1}{\sqrt{(x-1)^2 + (y-1)^2}} - \frac{1}{\sqrt{(x+1)^2 + (y-1)^2}} + \frac{1}{\sqrt{(x+1)^2 + (y+1)^2}} - \frac{1}{\sqrt{(x-1)^2 + (y+1)^2}} \right]$$

$$\begin{aligned}
 \therefore \vec{E}_p &= -\vec{\nabla} \cdot V \\
 &= \frac{q}{4\pi\epsilon_0} \left[-\frac{x-1}{((x+1)^2+(y-1)^2)^{\frac{3}{2}}} + \frac{x+1}{((x+1)^2+(y-1)^2)^{\frac{3}{2}}} - \frac{x-1}{((x+1)^2+(y+1)^2)^{\frac{3}{2}}} + \right. \\
 &\quad \left. \frac{x+1}{((x-1)^2+(y+1)^2)^{\frac{3}{2}}} \right] \vec{a}_x + \frac{q}{4\pi\epsilon_0} \left[-\frac{y-1}{((x-1)^2+(y-1)^2)^{\frac{3}{2}}} + \frac{y+1}{((x+1)^2+(y-1)^2)^{\frac{3}{2}}} \right. \\
 &\quad \left. - \frac{y+1}{((x+1)^2+(y+1)^2)^{\frac{3}{2}}} + \frac{y-1}{((x-1)^2+(y+1)^2)^{\frac{3}{2}}} \right] \vec{a}_y \\
 &\quad (x \geq 0 \text{ \& } y \geq 0)
 \end{aligned}$$