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Assignment 2 – ELEC ENG 3TQ3
Due Date Nov 12th 4:30 p.m.

NOTE: MATLAB can be used only to find values of Gaussian CDF if needed.

Assignment consists of 5 questions on two pages. All questions are mandatory.

1. (5 points) Let us consider breaking a chocolate bar of length L randomly into three pieces of lengths L1, L2 and L3 such that L=L1+L2+L3. Find
   a) Probability that L1 > L2+L3
   b) Expected value of L1
   c) Expected value of min(L1,L2,L3)

2. (5 points) John and Susan are going out on a first date. They agree to meet at Nathan Phillips Square at 8:00 p.m. Since it is their first date John decides that he will be their early in order to impress Susan and he plans to arrive at 7:45 p.m . Due to randomness in TTC performance the difference between his intended arrival time and actual arrival time is Gaussian distributed with mean 0 and variance 25. Susan decides to be fashionably late and she plans to arrive at 8:10 p.m. She decides to take a cab and hence the difference between her arrival time and intended arrival time is Gaussian distributed with mean 0 and variance 9.
   a) Find probability that John arrives before 8 p.m.
   b) Find probability that John arrives after 8:10 p.m.
   c) Find probability that John arrives at least 5 minutes before Susan. Hint: Z=Y-X can be viewed as a Gaussian RV with mean and variance that can be calculated.

3. (5 points) Let X be Gaussian distributed with mean 0 and variance 4. Let Y be Gaussian distributed with mean 5 and variance 9. Find a real number c such that that

$$P(X > c) + P(Y < c)$$

is the largest possible. Justify your answer. If you think such number cannot be found state it clearly and justify your answer.

4. (5 points) The starship Enterprise arrives at newly discovered planet Haldurian. The scientists find that there are 3 different genders on Haldurian with different height distributions. The height of gender A is Gaussian distributed with mean 6ft and 4 inches and variance 36. The height of gender B is Gaussian distributed with mean 7ft and 10

inches and variance 16. The height of gender C is Gaussian distributed with mean 5 ft and 10 inches and variance 25. Your data also indicated that 70% of Haldurian population is gender A, 20% population is gender B and, 10% population is gender C. Find probability that randomly chosen Haldurian is taller than 7 feet.

5. (5 points) Consider two random variables X and Y with join pdf such that

   a) Find c.
   b) Find marginal distributions of X and Y.
   c) Are X and Y independent? Justify your answer.

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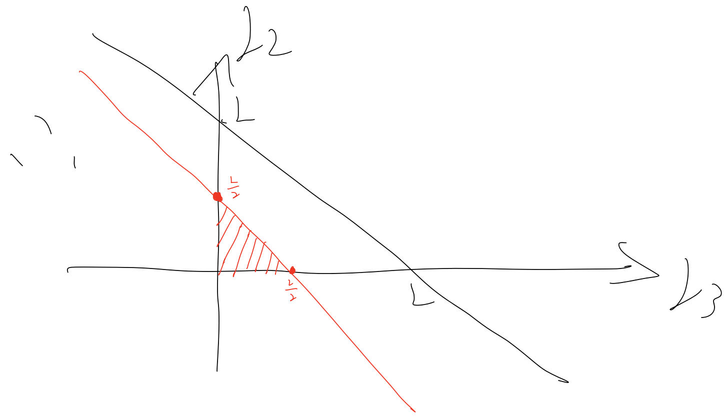
#1.

a) $\because L_3 + L_2 < L$; $L_2 > 0$; $L_3 > 0$ (x) (y)

$\because L_1 > L_2 + L_3$ $\therefore L_1 > \frac{L}{2}$ $\therefore L_3 + L_2 < \frac{L}{2}$

$\therefore$



$\frac{1-x}{n}$

$\therefore P(L_1 > L_2 + L_3) = (\frac{1}{2} \times \frac{L}{2} \times \frac{L}{2}) \div (\frac{1}{2} \times L \times L)$

$\therefore P(L_1 > L_2 + L_3) = (\frac{L^2}{8}) \div (\frac{L^2}{2}) = \frac{\cancel{L^2}}{\cancel{8}_4} \times \frac{\cancel{2}^1}{\cancel{L^2}} = \frac{1}{4}$

b) $\because$ The bar is breaking randomly

$\therefore$ Uniform distribution should be used, and there are 2 tests

$\therefore E(L_1) = \frac{L}{n+1} = \frac{L}{2+1} = \frac{L}{3}$

(c) $E\min(L_1, L_2, L_3) = \frac{L}{3} \div 2 = \frac{L}{3} \times \frac{1}{2} = \frac{L}{6}$

#2 a)

```
%Yichen Lu luy191 400247938
clear;clc;
mu = 0;
sigma = sqrt(25);
pd = makedist('Normal', 'mu',mu,'sigma',sigma);
y = cdf(pd,15);
y
```

```
y =

    0.9987
```

