A Semi-Parametric Two-Stage Estimator with Machine Learning to Solve Endogeneity *

[Preliminary version]

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Abstract

Based on the fast development in machine learning techniques, we propose a semi-parametric two-stage estimator to solve the endogeneity in models when implementing the usual linear-IV is impossible. Those models include binary discrete-choice with endogenous regressors and multinomial Logit model with zero market share. Our first stage is a flexible estimation of the expected outcomes, conditional on the regressors and instruments, whereas the second stage is a simple linear-IV regression with first-stage results inserted. After adding an analytical-calculated correction term, the first-stage error up to $o(N^{-1/4})$ does not affect the asymptotic distribution of the second-stage structural parameter estimates, allowing for a variety of non-parametric methods in the first stage. We demonstrate the behavior of our estimator through a pile of simulations. Moreover, we demonstrate the performance by a simple application using the publicly-available 2011 to 2015 transaction-level data between coal mining companies and coal-fired power plants in the US.

Keywords: Neyman Orthogonality, Machine Learning, Endogeneity, Binary Choice.
JEL classification codes: C14, C31, C36.

*All errors are our own.

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1 Introduction and Literature Review

Two-stage or two-step estimator is a type of estimator that depends on some preliminary results from the first step. As a well-developed tool, it has gained wide application in various subjects. Newey (1994) provides a summary regarding both the parametric and non-parametric first stage. They demonstrate that the second stage estimation can still be root-N consistent even when the first stage converges slower than root-N.

However, this advantage fails with the bias-variance trade-off in a typical machine learning estimator. The bias introduced into the model to reduce the probability of overfitting may convey into the second stage and generate unfavorable outcomes. Seminal work Chernozhukov et al. (2018) solves the problem by referring to the Neyman Orthogonality conditions and sample-splitting. After that, debiased machine learning forces its way into economic analysis and becomes a valid estimator. The widespread usage of their method starts by allowing a wide range of control variables in the treatment effect estimation. Empirical works like Dube et al. (2020), Burlig et al. (2020) and Farronato et al. (2020) have demonstrated the power of the method in Chernozhukov et al. (2018) when there are a large number of potential explanatory variables.

Succeeded by some theoretical improvements on finding the correction terms, like Chernozhukov et al. (2021), Chernozhukov et al. (2022a), and Chernozhukov et al. (2022b), the debiased machine learning gradually gains higher adaptability to various models. For example, Bakhitov (2022) incorporates the automatic debiased machine learning into demand estimation and provides the debiased estimation of own-price derivatives. Cha et al. (2021) combines the orthogonal greedy algorithm and high-dimensional AIC and then applies the method to the production analysis.

High-dimensional techniques are also involved in estimations with endogeneity. Some researchers attempt to use machine learning as the first-stage prediction in 2SLS to recover more correlation between endogenous variables and instruments. However, replacing 2SLS with biased predictions from machine learning would usually aggravate the performance. For instance, Lennon et al. (2021), through a simple decomposition of the bias, demonstrates that directly plugging in machine-learning predictions leads to a larger bias than ignoring the endogeneity. Instead, Belloni et al. (2011), Singh et al. (2019) and Chen et al. (2021) focus on using machine learning to generate the instruments. In particular, Chen et al. (2021) decides to use the first-stage machine-learning results as an in-

strumental variable, which they refer to as the most harmless machine-learning in 2SLS.

In this study, we propose a novel two-step semi-parametric estimator to settle the endogeneity when the direct usage of linear-IV fails. This two-step method combines with debiased machine learning whenever there is model selection or regularization. We do not search for a flexible approximation for the first stage or a selection of the optimal instruments. Alternatively, we target the problems which 2SLS or other linear GMM cannot tackle. We focus on nonlinear models that can be inverted, but some essential parts for inversion are unobserved. For example, if y_i^* is part of the observables, a nonlinear system may be expressed as a linear model $y_i^* = x_i'\beta_x + w_i'\beta_w + \eta_i$, with w_i the endogenous variables. These model types include binary choice with endogenous regressors, where we do not observe the latent linear structure or choice probability. Instead, we observe a binary response variable y_i.

Another possible example is the Logit model for market share, but we observe zero purchased quantity in the data. Logit model itself would not produce zero market share. Thus, zero shares impede the inversion (Nevo, 2000) by generating a negative infinite y_i^*. Usually, researchers deal with this by deleting those zero observations, imputing a fixed small number, or aggregating the purchase data into high-level until no zero quantity left. All of them may have the potential to generate bias.

This study shows that it is possible to construct a pseudo value $\tilde{y}_i$ of y_i^* from flexible methods. Both $\tilde{y}_i$ and y_i^* are unobserved and we know some information regarding $\tilde{y}_i$. In the binary choice model, we observe the actual 0 or 1 choice y_i. In the market share example, we have the purchase of a commodity subject to a sampling error (Gandhi et al., 2019), and we can estimate $\tilde{y}_i$. By inserting the first-stage machine-learning estimates $\tilde{y}$ into the original model, we could use the linear-IV estimation to deal with the endogeneity. However, naive plug-in estimator may introduce a serious econometric problem, especially when we use model selection techniques like Lasso, where debiased machine learning comes in.

Two methods are closely related to ours. The most closely related method is the control function, advocated by Petrin and Train (2010) and Wooldridge (2015). In the first stage, the researcher run an OLS regression of endogenous variables, w_i on the instruments (x_i, z_i). In the second stage, the parameters of interest are estimated via maximum likelihood or OLS, using the residuals from the first stage as an additional regressor. When the true form of $E[w_i|x_i, z_i]$ is accurately specified in

a parametric first stage, the control function estimator is root-N consistent. Our method, on the other hand, contains the usual linear control function as a special case. Briefly speaking, we can consistently achieve a comparable estimator after degenerating our first stage into a linear parametric estimator. Nevertheless, our setting permits more flexibility. Whereas control function puts pressure on the joint distribution of the structural error term with the residuals in the first stage, ours depends on the correct retrieval of $\tilde{y}_i$. By exploiting the debiased machine learning techniques, we allow for a complicated first stage with the error up to $o(N^{-1/4})$. Besides, our method can easily accommodate multiple endogenous variables since only a single variable, $\tilde{y}_i$, must be estimated in our first stage.

Another closed related method is the production function estimation with endogenous variables proposed in Levinsohn and Petrin (2003) and Ackerberg et al. (2015). Their method is an instrumental-free control function method, where the structural error term can be inverted out as an unknown function of capital and intermediate input. They overcome the endogeneity by obtaining a proxy of the structural error term in the first stage. Although both share the attribute of inversion, their method differs from ours. Their first-stage estimator enters into the original model as an extra regressor, and the endogeneity disappears after accounting for this extra regressor.

We organize the rest of the paper as follows. Section 2 shows the basic model and the algorithm. We illustrate it with two typical examples – the binary choice with endogenous regressors and the Logit model with endogenous variables and zero market shares. These two cases are the most important applications of our method, and we will revisit them throughout this study. Section 3 demonstrates that our algorithm is consistent with Chernozhukov et al. (2022a) and Chernozhukov et al. (2021). Hence, our estimator has the desirable asymptotic properties. Section 4 contains Monte Carlo simulations to demonstrate the performance of our estimator. Section 5 then discusses the advantages and potential problems. Section 6 exhibits a possible application for the binary choice model with the publicly-available individual transaction data between power plants and coal mines in the US. We study plants' purchase patterns in the spot market. Section 7 contains the conclusion and future research direction.

2 Model and Estimation

Here we restrict our attention to the i.i.d data only. Consider a linear function,

$$y_i^* = x_i'\beta_x + w_i'\beta_w + \eta_i \tag{1}$$

with exogenous variables x_i and endogenous variables w_i. Usually, we can solve this problem by 2SLS with excluded instruments z_i, while in certain circumstances, we may not observe y_i^*. Alternatively, we observe a y_i as an imperfect measure of y_i^*, and unfortunately, y_i cannot be expressed as a linear function of x_i and w_i. In this study, we show that a pseudo-outcome variable $\tilde{y}_i$ can be constructed through a nonlinear, nonparametric method with model selection or machine learning. Then we plug in the $\tilde{y}_i$ in the place of the unknown y_i^* and proceed on with the usual linear-IV setting.

A pseudo-outcome variable $\tilde{y}_i$ would satisfy the moment condition m_i such that, $E[m_i] = E[(\tilde{y}_i - x_i'\beta - w_i'\beta) \otimes (x_i', z_i')']$. Examples for this model include binary response and multinomial discrete choice using aggregate data with possibly zero market share, as illustrated below.

Case 1 – binary choice model.

