ELECENG 3TQ3 Project 2

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Task 1:

probability density function ~ mass ~

Task 1: Find the pdf of the received signal $r(t_j)$ and the pmf of the estimated sent signal s_j assuming $\tau = 0.5$ and $\sigma^2 = 0.25$.

$\because r(t_j) = s(t_j) + e(t_j)$

$\because e(t_j)$ is Guassian distributed $\because S(t_j)$ is binomial distributed

$\therefore$ pdf of $e(t_j)$ $\therefore$ Pmf of $S(t_j)$

$$f_e(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \qquad f_s(x) = p\,\delta(x) + (1-p)\,\delta(x-1)$$

$x \in R$

$\therefore$ Pdf of $r(t_j)$: Yichen Lu 400247738 luy191

$$f_r(x) = f_s(x) \otimes f_e(x)$$

"$\otimes$" means convolution

$$\therefore f_r(x) = \frac{1}{\sigma\sqrt{2\pi}}\left[p \cdot e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} + (1-p)\,e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}\right]$$

$\because \sigma^2 = 0.25;\ \tau = 0.5;\ \mu = 0$

$\therefore \sigma = 0.5$

$$\therefore f_r(x) = \sqrt{\frac{2}{\pi}}\left[p \cdot e^{-2x^2} + (1-p)\,e^{-2(x-1)^2}\right] \quad x \in R$$

Pmf of the estimated sent signal:

① $P(\hat{s}_j = 0) = P(r_{t_j} < \tau)$ $\because \tau = 0.5$

$= P(r_{t_j} < 0.5)$

$$= \int_{-\infty}^{0.5} \sqrt{\frac{2}{\pi}} \left[p e^{-2x^2} + (1-p) e^{-2(x-1)^2} \right] dx$$

② $p(\hat{s}_j = 1) = p(r_{tj} > \tau) \because \tau = 0.5$

$$= p(r_{tj} > 0.5)$$

$$= \int_{0.5}^{\infty} \sqrt{\frac{2}{\pi}} \left[p e^{-2x^2} + (1-p) e^{-2(x-1)^2} \right] dx$$

ANS:

① the pdf of the received signal $r(t_j)$

$$f_r(x) = \sqrt{\frac{2}{\pi}} \left[p \cdot e^{-2x^2} + (1-p) e^{-2(x-1)^2} \right] x \in R$$

② the pmf of the estimate sent signal $\hat{s}_j$

$$p(\hat{s}_j) = \begin{cases} \sqrt{\frac{2}{\pi}} \int_{-\infty}^{0.5} \left[p e^{-2x^2} + (1-p) e^{-2(x-1)^2} \right] dx & \hat{s}_j = 0 \\ \sqrt{\frac{2}{\pi}} \int_{0.5}^{\infty} \left[p e^{-2x^2} + (1-p) e^{-2(x-1)^2} \right] dx & \hat{s}_j = 1 \end{cases}$$

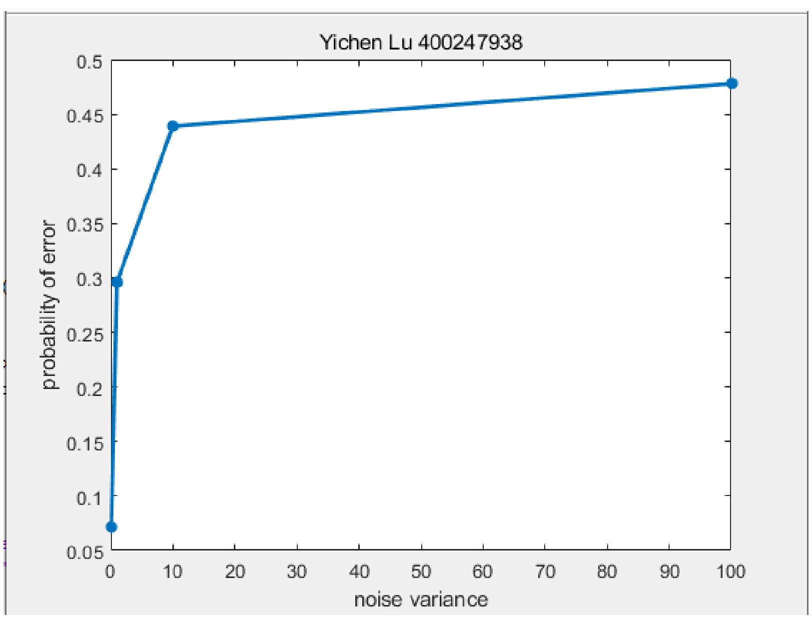
Task 2:

```
%Yichen Lu luy191 400247938
clc;
clear;

p=0.5;
tau=0.5;
variance=[0.1,1,10,100];
N=1000;
error=zeros(1,4);
s=randi([0,1],[1,1000]);

for i= 1:4
    e = randn(1,1000)*sqrt(variance(i));
    r = s+e;
    for j = 1:N
        if(s(j)==0 && r(j)>=tau) || (s(j)==1 && r(j)<tau)
            error(i) = error(i)+1;
        end
    end
end

figure
plot(variance,error/N,'LineWidth',2,'Marker','*')
title('Yichen Lu 400247938')
xlabel('noise variance')
ylabel('probability of error')
```



