

This project consists of four (4) Tasks

Consider the following experiment:

A robotic arm holding a saw is moving with a constant velocity, from left to right and back to starting position zero (0), along the wooden plank of length L . It will be repeating this process until you press the button, when the arm with the saw will cut the plank.

After that, the new plank of length L is placed, and the process is repeated until all N planks are cut. Obviously, the length of the cut is an outcome of a random event that can be any real number larger than 0 and smaller than L , with total of N different outcomes. Let us label the outcome (length) of the single experiment with $X(i)$, where $i=1, N$.

The outcome of this experiment in MATLAB can be produced by using the following command:
 $X = L * \text{rand}(N,1);$

The “rand” command is used to generate a random number in interval (0, 1), and multiplication by L creates a new random number in the interval (0, L).

Now, let us say we want to repeat this experiment 100 (N) times. In MATLAB we would implement this as:

```
N=100;      % number of experiments
L=25;      % length of the plank
```

```
% instead of a loop you can use X=L*rand(N,1)
```

```
For i = 1:N,
X(i) = L * rand;
End;
```

The “array X ” would contain outcomes of these 100 experiments (cuts).

Task 1: In your first task you are required to find how many times the length of the cut piece was larger than $0.1L$ and smaller than $0.15L$, using the aforementioned experiment. Explain the difference between the experimental result and the theoretical value (0.05).

We will refer to $N(100)$ experiments as one trial. Let us now introduce M to denote the number of trials. Also let m_i be the relative frequency (# of times/ N) of how many times the cut piece length was in the range of $(0.1L, 0.15L)$ in the i -th trial.

Repeat the overall experiment 100 times, i.e. generate 100 cuts 100 times and calculate average error and mean square error by counting how many times you got pieces in $(0.1L$ to $0.15L)$ range in each trial.

Average Error ($M=100$ = number of trials)

$$average = \frac{1}{M} \sum_{i=1}^M (m_i - 0.05)$$

Variance

$$var = \frac{1}{M} \sum_{i=1}^M (m_i - 0.05)^2$$

Task 2: Repeat **Task 1** for $N = 1000$ and $N = 10000$ (increase number of experiments not trials). Find both the average error and variance and explain the findings.

Does the Average Error increase or decrease with the number of experiments? Why?

Does the Variance increase or decrease as the number of experiments increases? Why?

Task 3: Let us now count the number of outcomes in particular intervals assuming N cuts:

- let $N1$ be number of times the value of X is in the range $(0, L/4)$;
- let $N2$ be number of times the value of X is in the range $(L/4, L/2)$;
- let $N3$ be the number of times the value of X is in the range $(L/2, 3L/4)$,
- and let $N4$ be the number of times value of X is in the range of $(3L/4, 1)$.

Although we intuitively expect that $N1 = N2 = N3 = N4 = N/4$, you will observe that these numbers will be different from the “expected” value.

In MATLAB, the “hist” command can be used to count the number of outcomes for the arbitrary number of bins. The more advanced version of “hist” command is called “histogram”, however for our purposes “hist” variant is sufficient. By using `hist(X, 4)`, MATLAB will count number of elements of vector X that belong to 4 bins of the same size, and represent it visually.

Generate random experiments consisting of $N = 100, 1000$, and 10000 cuts and divide the outcomes in 10 bins using:

```
L = 10;
N = 100;           % Note: replace N with 1000 and 10000;
X = L * rand(N,1);
P = hist(X,10);
```

The elements of vector P will be number of outcomes the value of X falls in a particular bin. For example:

```
L = 10;
N = 1000;
X = L * rand(1000,1);
hist(X,10);
p=hist(X,10);
```

```
p = 95 126 96 88 107 97 114 106 82 89
```

```
>> mean(p)
ans = 100
```

```
>> var(p)
ans = 177.3333
```

The “mean” command finds the average value which will be equal to N.
The “var” command measures the variance. We will be discussing later in the course that this variable is actually called, variance and that it measures uncertainty and/or the power of the noise/error process.

Your task will be to find the average and variance for different values of $N = 100, 1000, \text{ and } 10000$.

Does the average increase or decrease with N ? Explain your findings.

Does the variance increase or decrease with N ? Explain your findings.

Task 4: Self-study

Run the following commands:

```
N=100;           % number of experiments N
L=10; % length L

           %first set of experiments/cuts
X = L * rand(N,1);
           %second set of experiments/cuts
Y=L*rand(N,1);

Z=X+Y;

P1=histogram(X,10);      % P1, P2, P3 are vectors containing count outcomes for all three
                           experiments
P2=histogram(Y,10);
P3=histogram(Z,10);
D1=var(P1);              % D1, D2, D3 are mean square errors of three experiments
D2=var(P2);
D3=var(P3);
```

Compare and contrast values of $D1, D2$ and $D3$ and explain your findings.

Repeat this processes several times and observe ratios $D3/D2$ and $D3/D1$? What can you notice?