Suppose a consumer faces two alternatives – the inside good 1 and the outside option, good 0, which would always be purchased. A consumer decides whether to purchase a particular product 1 while keeping other daily consumption. The utilities consumer i derives from good 1 and good 0 are,

$$u_{i,1} = x_{i,1}'\beta_x + w_{i,1}'\beta_w + \eta_{i,1}$$
$$u_{i,0} = \eta_{i,0}$$

The outside option follows the general normalization that its non-random utility is 0. Consumer i would purchase good 1 whenever,

$$u_{i,1} \geq u_{i,0} \Rightarrow \eta_{i,0} - \eta_{i,1} \leq x_{i,1}'\beta_x + w_{i,1}\beta_w$$

In a binary choice model, we observe the binary choice y_i, which equals 0 and 1. Simply replacing y_i^* in 1 by y_i leads to the linear probability model (LPM). LPM is feasible for 2SLS but suffers from

a bunch of flaws[1]*. To overcome the drawbacks of LPM, threshold-crossing Probit or Logit becomes the mainstream in estimation. If there is no endogeneity between* w_i *and* η_i*, fully-parametrized maximum likelihood would be the most direct estimator*[2]*. However, this nonlinearity in* y_i *precludes the directly application of the linear-IV method to solve endogeneity.*

Although the binary choice model seems an elementary setting, a variety of other models are built on this basic logic. For instance, consider the multiple discrete-continuous model proposed in Bhat (2005) and Bhat (2008). With an outside option, a good j *is never purchased if the marginal utility of the first unit consumption is lower than a constant value from the outside option. This property breaks down the discrete part of multiple discrete-continuous models into a binary choice problem.*

Case 2 – Logit with zero market share and aggregate data.

Suppose there are T *markets. Each market has* M_t *individuals. In period* t*, they go to the market and purchase one unit of a good from alternatives* $j \in \{0, 1, 2, \cdots, J\}$*. 0 is the outside option. The random utility for good* j *is* $u_{ijt} = x_{ijt}\beta_x + w_{ijt}\beta_w + \eta_{ijt}$*. A consumer will choose the good* j *whenever* u_j *is the maximum. Conditional on the observables, with a random part* η_{ijt}*, a homogeneous consumer optimal choice ends up in a choice probability* s_{jt}*. The probability of observing the quantities purchased* $(Q_{0t}, Q_{1t}, Q_{2t}, \cdots, Q_{Jt})$ *inside a market* t *is,*

$$Pr_t(Q_{0t}, Q_{1t}, Q_2, \cdots, Q_{M_t}) = \frac{M_t!}{Q_{0t}!Q_{1t}!Q_{2t}!\cdots Q_{Jt}!} s_{0t}^{Q_{0t}} s_{1t}^{Q_{1t}} \cdots s_{Jt}^{Q_{Jt}} \tag{2}$$

Equation (2) gives $Q_{jt} = 0$ *with positive probability unless* $M_t \to \infty$*. However, a model with a Logit error* η *does not allow* $Q_{jt} = 0$*. To deal with the endogeneity, researchers like Nevo (2000) invert the system by taking* ln *of the observed shares. This inverting scheme makes it vulnerable to the potential zero market share, as* $\ln(x) \to -\infty$ *as* $x \to 0$*.*

Intuitively, our goal is to retrieve a $\tilde{y}_i$ from y_i, with $\tilde{y}_i$ mimicking the behavior of y_i^* in Equation (1). Nevertheless, y_i^* is a function of the unobserved η_i. Therefore, the first set of assumption – Assumption 1 below, unveils a condition under which the construction of $\tilde{y}_i$ is possible. Assumption 1(i) gives a sufficient condition about when the systematic part of η_i can be approximated well by

1. As summarized in Lewbel et al. (2012), the most commonly recognized drawback of LPM is that LPM may generate fitted probability below zero or above one. Alternatively, LPM can only approximate the probability for a limited set of regressors.

2. If full parameterization is too restrictive, Cosslett (1983) also provides distribution-free estimator.

a function of the observables.

Assumption 1.

(i) $\eta_i = h(\zeta_i) + \varepsilon_i$*, and* ζ_i *is the error term that can be defined implicitly by* $d(x_i, w_i, z_i, \zeta_i) = 0$*. Besides,* $E[h(x_i, w_i, z_i)|z_i, x_i] = 0$*.*
(ii) ε_i *is orthogonal to* $x_i, z_i, h(x_i, w_i, z_i)$*. The CDF of* ε_i*,* $F_\varepsilon(\cdot)$ *is known and strictly increasing for all* $\varepsilon_i \in \mathcal{R}$*, with finite first and second moments.*

Assumption 1(*i*) refers to the case where given (x_i, w_i, z_i), ζ_i is pinned down uniquely by a deterministic function $d(x_i, w_i, z_i, \zeta_i) = 0$. Assumption 1(*i*) is restrictive but may not be as restrictive as it appears. The restriction may preclude a more complicated determination of w_i. For instance, if w_i is the price, usually, we cannot allow for a Nash Bertrand competition with an additional error term in the marginal cost unless researchers are willing to risk estimating it semiparametrically first. As elucidated by Berry and Haile (2014) and Kim and Petrin (2019), estimating cost shock and a flexible pricing function is feasible but could be computationally intensive.

Assumption 1(*i*) implies that we can construct a $\tilde{y}$ such that $E[(\tilde{y}_i - x_i'\beta_x - w_i'\beta_w) \otimes (x_i', z_i)'] = 0$, at the true value (β_x^0, β_w^0). Assumption 1(*i*) contains the linear control function as a special case, where $\zeta_i = w_i - x_i'\tau_x - z_i'\tau_z$, and the joint distribution of (η_i, ζ_i) is assumed normal. Correspondingly, η_i can be decomposed into two parts as $\eta_i = a\zeta_i + \varepsilon_i$, with ε_i not correlated with any observable. Nevertheless, Assumption 1(*i*) allows more flexible patterns than a typical control function. w_i can be determined by an unknown nonlinear function without many parametric assumptions. We only need to guarantee that, at least, it can be approximated well by some methods.

As for the multi-product Nash Bertrand competition, which we commonly encounter in industrial organization, Kim and Petrin (2019) has studied the invertibility. First, they notice that by the identification results in Berry and Haile (2014), one can invert out the cost shock nonparametrically. Hence, they treat the supply side as if the η_i is the only error term and then naturally invert η_i out by a flexible function of observed characteristics. However, even though they start with an ambitious plan, their application focuses only on the low-dimensional additively separable case, without adjusting for the possibly high-dimensional machine learning first stage, where the convergence speed could be slower than $\sqrt{N}$.

Assumption 1(*ii*) claims that we need to know the distribution of ε_i to proceed with the inversion. One may consider the specification F_ε as a normalization for identification. This normalization

occurs in either the binary choice model estimation or the Logit-family with endogenous variables [3]. Then we continue with our two examples.

Case 1 continue

Suppose we observe $y_i = \mathbf{1}\{\eta_{i,0} - \eta_{i,1} \leq x'_{i,1}\beta_x + w_{i,1}\beta_w\} = \mathbf{1}\{h(x_{i,0}, w_{i,0}, z_{i,0}) + \varepsilon_{i,0} - h(x_{i,1}, w_{i,1}, z_{i,1}) - \varepsilon_{i,1} \leq x'_{i,1}\beta_x + w'_{i,1}\beta_w\}$. *Usual normalization of the outside option gives* $h(x_{i,0}, w_{i,0}, z_{i,0}) = 0$. *Assume that the CDF of the distribution of* $\varepsilon_{i,0} - \varepsilon_{i,1}$ *follows* Λ_ε. *Therefore, the conditional mean of the binary outcome variable is,*

$$\begin{aligned} E[y_i|w_i, x_i, z_i] &= E[y_i|w_i, x_i, h(x_i, w_i, z_i)] \\ &= Pr_i(y_i = 1|w_i, x_i, h(x_i, w_i, z_i)) = \Lambda_\varepsilon(x'_i\beta_x + w'_i\beta_w + h(x_i, w_i, z_i)) \end{aligned}$$

By Assumption 1(ii), Λ *is known and invertible. If we know the conditional expectation* $E[y_i|w_i, x_i, z_i]$, *we can invert the system, so that,*

$$\Lambda_\varepsilon^{-1}(E[y_i|w_i, z_i, x_i]) = x'_i\beta_x + w'_i\beta_w + h(x_i, w_i, z_i)$$

Denote $W_i = (x_i, w_i, z_i)$. *Then following Assumption 1(i), we have the moment condition* m_i *such that,*

$$m(W_i, \beta) = [\Lambda_\varepsilon^{-1}(E[y_i|w_i, x_i, z_i]) - x'_i\beta_x - w'_i\beta_w] \otimes \begin{pmatrix} x_i \\ \\ z_i \end{pmatrix}, \quad \text{with } E[m(W_i, \beta)] = 0. \tag{3}$$

In this case, $\tilde{y}_i = \Lambda_\varepsilon^{-1}(E[y_i|w_i, x_i, z_i])$.

