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(Answers are below)

ELEC ENG 3TQ3
Assignment 1
Due Date : Oct 4 – 11:59 p.m.

Q1. Suppose that for the general population, 1 in 5000 people carries the human immunodeficiency virus (HIV). A test for the presence of HIV yields either a positive (+) or negative (−) response. Suppose the test gives the correct answer 99% of the time. What is P[−|H], the conditional probability that a person tests negative given that the person does have the HIV virus? What is P[H|+], the conditional probability that a randomly chosen person has the HIV virus given that the person tests positive?

Q2. You have two biased coins. Coin A comes up heads with probability 1/4. Coin B comes up heads with probability 3/4. However, you are not sure which is which so you choose a coin randomly and you flip it. If the flip is heads, you guess that the flipped coin is B; otherwise, you guess that the flipped coin is A. Let events A and B designate which coin was picked. What is the probability P[C] that your guess is correct?

Q3. The basic rules of genetics were discovered in mid-1800s by Mendel, who found that each characteristic of a pea plant, such as whether the seeds were green or yellow, is determined by two genes, one from each parent. Each gene is either dominant d or recessive r. Mendel's experiment is to select a plant and observe whether the genes are both dominant d, both recessive r, or one of each (hybrid) h. In his pea plants, Mendel found that yellow seeds were a dominant trait over green seeds. A yy pea with two yellow genes has yellow seeds; a gg pea with two recessive genes has green seeds; a hybrid gy or yg pea has yellow seeds. In one of Mendel's experiments, he started with a parental generation in which half the pea plants were yy and half the plants were gg. The two groups were crossbred so that each pea plant in the first generation was gy. In the second generation, each pea plant was equally likely to inherit a y or a g gene from each first generation parent. What is the probability P[Y] that a randomly chosen pea plant in the second generation has yellow seeds?

Q4. Prove the following:

(a) $P[A \cup B] \geq P[A]$
(b) $P[A \cup B] \geq P[B]$
(c) $P[A \cap B] \leq P[A]$
(d) $P[A \cap B] \leq P[B]$

Q5. You have a six-sided die that you roll once. Let R_i denote the event that the roll is i. Let G_j denote the event that the roll is greater than j. Let E denote the event that the roll of the die is even -numbered.

(a) What is $P[R_3 | G_1]$, the conditional probability that 3 is rolled given that the roll is greater than 1?

(b) What is the conditional probability that 6 is rolled given that the roll is greater than 3?

(c) What is $P[G_3 | E]$, the conditional probability that the roll is greater than 3 given that the roll is even?

(d) Given that the roll is greater than 3, what is the conditional probability that the roll is even?

#1. $\because P(H) = \frac{1}{5000}$

$\because$ The test gives correct answer 99% of the time

$\therefore P(+|H) = 99\%$, $P(-|H) = 99\%$

a) $P(-|H) = 1 - P(+|H) = 1 - 99\% = 1\% = 0.01$

b) By Bayes' theorem

$$P(H|+) = \frac{P(+|H)\cdot P(H)}{P(+)} = \frac{99\% \times \frac{1}{5000}}{99\% \times \frac{1}{5000} + (1-99\%) \times \frac{4999}{5000}}$$

$$= \frac{99}{5098}$$

#2. The probability that your guess is correct

$$P[C] = P(A) \times P(tail) + P(B) \times p(head) = \frac{1}{2}\times\frac{3}{4} + \frac{1}{2}\times\frac{3}{4}$$

$$= \frac{3}{8} + \frac{3}{8}$$

$$= \frac{\cancel{6}\,3}{\cancel{8}\,4}$$

$$= \frac{3}{4}$$

A $(\frac{1}{2})$	B $(\frac{1}{2})$
head $(\frac{1}{4})$	head $(\frac{3}{4})$
Tail $(\frac{3}{4})$	Tail $(\frac{1}{4})$

#3.

$yg \backslash yg$	y	g
y	yy	yg
g	gy	$\cancel{gg}$

$$\therefore P(Y) = P(yy) + P(gY) + P(yg)$$
$$= \frac{1}{2}\times\frac{1}{2} + \frac{1}{2}\times\frac{1}{2} + \frac{1}{2}\times\frac{1}{2}$$
$$= \frac{3}{4}$$

#4.

a) $P[A \cup B] \geq P[A]$

proof:

$$P(A\cup B) = P((A\cup B)\cap A) + P((A\cup B)\cap A^c)$$
$$= P(A) + P((A\cup B)\cap A^c) \geq P(A)$$

b) $P[A\cup B] \geq P[B]$

proof:

$$P(A\cup B) = P((A\cup B)\cap B) + P((A\cup B)\cap B^c)$$
$$= P(B) + P((A\cup B)\cap B^c) \geq P(B)$$

c) $P[A \cap B] \le P[A]$

proof:

$$P(A) = P(A \cap (B \cup B^c)) = P(A \cap B) + P(A \cap B^c)$$

$$\therefore P(A \cap B) = P(A) - P(A \cap B^c) \le P(A)$$

d) $P[A \cap B] \le P[B]$

proof:

$$P(B) = P(B \cap (A \cup A^c)) = P(B \cap A) + P(B \cap A^c)$$

$$\therefore P(A \cap B) = P(B) - P(B \cap A^c) \le P(B)$$

#5.

a) $$P(R_3 \mid G_1) = \frac{P[R_3 \cap G_1]}{P(G_1)} = \frac{\frac{1}{6}}{\frac{5}{6}} = \frac{1}{5}$$

b) $$P(R_6 \mid G_3) = \frac{P[R_6 \cap G_3]}{P(G_3)} = \frac{\frac{1}{6}}{\frac{3}{6}} = \frac{1}{3}$$

c) $$P(G_3 \mid E) = \frac{P(G_3 \cap E)}{P[E]} = \frac{\frac{2}{6}}{\frac{3}{6}} = \frac{2}{3}$$

d) $$P(E \mid G_3) = \frac{P(G_3 \cap E)}{P(G_3)} = \frac{\frac{2}{6}}{\frac{3}{6}} = \frac{2}{3}$$