PHYS580 Lab 10, Oct 24, 2019

Assignment:

This lab emphasizes physics analysis more than substantive programming. You will be working on obtaining estimates of universal, critical exponents for two-dimensional percolation, a worthy task for physics. The provided sample programs (e.g., *make2a.m*) are already quite sophisticated, so there is no need to write your own algorithms. However, you will need to automate the analysis to collect sufficient statistics from a large number of percolation realizations (e.g., by revising *gridnormal.m*, or sending its output to file and post-processing the result). Regarding power law fits, check how fitting was done in earlier labs (e.g., in Lab 9 on random walks and self-avoiding walks).

The starter programs *gridnormal*.m and *make2a.m* simulate site percolation on a square lattice of user-given size L. Use these codes (or your own equivalent routines) to accomplish the two tasks below:

1. First, set the site occupation probability to $p = 0.593$ (very close the percolation threshold p_c for a 2D lattice), and study percolation on lattices of several different sizes (from $L \times L = 50 \times 50$ through 1000×1000 or so). Generate at least 20 realizations for <u>each</u> lattice size, and for each percolation record the *percolation probability* $P(p_c)$ and the *susceptibility* $S(p_c)$ (the latter is essentially the mean cluster size). Then, average P and S over the set of realizations, and determine how the averaged $P(p_c)$ and $S(p_c)$ depend on the lattice edge size L for asymptotically large L. This dependence is called *finite-size scaling* (you should find power-law behavior). Estimate the power-law exponents by linear least-squares fitting on log-log scale.

   Theoretically, the expected asymptotic behaviors are

   $$P(p_c) \sim L^{-\beta/\nu}, \qquad S(p_c) \sim L^{\gamma/\nu}$$

2. Next, generate percolation realizations for at least 10 different values of p around p_c on <u>large</u> lattices of fixed given size $L \times L$. Plot $P(p)$ and $S(p)$ (averaged over at least 20 realizations at each p) as a function of p, and show that for sufficiently large L these two quantities behave as power laws near (but *not too near*!) $p = p_c$. Specifically, $P(p) \sim (p - p_c)^\beta$ for $p > p_c$, and $S(p) \sim |p - p_c|^{-\gamma}$ on both sides of p_c. From your numerical data, estimate the critical exponents β and γ. Combine the results with your findings from part (1) to estimate ν as well.