Case 2 continue

We replace the η_{ijt} *in the random utility by its expression and then have* $u_{ijt} = x_{ijt}\beta_x + w_{ijt}\beta_w + h_i(x_{jt}, w_{jt}, z_{jt}) + \varepsilon_{ijt}$. *Following the routine assumption for the discrete choice model with aggregate data where* $\xi_{jt} = h_i(x_{jt}, w_{jt}, z_{jt}) = h(x_{jt}, w_{jt}, z_{jt})$, *we have* $\varepsilon_{ijt} \sim$ *T1EV and the aggregate market*

3. For the distribution-free estimator like Cosslett (1983), they still need to normalize parameters in front of some observables.

share becomes,

$$s_{jt} = \frac{\exp(x_{jt}\beta_x + w_{jt}\beta_w + h(x_{jt}, w_{jt}, z_{jt}))}{1 + \sum_{\ell=1}^{J} \exp(x_{jt}\beta_x + w_{jt}\beta_w + h(x_{jt}, w_{jt}, z_{jt}))},$$
$$s_{0t} = \frac{1}{1 + \sum_{\ell=1}^{J} \exp(x_{\ell t}\beta_x + w_{\ell t}\beta_w + h(x_{\ell t}, w_{\ell t}, z_{\ell t}))}$$

If we know the true s_{jt}, we can invert it (Nevo, 2000) by

$$log\left(\frac{s_{jt}}{s_{0t}}\right) = x_{jt}\beta_x + w_{jt}\beta_w + h(x_{jt}, w_{jt}, z_{jt})$$

And the implied moment condition m_i is,

$$m(W_i, \beta) = \left[log\left(\frac{s_{jt}}{s_{0t}}\right) - x_{jt}\beta_x - w_{jt}\beta_w\right] \otimes \begin{pmatrix} x_i \\ \\ z_i \end{pmatrix} \quad with\ E[m(W_i, \beta)] = 0. \tag{4}$$

However, the moment conditions in equation (3) and (4) are both infeasible. In (3), we do not observe the $E[y_i|w_i, x_i, z_i]$. What we observe is y_i. In the second case, we do not know the true s_{jt}. Our calculated $\tilde{s}_{jt} = \sum_{i=1}^{M_t} Q_{ijt} / \sum_{j=1}^{J} \sum_{i=1}^{M_t} Q_{ijt}$ may not be ready to use, since $\tilde{s}_{jt}$ can be 0 while the true s_{jt} cannot. Nevertheless, we may estimate them in the first stage and plug them in to use the linear-in-parameters moment condition with the known distribution of ε_i. Appendix A. shows through a simple example that a typical machine learning method would follow the observed data pattern and obtain the estimated $E[y_i|w_i, x_i, z_i]$, which is exactly what we want in equation (3) and (4).

The performance of this two-step estimator depends on the first-stage estimation of $\tilde{y}_i$. Newey (1994) demonstrates that the second-stage structural parameters β can still be root-N consistent even when the first-stage converges slower than root-N[4]. However, if we have a high dimensional space in which model a bias-variance trade-off is required to prevent overfitting, the original moment condition would fail to have root-N consistency unless using the orthogonal moment and sample-split (Chernozhukov et al., 2018). In this case, we can achieve the orthogonal moment condition by adding an analytical term ϕ, which can be constructed as a deterministic function of the estimated propensity score $\hat{\pi}_i$.

Suppose the moment condition is $m_i(W_i, \gamma, \beta)$. γ is an invertible function that maps π into $\tilde{y}_i$.

4. In their examples, kernel estimation can be adopted for a candidate of the first stage.

$\gamma_i = \gamma_i(\pi_i) = \tilde{y}_i$. π_i is the probability of choosing option 1 in Case 1, and market share $\hat{s}_{jt}$ in Case 2. Follow the notation in Chernozhukov et al. (2022a), the correction term ϕ equals,

$$\phi(W_i, \gamma) = \alpha(W_i)\rho(W_i, \gamma) = \frac{\partial \gamma(\pi_i)}{\partial \pi_i} \otimes \begin{pmatrix} x_i \\ z_i \end{pmatrix} \times (y_i - \pi_i) \tag{5}$$

$\rho(W_i, \gamma) = (y_i - \pi_i) = (y_i - \Lambda(\gamma_i))$. Adding ϕ in (5) to the moment condition, we obtain an orthogonal moment condition ψ such that,

$$\psi(W_i, \gamma, \beta, \gamma) = m(W_i, \gamma, \beta) + \phi(W_i, \gamma)$$

Theorem 1 shows the orthogonality of $\psi(W_i, \gamma, \beta, \gamma)$ regarding the first stage estimates π_i.

Theorem 1. *The addition of the ϕ term to m leads to the asymptotic orthogonality. And it has $\int_W \phi(W, \gamma_0, \alpha_0) F_0(dW) = 0$.*

Proof. *With $\tilde{y}_i = \gamma(\pi_i)$, the population moment condition equals*

$$\begin{aligned}
\psi(W_i, \gamma, \beta, \pi_i) &= E\Bigg[\bigg(\frac{\partial \gamma(\pi_i)}{\partial \pi_i} \times (y_i - \pi_i) + \gamma(\pi_i) - x_i'\beta_x - w_i'\beta_w\bigg) \otimes \begin{pmatrix} x_i \\ z_i \end{pmatrix}\Bigg] \\
&= E\Bigg[E\Bigg[\bigg(\frac{\partial \gamma(\pi_i)}{\partial \pi_i} \times (y_i - \pi_i) + \gamma(\pi_i) - x_i'\beta_x - w_i'\beta_w\bigg)\Bigg|W\Bigg] \otimes \begin{pmatrix} x_i \\ z_i \end{pmatrix}\Bigg] \\
&= \int_W \bigg(\frac{\partial \gamma(\pi_i)}{\partial \pi_i} \times (E[y_i|W_i] - \pi_i) + \gamma(\pi_i) - x_i'\beta_x - w_i'\beta_w\bigg)\Bigg|W\Bigg] \otimes \begin{pmatrix} x_i \\ z_i \end{pmatrix} dF(W) \\
&= \int_W \bigg(\frac{\partial \gamma(\pi_i)}{\partial \pi_i} \times (\pi_i^0 - \pi_i) + \gamma(\pi_i) - x_i'\beta_x - w_i'\beta_w\bigg)\Bigg|W\Bigg] \otimes \begin{pmatrix} x_i \\ z_i \end{pmatrix} dF(W)
\end{aligned}$$

Take the derivatives with respect to π_i and evaluate at the true value (β^0, π_i^0). We obtain,

$$\frac{\partial \psi(W_i, \gamma, \beta, \pi_i)}{\partial \pi_i} = \frac{\partial}{\partial \pi_i}\bigg[\frac{\partial \gamma(\pi_i)}{\partial \pi_i}(\pi_i^0 - \pi_i) + \gamma(\pi_i)\bigg] = \frac{\partial \gamma^2(\pi_i)}{\partial \pi_i^2}(\pi_i^0 - \pi_i)$$

which equals to 0 when evaluated at $\pi_i = \pi_i^0$, for all possible $(x_i, w_i, z_i) \in \mathcal{X} \times \mathcal{W} \times \mathcal{Z}$. It implies that the Gateaux derivative of moment condition $\psi(\cdot)$ with respect to π_i is zero. Since

$E[y - \pi_0|W_i] = E[y - E[y|W]|W] = 0$ and $\partial\gamma(\pi)/\partial\pi$ is a function of W, we have $E[\phi(W, \gamma_0)] = \int_W \phi(W, \gamma_0, \alpha_0)F_0(dW) = 0$. □

Case 1 continue

For the binary choice model, $\pi_i = Pr_i(y_i = 1|W_i)$*, and* γ *is* $\Lambda_\varepsilon^{-1}(\cdot)$*. Thus, we have the* ϕ *as,*

$$\phi(W_i, \gamma) = \frac{y_i - E[y_i|W_i]}{\Lambda_\varepsilon'(\Lambda_\varepsilon^{-1}(E[y_i|W_i]))} \otimes \begin{pmatrix} x_i \\ z_i \end{pmatrix} \tag{6}$$

Then the orthogonal moment condition ψ *becomes,*

$$E\left\{ \left(\frac{y_i - E[y_i|W_i]}{\Lambda_\varepsilon'(\Lambda_\varepsilon^{-1}(E[y_i|W_i]))} + \Lambda_\varepsilon^{-1}(E[y_i|W_i]) - x_i'\beta_x - w_i'\beta_w \right) \otimes \begin{pmatrix} x_i \\ z_i \end{pmatrix} \right\} = 0$$

Case 2 continue

For the discrete choice, we assume that the outside option is a "safe good" defined in Gandhi et al. (2019). s_{0t} *is always positive and far from 0* [5]*.* $y_{jt} = s_{jt}$ *and* $\pi_{jt} = \tilde{s}_{jt}$*. Then we have the correction term,*

$$\phi(W_{jt}, \gamma) = \frac{1}{\tilde{s}_{jt}}(s_{jt} - \tilde{s}_{jt}) \otimes \begin{pmatrix} x_i \\ z_i \end{pmatrix}$$

Then the moment condition ψ *in the second stage becomes,*

$$E\left\{ \left[\frac{1}{\tilde{s}_{jt}}(s_{jt} - \tilde{s}_{jt}) + log\left(\frac{\tilde{s}_{jt}}{s_{0t}}\right) - x_{jt}\beta_x - w_{jt}\beta_w \right] \otimes \begin{pmatrix} x_i \\ z_i \end{pmatrix} \right\} = 0$$

In practice, we can implement our estimation in a few straightforward steps through standard statistical software. Furthermore, as indicated in Chernozhukov et al. (2018) and Cattaneo et al. (2019), sample splitting would further decrease the overfitting bias. To implement, we divide the sample into L different sets, $\ell \in \{1, 2, \cdots, L\}$[6]. I_ℓ contains the data in ℓ and $\hat{\gamma}_\ell$ are the

5. This is a reasonable assumption. Usually the outside option is defined as a composite good or the all-other goods, and usually it accounts for more than 50% of the observed market share.