Task 3:

```
%Yichen Lu luy191 400247938
clc;
clear;
word='YichenLuisthebest';
l=length(word);
binary=dec2bin(word,8);   %transfer the words into ASSCLL form
tempholder=zeros(l,8);
%constant
t = [0.25 0.5 0.75];
variance=[0.1 1 10];
error= zeros(length(t),length(variance));
times =5000;

%transfer string into integer
for i=1:l
    for j=1:8
        if binary(i,j)=='0'
            tempholder(i,j)=0;
        end
        if binary(i,j)=='1'
            tempholder(i,j)=1;
        end
    end
end



%main part
for i =1:length(t)            %go through tau
    for j=1:length(variance)  %go through variance
        wrong=0;          %incorrect numbers holder
        for x=1:times
            e=randn(1,8)*sqrt(variance(j));   %noise function
            for k = 1:l
                eachletter=tempholder(k,:);       %for each row
                r=eachletter+e;
                wrongbit=false;
                for n=1:8                    %run through every bit of the letters
                    if(eachletter(n)==0&&r(n)>=t(i)||eachletter(n)==1&&r(n)<t(i))
                        wrongbit=true;
                    end
                end
                if wrongbit
                    wrong=wrong+1;
                end
            end
        end
        error(i,j)=wrong/times;          %get average error
        fprintf("when tau=%.2f and variance=%.1f, the average number of incorrect words is %.1f out of %d. \n",t(i),variance(j),error(i,j),l);
    end
end
```

```
when tau=0.25 and variance=0.1, the average number of incorrect words is 10.7 out of 17.
when tau=0.25 and variance=1.0, the average number of incorrect words is 16.2 out of 17.
when tau=0.25 and variance=10.0, the average number of incorrect words is 16.8 out of 17.
when tau=0.50 and variance=0.1, the average number of incorrect words is 6.3 out of 17.
when tau=0.50 and variance=1.0, the average number of incorrect words is 16.1 out of 17.
when tau=0.50 and variance=10.0, the average number of incorrect words is 16.8 out of 17.
when tau=0.75 and variance=0.1, the average number of incorrect words is 10.7 out of 17.
when tau=0.75 and variance=1.0, the average number of incorrect words is 16.2 out of 17.
when tau=0.75 and variance=10.0, the average number of incorrect words is 16.8 out of 17.
```

In conclusion, the tau is the same for all numbers in the same row of the resulting matrix, and the variance is the same for all numbers in the same column.

I ran the test 5000 times in total by using my code, and the resulting matrix is the average of all the test results. We can observe that both tau and the variance of the noise have an impact on the number of errors that occur from the output result.

In terms of tau, we can see that when tau equals 0.5, it has the lowest the average number of incorrect words. However, when tau is 0.25 or 0.75, the average number of incorrect words is almost identical, although it is greater than when tau is 0.5.

When it comes to variance, there is a clear tendency that the larger the sigma, the greater the chance of inaccuracy. When variance = 0.1 with the same tau,0.25, average error = 10.7, when variance = 1, average error = 16.2, and when variance = 10, average error = 16.8.

Thus, the number of correctly decoded letters did depend on tau and sigma.