PHYS580 Lab 3, Sep 5, 2019

Assignment:. Today's lab is a preliminary to the main portion of Chapter 3, where we will investigate striking features that arise from nonlinearity coupled with openness of the system (dissipation and driving). In this lab, however, either nonlinearity or openness is turned off; so you are basically preparing the numerical apparatus for the richer physics that arises in the next lab. So the assignment here is a useful and necessary prerequisite. The starter code provided on the course web site uses the Euler approximation, which you will find *inadequate* in (1) below.

1. First, take the linear oscillator with no damping and no driving force (i.e., completely harmonic). Investigate the stability and accuracy of three numerical methods: Euler, Euler-Cromer, and Runge-Kutta 2nd order. Demonstrate your conclusions with a few chosen parameters and plots. Be sure to include the cumulative (global) error analyses for both θ and E (energy) in your report, comparing the different methods to each other and to the theoretical expectations. Do NOT use Matlab built-in functions for these approximations, rather, code the respective algorithms into your programs explicitly.

2. Second, still for the linear oscillator, turn on some damping and/or driving force. Using the method and parameters you determined in (1) to be adequate, demonstrate the overdamped, underdamped, and resonant regimes (cf. Section 3.2 of the textbook). Set l = 9.8 m to simplify things a bit.

3. Finally, take the physical oscillator with unapproximated nonlinear equation of motion (i.e., the full sin θ term) , but without damping and driving force. Investigate the dependences, if any, of the period (if periodic) and wave form on the amplitude of oscillation. Compare with the exact result for the period:
$$T = 4\sqrt{l/g}\,K\left(\sin\left(\theta_m/2\right)\right) = 2\pi\sqrt{l/g}\left[1+\frac{1}{16}\theta_m{}^2+\frac{11}{3072}\theta_m{}^4+\ldots\right]$$
Here, the right-hand side explicitly shows the first three terms of $K(a)$, the complete elliptic integral of 1st kind, when expanded into powers of the oscillation amplitude θ_m .