6. A full leave-one-out jackknife estimator like Cattaneo et al. (2019) could be too time-consuming in this case.

estimates using all sample points not contained in I_ℓ. We summarize our estimation procedure in the algorithm below.

Algorithm. *We present the algorithm as:*

> ***Step 1.*** *For each splitted sample, $\ell \in \{1, 2, \cdots, L\}$. Obtain an estimate $\hat{\pi}_\ell$. Usually $\hat{\pi}_\ell$ is probability or propensity score and γ is a deterministic function of $\hat{\pi}$. $\hat{\pi}_\ell$ can be estimated using a flexible nonparametric machine learning or flexible parametric model.*
>
> ***Step 2.*** *Construct the moment condition by,*
>
> $$\hat{m}(W, \gamma, \beta) = \frac{1}{n} \sum_{\ell=1}^{L} \sum_{i \in I_\ell} m(W_i, \hat{\gamma}_\ell, \beta)$$
>
> *where $\hat{\gamma}_i$ serves as $\tilde{y}_i$. If there is model selection or parameter penalization for bias-variance trade-off in **Step 1,** then one needs to refer to the debiased moment conditions,*
>
> $$\begin{aligned} \hat{\psi}(W, \gamma, \beta) &= \hat{m}(W, \gamma, \beta) + \hat{\phi}(W, \gamma) \\ &= \frac{1}{n} \sum_{\ell=1}^{L} \sum_{i \in I_\ell} m(W_i, \hat{\gamma}_\ell, \beta) + \frac{1}{n} \sum_{\ell=1}^{L} \sum_{i \in I_\ell} \phi(W_i, \hat{\gamma}_\ell) \end{aligned}$$
>
> ***Step 3.*** *Solve for the $\hat{\beta}$, as the minimizer of,*
>
> $$\hat{\beta} = \underset{\beta_x, \beta_w}{arg\ min}\, \hat{m}(W, \gamma, \beta)' \hat{\Upsilon} \hat{m}(W, \gamma, \beta), \quad or\ \hat{\beta} = \underset{\beta_x, \beta_w}{arg\ min}\, \hat{\psi}(W, \gamma, \beta)' \hat{\Upsilon} \hat{\psi}(W, \gamma, \beta)$$
>
> *$\hat{\Upsilon}$ is the weighting matrix. If we have linear m, then the 2SLS have a simple analytical solution. Additionally, we can use two-step GMM in Step 3 and have $\hat{\Upsilon} = \hat{\Psi}^{-1}$ to improve efficiency.*

The usual GMM standard errors are asymptotically valid if we use the orthogonal moments. However, they may have worse finite-sample performance. More discussion of asymptotic results can be found in Section 3.

Case 1 continue

Consider a special case with ε_i follows i.i.d normal standard normal distribution Φ. Then, we cal-

As pointed out by Chernozhukov et al. (2022a), L = 5 or L = 10 would be a reasonable choice to balance the computational burden and bias reduction. L = 10 works better for the smaller data sets.

culate the $\hat{\gamma}$ in step 1, obtain the predicted probabilities $\hat{\pi}_i$, and insert it back to get $\tilde{y}_i = \hat{\gamma}(\pi_i) = \Phi^{-1}(\hat{\pi}_i)$. In step 2, if model selection techniques are adopted, the pseudo-outcome variable $\tilde{y}_i$ then becomes $\tilde{y} = (y_i - \hat{\pi}_i)/(\phi(\Phi^{-1}(\hat{\pi}_i))) + \Phi^{-1}(\hat{\pi}_i)$.

Including Λ imposes a parametric assumption on the random error. However, it may be less restrictive after considering the extra cost we encounter for having a distribution-free estimation scheme. Theoretically, one could refer to the distribution-free maximum likelihood proposed by Cosslett (1983) or the Special Regressor method advocated in Dong and Lewbel (2015) to have the distribution estimated at the same time as the parameters. Both of them suffer from non-trivial large-support requirements – Cosslett (1983) calculates the cumulative probability for the observed points only, and similar to the probability density in Lewbel (2012) and Dong and Lewbel (2015)[7].

Our proposed technique is different from the so-called forbidden regression. The forbidden regression has different meanings. J. Wooldridge (2001) considers the forbidden regression as inserting a nonlinear function of OLS-predicted $\hat{w}_i$ into the second stage since it is incorrect to assume that "the linear projection of the square is the square of the linear projection". In Angrist and Krueger (2001) and Chen et al. (2021), forbidden regression resorts to regressing y_i on the $\hat{w}_i$, where $\hat{w}_i$ is estimated using a nonlinear regression method. Angrist and Krueger (2001) points out that this would not lead to consistent estimation. All of those focus on fitted values entering as regressors, which is different from what we propose. Besides, our main focus does not lie in improving the linear-IV performance. Instead, we want to extend the linear-IV method to those that previously could not use linear-IV by constructing a pseudo dependent variable $\tilde{y}_i$.

3 Asymptotic Results for the Debiased Two-Stage Estimator

Firstly, we state several assumptions. Basically, they are the similar set of conditions required in Chernozhukov et al. (2022a) for the method of moments and Chernozhukov et al. (2021) for the generalized linear model.

Assumption 2. ***Bounded π_i:*** *π_i^0 is between $(\bar{c}, 1-\bar{c})$ for some $\bar{c}$ uniformly over the support of $W \in \mathcal{X} \times \mathcal{W} \times \mathcal{Z}$.*

7. Although, according to Dong and Lewbel (2015), a symmetric-support condition can be an alternative, the importance of their large-support conditions has been re-emphasized by a bunch of simulations in Bontemps and Nauges (2017).

Assumption 3.

(i) the weighting matrix, $\hat{\Upsilon} \xrightarrow{p} \Upsilon_0$*, and* Υ_0 *is positive definite;*

(ii) $E[m(W_i, \gamma_0, \beta)] = 0$ *holds if and only if* $\beta = \beta_0$*;*

(iii) $\beta \in \mathcal{B}$ *and* $\mathcal{B}$ *is a compact set, and* $E[sup_{\beta \in \mathcal{B}} ||m(W, \gamma_0, \beta)||] < \infty$*;*

(iv) there is a $C > 0$ *and* $\hat{M} = O_p(1)$ *such that for* $||\gamma - \gamma_0||$ *small enough and all* $\tilde{\beta}, \beta \in \mathcal{B}$*, we have*

$$||\hat{m}(W, \gamma, \tilde{\beta}) - \hat{m}(W, \gamma, \beta)|| \leq \hat{M}||\tilde{\beta} - \beta||^{1/C}$$

Assumption 4.

(i) $E[||m(W_i, \gamma_0, \beta_0)||^2] < \infty$ *and there exists* $C > 0$ *such that* $E[||m(W, \gamma, \beta)||^2] \leq C||\rho||^2$*;*

(ii) $\alpha_0(W_i)$ *and* $E[||\rho(W_i, \pi_0)||^2|W]$ *are bounded;*

(iii) There is a $C > 0$ *such that for all* $||\pi_\ell - \pi_0||$ *small enough, such that (1)* $E||\hat{\alpha}_\ell(W_i, \hat{\pi}_\ell) - \alpha_0(W_i, \pi_0)||^2 \leq C||\hat{\pi}_\ell - \pi_0||$*; (2)* $||\hat{\gamma}(W, \hat{\pi}_\ell) - \gamma_0(W, \pi_0)|| \leq C||\hat{\pi}_\ell - \pi_0||$*; (3) For the* α *corresponding to j-th moment condition,* $||\hat{\alpha}_{j,\ell}(W, \hat{\pi}_\ell) - \alpha_{j,0}(W, \pi_0)|| \leq C||\hat{\pi}_\ell - \pi_0||$*.*

(iv) Faster than $N^{-1/4}$ *convergence of estimated* π*, or* $||\hat{\pi}_\ell - \pi_0|| = o_p(N^{-1/4})$*.*

(v) $E[||\psi(W_i, \gamma, \alpha_0, \beta_0)||^2] < \infty$*. For all* γ *with* $||\gamma - \gamma_0||$ *small enough.*

$$||E[\psi(W_i, \gamma, \alpha_0, \beta_0)]|| = \left|\left| \int [m(W_i, \gamma, \beta) + \alpha(W_i)\rho(W_i, \pi)] F_0(dW) \right|\right| \leq C||\gamma - \gamma_0||^2$$

(vi) $\int ||m(W_i, \hat{\gamma}_\ell, \hat{\beta}) - m(W_i, \hat{\gamma}_\ell, \beta_0)||^2 F_0(dW) \xrightarrow{p} 0$

Assumption 2 requires the propensity score π_i to be bounded from below and above. Assumption 2 is a common technical regularization condition for literature that involves propensity score inversion since otherwise, the function would blow up as it approaches 0 or 1. Assumption 2 helps regularize the function γ and α and is necessary even in the work that does not explicitly have propensity score entering into the estimation, like Singh and Sun (2021).

