Here we briefly describe the methods for EEG data synthesis and the proposed threshold-free wavelet packet transform-based denoising (TF-WPTD) method. Detailed descriptions will be available in the full paper once been published.

Data synthesis

We synthesised a total of 300 ERPs (150 ``success" and 150 ``failure") at a range of controlled signal-to-noise ratios (SNRs). Following previous studies, we modelled the multi-channel ERP as a weighted sum of source ERP components plus noise [1]–[4]. We synthesised the ERPs following Yeung et al.'s method [1] and controlled the SNR by manipulating the amount of noise. The source ERP components and their parameters were derived from various empirical sources [5]–[8], as summarised in Table 1.

Table 1: Source ERP components and their parameters. FRN denotes Feedback-Related Negativity.

Component	In	Peak amplitude (uV)	Frequency (Hz)	Latency (ms)	Jitter (ms)
N1	success, failure	4	8	90	12
P2	success, failure	6	4	180	24
P3	success, failure	5	2	300	36
FRN	failure only	6	4	180	24
P3a	failure only	5	2	300	36

Threshold-free wavelet packet transform-based denoising (TF-WPTD)

We developed a novel, threshold-free wavelet packet transform-based denoising (TF-WPTD) method. We first decomposed EEG recordings (for each channel and trial) into wavelet coefficients using the maximal overlap discrete wavelet packet transform (MODWPT) [9]. Then, for each coefficient, we used a Student's two-tailed t-test to determine whether the mean across trials was nonzero. We used a cluster-wise permutation method to correct for multiple comparisons [10]. We generated a coefficient mask based on the statistical test results – a coefficient with a mean statistically significantly different from zero was marked as one associated with ERPs, the rest were marked as ones associated with noise. Finally, with the coefficient mask, we denoised any given EEG trial by first decomposing it into wavelet coefficients, then applying the mask and zeroing all coefficients not associated with ERPs, and reconstructing the denoised trail using the masked coefficients.

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