a3_sub

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Question 1

Following Inductive Loop Invariants, we have:

$$\frac{n \geq 0 \ \wedge r = 0 \wedge i = 0 \ \wedge p = 1 \Longrightarrow I[0/r, 0/i, 1/p] \quad \dfrac{I \wedge i \neq n \Longrightarrow I[(r+p)/r, 2p/p, (i+1)/i}{\{I \wedge i \neq n\}\ r = r - p; p = 2p; r = r + p; i = i + 1\ \{I[(r+p)/r, 2p/p, (i+1)/i\}} \quad I \wedge i = n \Longrightarrow r = 2^n - 1}{\{n \geq 0 \ \wedge r = 0 \wedge i = 0 \ \wedge p = 1\}\ while\ i \neq n\ do\ P\ \{r = 2^n - 1\}}$$

Then we have the three constraints:

1. $n \geq 0 \ \wedge r = 0 \wedge i = 0 \ \wedge p = 1 \Longrightarrow I[0/r, 0/i, 1/p]$
2. $I \wedge i \neq n \Longrightarrow I[(r+p)/r, 2p/p, (i+1)/i$
3. $I \wedge i = n \Longrightarrow r = 2^n - 1$

The program is correct if I is valid for the three constraints.
We can use $p = 2^i \wedge r = 2^i - 1 \wedge i \leq n$ as I.
For the first constraint:

$$n \geq 0 \ \wedge r = 0 \wedge i = 0 \ \wedge p = 1 \Longrightarrow p = 1 \wedge r = 0 \wedge i = 0 \leq n$$

For the second constraint:

$$p = 2^i \wedge r = 2^i - 1 \wedge i \leq n \wedge i \neq n \Longrightarrow p = 2^i \wedge r = 2^i - 1 \wedge i < n \Longrightarrow r = 2^i - 1 + 2^i = 2^{(i+1)} - 1 \wedge p = 2^{(i+1)} \wedge (i+1) \leq n$$

For the third constraint:

$$p = 2^i \wedge r = 2^i - 1 \wedge i \leq n \wedge i = n \Longrightarrow r = 2^n - 1$$

Thus, the three constraints are satisfied when we leverage $p = 2^i \wedge r = 2^i - 1 \wedge i \leq n$ as I. Now we prove the program is valid.