Assumption 3(*i*), 3(*ii*) and 3(*iii*) are typically identification conditions in the extreme estimator. We need these for consistency proof. According to Newey (1991), Assumption 3(*iv*) can be replaced by $\mathcal{B}$ is convex, $m(W_i, \gamma, \beta)$ is continuously differentiable and $\partial \hat{m}(W, \beta)/\partial \beta$ is dominated by a stochastically bounded sequence. Both Assumption 3(*iv*) and the alternative are easily satisfied whenever we have a moment condition linear in β.

Assumption 4 is more specific. Assumption 4(*i*) requires the finite second moment of m. For this

case, it refers to $E[||m(W,\gamma_0,\beta_0)||^2] = E[||h_i \otimes (x_i,z_i)'||^2] < \infty$, a typical assumption for any linear-IV estimator to obtain the asymptotic distribution. Assumption 4(*ii*) is purely technical, and we maintain it for simplicity.

Recall that both γ and α are deterministic functions of π, Assumption 4(*iii*) requires the mean square continuous of α and Lipschitz continuity of α and γ, which hold with Assumption 2. In the binary choice case, we have the quantile function (inversed CDF) $\gamma = \tilde{y}_i = \Lambda_\varepsilon^{-1}(\pi_i)$ and the inverse propensity score $\alpha = (1/(\Lambda_\varepsilon'(\Lambda_\varepsilon^{-1}(\pi_i)))) \otimes (x_i', z_i')$. By Assumption 2, the continuity follows directly after the elimination of the points where $\pi_i \to 0$ or $\pi_i \to 1$ and both γ and α diverge rapidly. A similar situation applies to the zero-share case. We need the true market share to be bouneded away from both 0 and 1.

Assumption 4(*v*) bounds the expectation of the debiased moment condition. Assumption 4(*vi*), in this case, is equivalent to $\hat{\beta}_\ell$ is a consistent estimator for the linear-IV model with $\hat{\gamma}_\ell$ inserted and the linear moment m is mean-square continuous in β – a condition that is easy to meet with linear moment m.

Assumption 4(*iv*) requires the rate of convergence for the estimated π_i is faster than $N^{-1/4}$. As illustrated in Chernozhukov et al. (2021), unfavorable nonlinearity in ρ regarding γ eliminates the possibility of double robustness[8]. Moreover, it requires a faster convergence rate in the first stage, which is easily stated but may be hard to verify in practice. Fortunately, the $o(N^{-1/4})$ convergence is attainable in nonparametric non-endogenous settings theoretically.

The major problem slowing down the convergence rate is that the estimated $\hat{\gamma}$ needs not to have the correct structural specification. For instance, higher-order polynomials can approximate the underlying γ_0 while the γ_0 itself needs not to be a linear combination of the exact polynomials. With this, the traditional consistency is no longer applicable. Instead, after assuming an optimal solution inside the restricted function set, the error term can be bounded by oracle inequalities. For the lasso-type estimator, after the influential work by Bickel et al. (2009), further research like Caner and Kock (2019) has shown the oracle inequalities in linear-GMM with Lasso. Their results have been used in Bakhitov (2022) to demonstrate the mean square convergence of α. For the deep neural nets, the bounds provided in Farrell et al. (2021) are then used for the error in Chernozhukov

8. Double robustness is an encouraging property widely existing in treatment-effect estimation. One needs to have both γ and ρ entering linearly for the double robustness to hold, which is impossible in our setting. Either γ or ρ can be made linearly in m and ϕ, but not both.

et al. (2021).

While a variety of papers provide well-developed results for linear regression, only a few focus on a generalized linear model or maximum likelihood. Alquier et al. (2019) give a version of tight oracle inequality for penalized maximum likelihood, but their analysis is mainly restricted to the case that the regressors follow a Gaussian distribution. Blazère et al. (2014) show the asymptotic oracle inequality of the generalized linear model for a grouped Lasso, incorporating the binary response model with known parametric error. Neural nets is considered a sieve estimation but with sieves determined within the data. Chen and White (1999) studies the shallow neural networks with smooth activation functions and provide bounds for the estimation error. Farrell et al. (2021) demonstrate the performance of deep neural nets with ReLU activation function and Lipschitz loss functions in generalized linear regression, including logistic[9]. They show a non-asymptotic oracle inequality, which attains a convergence rate of $o(N^{-1/4})$ under some smoothness conditions, rendering it a reasonable alternative for the first stage.

Then we reach the central theorem for this study. Theorem 2 states that under the assumptions mentioned above, we would have root-N consistent and asymptotic normality of β, with the propensity score estimated by high-dimensional techniques.

Theorem 2. *After all the Assumptions 1 - 4 and the zero-derivative property illustrated in Theorem 1 are satisfied*[10]*, we would have* $\hat{\beta} \xrightarrow{p} \beta_0$ *and the asymptotic normality of the structural parameters* $\boldsymbol{\beta}$ *as,*

$$\sqrt{N}(\hat{\boldsymbol{\beta}} - \boldsymbol{\beta_0}) \xrightarrow{d} N(0, \boldsymbol{V}), \quad \hat{\boldsymbol{V}} \xrightarrow{p} \boldsymbol{V}, \quad \hat{\boldsymbol{V}} = (\hat{G}'\hat{\Upsilon}\hat{G})^{-1}\hat{G}'\hat{\Upsilon}\hat{\Psi}\hat{\Upsilon}\hat{G}(\hat{G}'\hat{\Upsilon}\hat{G})^{-1}$$

$$\hat{G} = \partial\hat{m}(W, \gamma, \hat{\beta})/\partial\beta.$$

$\hat{\Psi} = \frac{1}{n}\sum_{\ell=1}^{L}\sum_{i\in I_\ell}\hat{\psi}_{i\ell}\hat{\psi}'_{i\ell}$. Υ *serves as the weighting matrix for GMM type estimators, and equals to* $\hat{\Upsilon} = (Z'Z)^{-1}$ *in the 2sls.*

Proof. See Appendix B.. □

9. And they mention that their results can even cover multinomial logistic regression with multiclass classification.
10. For Case 2, we need an extra assumption that the number of individual consumers in each market would also go to infinity with N.

4 Monte Carlo Simulations

In this section, we evaluate the finite-sample performance of our proposed algorithm using a variety of data generating processes, for both the binary choice and zero market shares.

4.1 Example 1: binary choice with endogenous variables

We assume that our binary choice model as the discrete part of the multiple discrete-continuous choice model in Bhat (2018)[11]. Suppose the error term η_i can be decomposed into two parts, $\theta_i = \xi_i + \varepsilon_i$. ξ_i is the systematical error that causes endogeneity, and ϵ_i is the random part. A consumer i would choose to purchase good 1 with the outside option good 0 if and only if,

$$x'_{i,1}\beta_x + w'_{i,1}\beta_w - \log(h_{i,1}) + \xi_{i,1} + \varepsilon_{i,1} \leq x'_{i,0}\beta_x + w'_{i,0}\beta_w - \log(h_{i,0}) + \varepsilon_{i,0}$$

More precisely, we focus on disutility rather than the utility. $h_{i,0}$ comes from the capacity constraint of a multiple discrete-continuous optimization. Our goal in first stage is to provide a flexible estimation of the choice probability, or, $\Lambda_{\varepsilon_{i,1}-\varepsilon_{i,0}}\big[x_{i,0} - x_{i,1})'\beta_x + (w_{i,0} - w_{i,1})'\beta_w - (\log(h_{i,0}) + \log(h_{i,1})) - \xi_{i,1}\big]$. we test different nonparametric and machine learning methods, including polynomial approximation, elastic nets (EN), deep neural nets (DNN), etc.

4.1.1 w_i is generated linearly

In the most basic case, we have w_i determined by a linear separable function,

$$w_{i,j} = x'_{i,j}\tau_x + z'_{i,j}\tau_z + \xi_{i,j} + \upsilon_{i,j} \tag{7}$$

υ may simply follows a Normal distribution with $\mu = 0$ and $\sigma = 0.2$. z_i are the excluded instruments. z_{ij} enters as the determinants for w_i and also as exclusive moment conditions in the linear-IV part.

Issues regarding numerical stability need to be emphasized before proceeding on. In the binary response model with endogenous regressors, our correction term has probability density in the denominator. Even without correction, we still have the inverse of the CDF. When implemented, we adopt the ad hoc trimming or censoring and delete the estimated $\hat{\pi}_i$ with extreme values. These are

11. In Bhat (2018), the baseline utility governing the discrete choice can be different from the continuous part; thus, we may consider them separately.

standard empirical practices in economic research, though they come with limited theoretical justifications[12]. Moreover, as a common implicit requirement for moments involving inverse propensity scores, we must have enough variation in the data, where we cannot have most choice probabilities clustering at around 0 and 1.

