

Laboratory 4 Discussion

A short discussion about how the single-sided spectrum is plotted in Lab 4, i.e.,

```
f = Fs/2*linspace(0,1,L/2+1)
plot(f,2*abs(Y(1:L/2+1)))
```

About the divide by L. This is what they refer to as fft "scaling". Depending on how you plan to use the fft results, people may use different scaling factors.

One of the reasons that this is needed can be seen from the definition of the DFT. Say we take L samples from a signal and do the DFT. Consider what the $k=0$ value of the DFT is (which would be some measure of the DC component of the signal). From the $X[k]$ definition:

$$X[0] = \text{sum (from } n=0 \text{ to } L-1) \text{ of } x[n]$$

Now if we take twice as many samples, then the magnitude of $X[0]$ will probably roughly double since we are now summing from $n=0$ to $2L-1$. But what we really want for $X[0]$ is the DC component of the signal, i.e., it would be:

$$1/L \times \text{sum (from } n=0 \text{ to } N-1) \text{ of } x[n]$$

This is why we are dividing by L, i.e., to get the proper normalization regardless of how many samples we take of the signal. Then we can read the values of frequency components directly from the fft graph. There are other ways of scaling, e.g., $\text{fft}(x)*T$. This way more properly graphs the magnitude of the frequency components in each frequency "bin" and better approximates the FT of $x(t)$. Some people argue that this is the proper scaling method.

When the signal is sampled at a rate of F_s , the highest frequency component that can be seen is $F_s/2$, and the frequency bins start at 0 (DC) and go to $L/2+1(F_s/2)$. This is because when L is even, the fft returns a $1 \times L/2+1$ vector where $L/2+1$ is the Nyquist frequency. The output vector entries starting at index $L/2+2$ are the negative complex conjugates of the positive frequency ones. The above description shows where

$$f = F_s/2 \times \text{linspace}(0,1,L/2+1)$$

comes from. `linspace` just creates a uniformly spaced vector of length $L/2+1$ from 0 to 1. Multiplying by $F_s/2$ gives the frequency bins associated with the single-sided spectrum.

$$\text{plot}(f,2*\text{abs}(Y(1:L/2+1)))$$

Here, we are plotting the magnitude of the sinusoids. We multiply by 2 since there is one positive frequency and one negative frequency component with the same magnitude (they are complex conjugates).

One must be very careful about reading amplitudes from the fft. Depending on system parameters, for example, a sinusoid's frequency may lie between two fft frequency bins. In that case, reading the sinusoid amplitude may be more difficult.