PHYS580 Lab 4, Sep 12, 2019

Assignment: In today's lab, you will investigate some of the striking features that arise from non-linearity when coupled with openness of the system (with dissipation and driving). Specifically, you will simulate a physical pendulum with both dissipation and a driving force. As we already know from Lab 3, for systems which encompass periodic regimes, the minimum acceptable approximation is the Euler-Cromer method (but higher order is still preferable).

1. Modify the provided starter code (or write your own using any *appropriate* approximation) to study the Poincaré section of the phase portrait and those of the time evolution of the dynamical variables θ, ω, and E (energy). Consider both θ-ω and ω-E pairings (2D). Produce the Poincaré sections for some model parameters in the normal periodic (with period $T=2\pi/\Omega$, where Ω is the driving force angular frequency), chaotic, and period doubling (with period $2T$) regimes, choosing the sectioning and driving frequencies to be the same in all cases. Set the sectioning phase to 0 (i.e., do *in phase* sectioning). What are the characteristics of the different regimes reflected in these results? Explain how you can be sure that your computational parameters are sufficient to reach your conclusions..

2. Second, investigate the effects, if any, of using different initial conditions $\theta(0)$ and $\omega(0)$, while keeping all other parameters the same as in (1). Explicitly compare results with those of part (1).

3. Third, do the same as in (1) but with changing the phase cut to $\pi/2$, π, and 37°. Compare the results with those of parts (1) and (2).

4. Lastly, keeping all the other parameters the same as in (1), use an arbitrary sectioning frequency that differs from the driving frequency Ω and is irrationally related to or incommensurate with it. As an example, you can try $\omega_0 = (g/l)^{1/2}$ (the natural angular frequency of the linearized oscillator) or $\omega_1 = (g/l)^{1/2}\pi$ / $[2K(\sin(\theta_m/2))]$ (that of the non-linear oscillator without dissipation or driving force where θ_m is the amplitude). Discuss the results.