Table 1 to 2 illustrate the simulation with w_i determined by (7). The standard errors are the empirical from the 1000 repetitions of simulations. Raw indicates the use of the moment condition 11, while debiased as. Table 2 includes the case where we have 100 more invalid instruments generated as i.i.d standard Normal series. They do not enter into the determination of w_i. While not accounting for endogeneity results in biased results, all other methods in Table 1 work reasonably well. Both bias and standard errors slightly increase when we switch from linear control function or Logistic regression in the first stage to the DNN. In general, all the performances are fairly acceptable.

Table 2 shows the estimation when we adopt Lasso penalization to tackle a large number of instruments. Table 2 indicates that using orthogonal moment accompanied by Lasso would decrease the bias at the cost of a slight variance increase. If the sparsity pattern holds, we can extend it to a high-dimensional case where the number of potential instruments grows higher than the sample size. Table 3 focus on the case where we have slightly misspecification. In this case, the w_i is determined by $w_{i,j} = x'_{i,j}\tau_x + z'_{i,j}\tau_z + 0.5\xi_{i,j} + z_{i,j,3} \times \xi_{i,j} + 0.5\nu_{i,j} + 0.5\nu_{i,j} \times \xi_{i,j}$. With this misspecification, control function generates more bias for β_w. Nevertheless, the level of bias fluctuates across different DNN settings, indicating the sensitivity of our model towards the precision of first-stage prediction.

We need to emphasize that our method is sensitive to overfitting. We want the machine learning to pick up all the variations in the error term correlated by the observables while not to fit too closely to the data. Sticking to a reasonable tuning parameter would be crucial. For example, we fix λ_R at a minimal value for the elastic net. Ten-fold cross-validation to choose λ_L could be too time-consuming. Instead, we adopt the Generalized Information Criterion (GIC) developed in Zhang et al. (2010) and Fan and Tang (2013). Given a candidates λ_L, GIC takes the form

12. For instance, Chernozhukov et al. (2018) drops the observations in group Z = 0 with propensity score lying outside the range in group Z = 1. Trimming has been criticized a lot for its randomness, loss of information, and lack of theoretical justification. Li et al. (2019) down-weights the observations with extreme propensity scores in a treatment effect estimation framework. Singh and Sun (2021) refers to the automatic debiased machine learning to directly approximate the propensity score function. However, what they have proposed seems not applicable to our setting.

$GIC(\lambda_L) = 1/n\{-2\sum_{i=1}^{n}\ell_i(y_i, x_i, w_i, z_i) + a_n \times \#\text{ of df}\}$, where ℓ_i is the log-likelihood for sample point i, # of df means the degree of freedom[13] and $a_n = \ln(\ln(n)) \times \ln(\#\text{ coefficients})$, for a variety of maximum likelihood models, as in Fan and Tang (2013). We iterate over all candidates of λ_L and choose the λ_L that gives the smallest GIC after all the regressors are standardized[14].

4.1.2 w_i is generated nonlinearly

Suppose we have w_i as a nonlinear function, with interaction terms between observables and unobservables entering. When estimating, we do not know the exact functional form for w_i. Instead, we assume that we know that $[x_{i,1}, z_{i,2}, w_i]$ would enter as a nonlinear function which a series of basis functions can approximate. We use penalized sieve maximum likelihood in the first stage.

The endogenous w_i is generated following,

$$w_{i,j} = \frac{1}{3}x_{i,j,1} + \frac{1}{3}z_{i,j,2} + 0.1z_{i,j,2}^2 + \frac{1}{6} \times z_{i,j,2} \times \xi_{i,j} + \frac{1}{3}x_{i,j,1} \times \xi_{i,j} + \frac{2}{3} + \frac{1}{15}N(0,1) \tag{8}$$

Even though (8) contains a nonlinear part, ξ_i is still an implicit function of all other observables. With this, we may have misspecification in the linear control function (Petrin and Train, 2010). We use B-spline as an approximation for the unknown nonlinear approximation on $[x_{i,1}, z_{i,2}, w_i]$. Appendix C. contains detailed information for implementation.

Table 4 to 5 show the performance under different sample sizes. Degree-2 B spline generates 216 nonlinear parameters. The GIC selects a large λ_L, approximately between 30 to 80 for most of the simulated samples when n = 2880, and an even large λ_L for n = 570. As expected, the linear control function generates biased estimates due to misspecification, especially for β_w. With EN, the results from B-spline are generally close to the true value when n = 2280, demonstrating our estimator's superiority in capturing the underlying nonlinearity and irregularity in constructing $w_{i,j}$. The debiased machine learning combined with sample splitting gives the best estimates across all the cases. However, the encouraging performance exacerbates when the sample size decreases to approximately two times the number of parameters. Although the bias of the debiased sample-

13. Zou et al. (2007) points out that the degree of freedom for Lasso can be estimated consistently by the number of non-zero parameters. The degree of freedom of elastic net takes a slightly complicated form following Zou and Hastie (2005).

14. GIC-based tuning parameter selection would be time-saving, since for each λ_L, we only need to solve for the corresponding coefficients from optimization once. In our application, when we stick to a very small λ_r, though Zou et al. (2007) points out that the number of non-zero parameters is not an unbiased estimator for the elastic net, the discrepancy is neglectable. So we use the number of non-zero parameters for simplicity.

splitting estimator is still smaller than the linear control function for β_w, the discrepancy reduces. This is not surprising since those big-data techniques typically enjoy a good performance with sufficient information. It requires a large sample size for the underlying pattern to be detected.

As is well-known in the sieve approximation literature, curse of dimensionality is the main issue preventing the wide application of nonparametric techniques. Consider the case in Equation (8). The need of inverting ξ_i renders all the variables inside as nonlinear. To reduce the large dimension, we can adopt a partial-linear scheme. Specifically, for a partial linear model like $w_i = x_i^{(1)}\theta_x + z_i^{(1)}\theta_z + f_N(x_i^{(2)}, z_i^{(2)}, \xi_i)$, we can rewrite our polynomial approximation as a function on $(w_i - x_i^{(1)}\theta_x - z_i^{(1)}\theta_z, x_i^{(2)}, z_i^{(2)}, \xi_i)$, conditional on the $(\hat{\theta}_x, \hat{\theta}_z)$ off the current iteration.

Appendix D. contains simulations that are omitted in the main text, including the case where we have very weak instrumental variables or misspecification in the model determining w_i.

4.2 Example 2: zero market shares with aggregated data

The settings we use in the simulations for zero market shares originate from Gandhi et al. (2019), where sampling error causes the zero shares observed in the aggregated quantity demanded[15]. The multinomial model gives the underlying choice probabilities, but the observed quantities come from purchases from a finite number (for instance, $M_t = 1000$) of consumers, which is probably zero. As discussed in Section 2, zero market shares would prevent linear-IV's inversion from dealing with endogeneity.

We test a variety of machine-learning algorithms as the first stage, including EN for random-coefficient Logit discussed in Horowitza and Nesheimb (2021), and the newly-developed ResLogit in Wong and Farooq (2021). Specifically, Wong and Farooq (2021) provides a partial-linear model with multi-layer neural nets approximating the nonlinear part. In their algorithm, the product characteristics of good $j' \in \{0, 1, 2, \cdots, J\}$ join together to determine the choice probability of good j. Wong and Farooq (2021) refers to this as a valid and computationally-feasible method to approximate unknown nested structure. Details for implementation can be found in Appendix C.. The subsections below contain different settings and tables for simulated results.

15. In contrast to the situation where there is an underlying structure (Dubé et al., 2021)

4.2.1 w_i is generated linearly

First, let us consider the most basic case where w_i is a linear function, $w_{i,j} = x'_{i,j}\tau_x + z'_{i,j}\tau_z + \xi_{i,j} + \nu_{i,j}$. The number of goods inside a market is 6, with good 0 as the always-purchased outside option. Approximately 35% of the markets experience zero purchased quantities of inside goods. The performance of different estimators can be found in Table 6 and 7. While ignoring the endogeneity generates the largest bias, eliminating the zero shares may also induce imprecision. All other methods in Table 6 land in reasonable estimates. Debias do not matter since all the first-stage would have root-N consistency.

Table 7 contains a setting similar to Table 2, where we have 300 repetitions of simulations and 100 irrelevant excluded instruments. Even the linear-IV method based on the true market share would give bias up to 0.0439 for β_p. Dropping the zero market share or not bothering with endogeneity generates a larger bias than in Table 6. MLE with control function, on the other hand, performs the best since it satisfies the linearity and manages to circumvent the extra bias caused by linear-IV. Among all the methods with Lasso as the first stage, biases would be corrected whenever we switch from Raw to Debiased and from L = 1 to $L > 1$. Their bias is similar to the true market share but with a more significant standard error, as expected[16].