$\therefore$ P(John arrives before 8p.m.) = 0.9987 = Gaussian CDF

b)

```
%Yichen Lu luy191 400247938
clear;clc;
mu = 0;
sigma = sqrt(25);
pd = makedist('Normal', 'mu',mu,'sigma',sigma);
y = cdf(pd,25);
P = 1-y
```

```
P =

   2.8665e-07
```

$\therefore$ P(John arrives after 8:10 p.m.) = $2.8665\times10^{-7} \approx 0$ = Gaussian CDF

c)

```
%Yichen Lu luy191 400247938
clear;clc;
mu = 0-0;
sigma = sqrt(34);
pd = makedist('Normal', 'mu',mu,'sigma',sigma);
y = cdf(pd,5);
ANS = 1-y
```

```
ANS =

    0.1956
```

$\therefore$ P(John arrives at least 5 minutes before Susan) = Gaussian CDF = 0.1956

#3. Assume such number exist that

$\because X: \mu=0,\ \sigma^2=4,\ \sigma=2$

$Y: \mu=5,\ \sigma^2=9,\ \sigma=3$

$P(X>c)+P(Y<c)=1$

$1-P(X<c)+P(Y<c)=1$

$$1-P\left(c<\frac{z}{2}\right)+P\left(z<\frac{c-5}{3}\right)=1$$

$$P\left(z<\frac{c}{2}\right)=P\left(z<\frac{c-5}{3}\right)$$

$$\frac{c}{2}=\frac{c-5}{3}$$

$$c=-10$$

#4, ∵ 1 foot = 12 inches

∴ A: 6 ft and 4 inches $= 6 \times 12 + 4 = 72 + 4 = 76$ inches

B: 7ft and 10 inches $= 7 \times 12 + 10 = 84 + 10 = 94$ inches

C: 5 ft and 10 inches $= 5 \times 12 + 10 = 60 + 10 = 70$ inches

7 ft $= 7 \times 12 = 84$ inches

```
%Yichen Lu luy191 400247938
clear;clc;
mu = 76;
sigma = sqrt(36);
pd = makedist('Normal', 'mu',mu,'sigma',sigma);
x = cdf(pd,84);
A = 1-x;

mu = 94;
sigma = sqrt(16);
pd = makedist('Normal', 'mu',mu,'sigma',sigma);
y = cdf(pd,84);
B = 1-y;


mu = 70;
sigma = sqrt(25);
pd = makedist('Normal', 'mu',mu,'sigma',sigma);
z = cdf(pd,84);
C = 1-z;

ANS = 0.7*A + 0.2*B + 0.1*C;
ANS
```

```
ANS =

    0.2629
```

I use the MATLAB code to calculate the CDF

∴ $P(A>7) = 1 - P(A<7) = 1 - 0.9088 = 0.0912$

$P(B>7) = 1 - P(B<7) = 1 - 0.0062 = 0.9938$

$P(C>7) = 1 - P(C<7) = 1 - 0.9974 = 0.0026$

∴ P(randomly chosen Haldurian is taller than 7 feet) = 0.263

∴ $P = 0.7\,P(A>7) + 0.2\,P(B>7) + 0.1\,P(C>7) = 0.263$

#5.

a) Joint Pdf

$$f_{X,Y}(x,y) = \begin{cases} cx^2y & 0 \le y \le x \le 2 \\ 0 & \end{cases}$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dx\,dy = 1$$

$$\therefore \int_0^2 \int_0^x cx^2y\,dx\,dy = 1$$

$$\therefore \frac{c}{2}\int_0^2 x^4\,dx = 1$$

$$\frac{c}{2}\frac{x^5}{5}\Big|_0^2 = 1$$

$$\frac{32c}{10} = 1$$

$$c = \frac{5}{16} \approx 0.3125$$

b) Marginal distributions of X & Y

$\because f_X(x) = \int_{-\infty}^{\infty} f(x,y)\, dy$

$= \int_0^x \frac{5}{16} x^2 y \, dy$

$= \frac{5}{32} x^4$

$f_Y(y) = \int_{-\infty}^{\infty} f(x,y)\, dx$

$= \int_0^2 \frac{5}{16} x^2 y \, dx$

$= \frac{5}{16} y \cdot \frac{x^3}{3} \Big|_0^2$

$= \frac{5}{6} y$

c) Independence of x and y

If x and y are independent

$f_{XY}(x,y) = f_X(x) \cdot f_Y(y)$

$f_X(x) \cdot f_Y(y) = \frac{5}{32}x^4 \cdot \frac{5}{6}y = \frac{25}{192}x^4 y$

$f_{XY}(x,y) = \frac{5}{16}x^2 y$

$\therefore \frac{25}{192}x^4 y \neq \frac{5}{16}x^2 y$

$\therefore$ X and Y are not independent