4.2.2 w_i is generated by Bertrand competition

In industrial organization, the most popular model for the endogenous prices w_i is the Nash-Bertrand competition. In each market t, the price vector $\boldsymbol{p}_t = (w_{1t}, w_{2t}, \cdots, w_{Jt})'$ is the solution of a nonlinear system,

$$\boldsymbol{p}_t = \underbrace{\boldsymbol{z}_t\tau + \boldsymbol{\iota}_t}_{\text{Marginal Cost}} - \left[\boldsymbol{\Omega}_t \circ \left(\frac{\partial \boldsymbol{S}_t}{\partial \boldsymbol{p}_t}\right)'\right]^{-1} \boldsymbol{S}_t \tag{9}$$

where $\boldsymbol{S}_t = (s_{1t}, s_{2t}, \cdots, s_{Jt})'$, $\boldsymbol{z}_t = (z_{1t}, z_{2t}, \cdots, z_{Jt})'$, $\boldsymbol{\iota}_t = (\iota_{1t}, \iota_{2t}, \cdots, \iota_{Jt})'$, $\boldsymbol{\Omega}_t$ is the $J \times J$ ownership matrix with $\boldsymbol{\Omega}_{t,ij} = 1$ if product i and j belong to the same company. Here we assume $J = 4$ with an outside option. In this setting, approximately 25% of the markets has zero shares. Since we only have three inside goods, zero-share on either of them could be an important concern and dropping any point may hurt the performance. The equilibrium prices are generated by fixed-point iteration. Kim and Petrin (2019) shows that under multi-product Nash Bertrand competition,

16. Because we have linear-IV as the second-stage estimator, the best we can achieve would be the bias when the true share is used. Further studies may extend this to incorporate some instrumental variable selection techniques.

the price of good j at market t can be expressed as a good-specific function such that,

$$p_{jt} = p_j(\boldsymbol{x}_t, \boldsymbol{z}_t^{(2)}, \boldsymbol{\iota}_t + \boldsymbol{z}_t^{(1)}, \nu_j(\boldsymbol{\xi}_t)) \tag{10}$$

if there exists a $z_{jt}^{(1)}$ as a perfect substitute to cost shocks ι_t. The last term in equation (10) gives an a one-to-one mapping of $\tilde{\xi}_{jt}$. The function p_j is specific to a commodity j. Kim and Petrin (2019) illustrates that one can obtain $\tilde{\xi}_{jt}$ by inverting the commodity-specific p_j that $\tilde{\xi}_{jt} = \nu_j(\boldsymbol{\xi}_t) = p_j^{-1}(\boldsymbol{x}_t, \boldsymbol{z}_t^{(2)}, \boldsymbol{\iota}_t + \boldsymbol{z}_t^{(1)}, p_{jt})$.

For the simulations below, we assume to know the cost shocks $\boldsymbol{\iota}_t + \boldsymbol{z}_t^{(1)}$. This simplified assumption comes from the supply-side identification results in Berry and Haile (2014), where they point out that the cost shock can be identified without specifying a supply-side model under the presence of an index variable $z_{jt}^{(1)}$. $\iota_{jt} + z_{jt}^{(1)}$ can be expressed as a function of s_t and p_t. Then, one can construct moment conditions and estimate via NPIV methods. Due to the lack of a computationally-feasible method for supply-side NPTV estimation, for simplicity, we omit the procedure of estimating and treat the $\boldsymbol{\iota}_t + \boldsymbol{z}_t^{(1)}$ as observed.

We attempt several methods for approximating the market shares. First, We use the ResLogit (Wong and Farooq, 2021). Because all the characteristics for the J-1 commodities launch into the function p_j, a sieve approximation based on tensor-product becomes almost infeasible even in a parsimonious setting[17]. Hence, we use the first and second-order interactions, which may not give a good approximation but is computationally feasible.

Table 8 contains the simulation results. In this case, all the methods except the case with true market share have some misspecification. The control function incorrectly retains the separable linearity. Res-Logit restricts how the nonlinearity enters into the model. Polynomials generated by higher-order interaction lose the usual good properties in B-spline or Chebyshev polynomials. In any case, considering endogeneity would never make things worse. If our machine learning estimation is not flexible enough, the estimated β would still be biased and may be worse than just ignoring the zero market shares. Methods like the Res-Logit with Layer 1 could be worse than the linear control function, though Res-Logit with Layer 1 itself is more flexible than the linear control function. Nevertheless, the debiased moments would unambiguously improve performance,

17. Consider we have the one-dimension x_{jt}, $z_{jt}^{(2)}$ and $\iota_{jt} + z_{jt}^{(1)}$. With J = 4, p_j is a flexible function of $(x_{1t}, x_{2t}, x_{3t}, z_{1t}^{(2)}, z_{2t}^{(2)}, z_{3t}^{(2)}, \iota_{1t} + z_{1t}^{(1)}, \iota_{2t} + z_{2t}^{(1)}, \iota_{3t} + z_{3t}^{(1)}, p_{jt})$. For degree-2 Chebyshev polynomials, there are $3^{10} = 59049$ bases.

especially for the polynomial case with the Lasso penalty.

Overall, machine learning itself may not be a panacea. We regard a careful implementation of those methods as a necessity for the success of our estimator. Moreover, researchers need to propose a feasible method to solve the high-dimensional problem before the identification results in Kim and Petrin (2019) can be applied empirically to a supply-side with Nash Bertrand competition.

5 Advantages and Problems

In this section, we discuss our estimator's advantages and potential problems. Our method treats the control function method (Petrin and Train, 2010) as a special case. If we assume w_i is determined by a low-dimensional linear function, then our estimated probability is root-N consistent. With correct normalization, the plug-in estimator gives root-N consistency for the structural parameters, similar to the control function. On the other hand, our method does not stipulate the exact formula of w_i. w_i can even be determined by an implicit function. All we need is a reasonable probability estimation. The analytical construction of orthogonality moments equips the first-stage probability estimation with flexible machine learning methods, which may be hard to achieve in the control function.

As for the problems, numerical instability is the first issue we encounter. The method includes inverting a system of probabilities, shares, or propensity scores, regardless of the correction. If probabilities are close to 0 or 1, they would have large leverage in the second stage. For instance, consider the case where ε_i follows a standard normal distribution, and the deterministic part $X\beta$ takes either a very large or small value. Then the true probabilities would be concentrated near 0 or 1, generating unstably large inversion value of both $\Lambda^{-1}(\cdot)$ and $\Lambda'(\Lambda^{-1}(\cdot))$. This problem cannot be relieved in a large sample.

This numerical instability leads to a data-dependent performance of our method, which is common in the well-developed inverse propensity score weighted estimators[18]. Thus, our estimator works better when the difference in product characteristics enters as X rather than the characteristics themselves. Also, performance may improve if more structures are included in the first stage.

18. This is why the Assumption 2 is always a pre-requirement for these types of estimators. Besides, Singh and Sun (2021) refer to techniques of automatic debiased machine learning and double robustness to circumvent the need of directly estimating the propensity score. However, their methods are not applicable in our case. We still need to bear the risk of inserting estimated probabilities.

Secondly, as discussed in section 3, our estimator does not have double-robustness as in Bakhitov (2022), which requires linearity in γ and the correction term simultaneously. This lack-of-double-robustness triggers a high-standard requirement on the first stage propensity score estimation. Not enough flexibility may cause it inadequate to endogeneity, while too much flexibility may result in overfitting. For the sieve-based polynomial approximation, tensor-based parameters may quickly rendering it impractical. Besides, researchers usually resort to nonparametric or machine learning methods when they are reluctant to impose model restrictions, which makes justifying the performance of those techniques hard.

In summary, flexibility can lead to instability, especially at the comparison of our estimator to the linear control function. To obtain good performance, the first-stage machine learning method needs special attention and carefully tuning.

6 Illustrative Application

Now we consider a simple application of our method in the context of coal purchases of power plants in the US from Year 2011 to 2015. We use the public data available on the EIA website. EIA form 923 [19] contains the monthly transaction-level coal procurement data for all the regulated power plants in the US. A transaction record carries some detailed information, for instance,

"Colbert, a power plant operated by Tennessee Valley Authority, on January 2012, purchased in the spot market of coals from the Black Thunder mine at Gillette, WY, owned by Arch Coal. The purchase involves 297.0 short tons, with average heat at 18.000 MMBtu per short ton, 0.23% sulfur content and 3.8% ash content. In total, it paid 247.8 cents per MMBtu for the coal to deliver to its site by railroad."

Note that the fuel cost is an aggregated cost, including both the free-on-board price and the fees for transportation. Coals are traded either by long-term contracts or in the spot market[20]. Train and Wilson (2011) shows that power plants' purchasing behavior falls into the class of multiple discrete-continuous choice model[21]. For simplicity, we only focus on the discrete-choice part – power plants'

19. EIA Form 923 is available here: https://www.eia.gov/electricity/data/eia923/.

20. Long-term contracts and spot market co-exist in the coal transactions. Long-term contracts lasts for at least a year, where power plants may order coal delivery every month. Delivery prices are adjusted following a pre-determined scheme in the contract. Further information can be found in Joskow (1985).

21. Coal-fired power plants decide which source of coals they purchase from and how much to purchase from each source, which perfectly fits in the scope of multiple discrete-continuous choice (Bhat, 2005)

binary decision on purchasing from a specific coal source j in the spot market, given the long-term contracts they currently hold as the outside options. Moreover, we aggregate the large number of coal varieties into six composite products grounded on their place of origin – (i) tier 1 SPRB; (ii) tier 2 SPRB; (iii) Other PRB coal;[22] (iv) Appalachia; (v) Other western; (vi) Mid continent[23]. The choice set of plant i in the spot market is assumed constant across time and constituted by the purchase history. Specifically, if plant i purchased j in the spot market in April 2014, then j is in the choice set for plant i for the entire time periods.

To proceed on, besides EIA-923, we also refer to the EIA-860 for plant related attributes, like the nameplate capacity and the installation of capital-intensive environmental equipment[24]. Table 9 contains the averaged observable characteristics, where one can see that coals from distinct regions have different characteristics. For instance, tier 1 SPRB, compared with the rest of PRB region, has higher energy content and lower sulfur content. Correspondingly, the delivery price for tier 1 SPRB coal is higher.

For each plant-month pair, we model the behavior of each plant as a binary choice, with j denotes the inside option from the spot market and the outside option with subscript 0 refers to the quantity-averaged characteristics from the currently-hold long-term contract. We construct a model following a Probit setting where a power plant's purchase decision y_{ijt},

$$y_{ijt} = 1\{X_{ijt}\beta + \xi_{ijt} + \varepsilon_{ijt} \geq X_{i0t}\beta + \xi_{i0t} + \varepsilon_{i0t}\} \tag{11}$$

If $\varepsilon_{i0t} - \varepsilon_{ijt}$ follows a standard normal distribution, then the probability of j is being chosen given the outside option is $\Phi((X_{ijt} - X_{iot})\beta + (\varepsilon_{ijt} - \varepsilon_{i0t}))$, which depends on the difference in characteristics. One important drawback for this individual transaction data is that we do not observe the transaction prices whenever the transaction breaks down, since the power plants only need report its successful purchases. While the physical characteristics of coals stays stable, the spot

22. PRB coal refers to the low-sulfur content subbituminous coal produced in WY or MT, along the powder river basin. Tier 1 SPRB includes the largest coal mines in SPRB, like North Antelope Rochelle, Black Thunder, Antelope and Jacobs Ranch. Tier 2 SPRB locates on the north of Tier 1, indicates coal mines like Caballo, Belle Ayr, Coal Creek and Cordero Rojo. Tier 1 and 2 SPRB coal have higher quality than other PRB coal.

23. Appalachia refers to the high-sulfur bituminuous coal produced along the Appalachia mountain. Other western mainly points to the coal produced in Southern Wyoming and Rockies. Coals mainly produced in Illinois Basin is ascribed as Mid continent.

24. The environmental equipment mainly refers to the Wet Scrubber Flue Gas Desulfurization (abbreviated as Scrubber), an apparatus used to remove the sulfur in the gas waste, helping the plant to meet the requirement of the Clean Air Act. Installation of scrubber is expensive, while its operation is less costly (Cicala, 2015). We expect that a power plant better equipped with scrubbers to be less sensitive towards sulfur content.

price needs to be imputed, following the appendix in Jha (2022) [25]. Simply speaking, the price is approximately calculated by a reduced-form forecast problem of log price on other characteristics and the price from the closest plant in physical distance. Denote that plant using superscript D, the reduced-form regression takes the form below[26],

$$\ln(p_{ijt}) = \kappa_{jt} + \mu_i + \ln(p_{ijt}^D)\varrho_1 + X_{2,ijt}\varrho_2 + \sum_{r=1}^{4}[X_{2,ijt,r} - X_{2,ijt,r}^D]^2\varrho_{3,r} + \upsilon_{ijt} \tag{12}$$

Next, we implement several models and compare them with the same coal-purchase data set. In this case, X_{ijt} contains transaction-specific characteristics, like the difference in heat content, ash content, sulfur content, and sulfur content interacting with the scrubber installation. The potentially endogenous variable is the ln price per ton. The excluded instruments we refer to are the physical distance calculated by Haversine formula from the power plant to the mine county and the proportion of coal generating capacity inside a plant.

In Table 10, Model (1) is the Probit without considering the endogeneity of p_{ijt}; Model (2) is the linear control-function procedure[27]; Model (3) is our estimator, where the first stage machine learning is a tensor-product-based degree-2 Chebyshev polynomials, and $\lambda_R = 0.000001$, λ_L is obtained from GIC as in the simulation; Model (4), instead, has the first stage as the DNN with three layers and $(30, 10, 2)$ nodes in each layer. L = 10 for both Model (3) and (4), and both of them are truncated to eliminate the extremely large correction terms.

Table 10 shows the estimated results from different methods. Price coefficient would be positive without explicitly accounting for the endogeneity, while all other methods give a negative price coefficient. Model (2) and Model (3) gives similar estimated parameters for $\ln(p_{ijt})$, which indicates that there may not be much nonlinearity in the current model. Model (4) gives a larger estimates on the price coefficient[28]. As expected, due to the introduction of machine learning first-stage, the standard error increases.

25. When a full supply-side model is available, those missing prices can be calculated using the equilibrium of a structural model (Song, 2022). However, we do not specify a supply model in this illustration, so the missing price must be imputed separately and outside the system (Jha, 2022).

26. where $X_{2,ijt}$ contains heat, sulfur, ash content, and also the Haversine distance.

27. The procedure is the same as https://www.stata.com/manuals/rivprobit.pdf

28. Note that the variables in Table 10 are all the difference of product j with the outside option, like $\ln(p_{ijt}) - \ln(p_{i0t})$. So even a large coefficient may not result in a significant effect.

7 Conclusions

In this study, we propose a new machine-learning-based semi-parametric estimator. We demonstrate the applicability to the binary discrete choice model with endogenous variables and Logit model with zero market shares. We show that in those cases, the correction term for Neyman orthogoanlity can be calculated analytically with the estimated probabilities from the first stage. By construction, first-stage error up to $o(N^{-1/4})$ would not affect the asymptotic distribution of the second-stage parameter estimates. And it can be easily implemented in standard statistical software (Stata, Python, R, etc.). Simulations under a collection of settings demonstrate that our estimator serve as a flexible alternative to the control function, epscially when there is unknown nonlinearity in determining the endogenous variables. Furthermore, we provide an illustrative application using the individual coal transaction data from Year 2011 to 2015, where the decision of power plants is simplified as a binary choice inside the spot market, in light of the currently-hold long-term contracts.

Further research directions may include the possibility of finding this type analytical correction term for a broader class of models; or the extension of this type of method into more structural model like the original multiple discrete continuous model.

		Not Consider Endogeneity	Linear Control Function	Raw L = 1	Raw L = 5	Raw DNN L = 1	Raw DNN L = 5	Debiased L = 1	Debiased L = 5	Debiased DNN L = 1	Debiased DNN L = 5
β_0	Mean Bias	0.0181	0.0088	0.0065	0.0062	-0.0270	-0.0257	-0.0020	0.0062	-0.0136	0.0090
	Std	0.4833	0.1600	0.1500	0.1502	0.1926	0.2255	0.1627	0.1642	0.1630	0.1582
	Median Bias	0.0689	0.0000	0.0027	0.0024	-0.0321	-0.0270	-0.0056	0.0035	-0.0184	0.0081
	Coverage	0.9570	0.9540	0.9460	0.9500	0.9480	0.9510	0.9500	0.9470	0.9480	0.9470
β_1	Mean Bias	-0.6266	-0.0020	-0.0034	-0.0059	0.1649	0.2663	-0.0521	-0.0278	-0.0500	0.0420
	Std	0.2845	0.2274	0.2244	0.2234	0.2559	0.2609	0.2776	0.2902	0.2790	0.2956
	Median Bias	-0.6167	-0.0093	-0.0057	-0.0095	0.1764	0.2657	-0.0697	-0.0471	-0.0543	0.0267
	Coverage	0.4300	0.9450	0.9440	0.9490	0.9040	0.8370	0.9400	0.9420	0.9460	0.9440
β_2	Mean Bias	0.1483	0.0010	0.0013	0.0018	-0.0140	-0.0279	0.0126	0.0073	0.0154	-0.0068
	Std	0.1299	0.0718	0.0706	0.0705	0.0812	0.0864	0.0855	0.0888	0.0838	0.0859
	Median Bias	0.1499	0.0016	0.0019	0.0035	-0.0150	-0.0265	0.0158	0.0120	0.0179	-0.0057
	Coverage	0.8010	0.9490	0.9480	0.9440	0.9420	0.9380	0.9390	0.9500	0.9390	0.9420
β_w	Mean Bias	0.3119	0.0021	0.0032	0.0049	-0.0160	-0.0270	0.0307	0.0157	0.0348	-0.0252
	Std	0.1059	0.0993	0.0988	0.0982	0.1119	0.1105	0.1249	0.1313	0.1271	0.1365
	Median Bias	0.3121	0.0044	0.0063	0.0104	-0.0197	-0.0288	0.0336	0.0223	0.0291	-0.0274
	Coverage	0.1860	0.9480	0.9450	0.9480	0.9540	0.9380	0.9440	0.9480	0.9420	0.9470

Table 1: w_i is determined by a simple linear regression, n = 2280, true $\beta_0 = (-0.2, -1, 0.2, 0.7)'$, true $\tau_0 = (3, 2, -0.5, 0.5, 0.5)'$ for $(1, x_i, z_i)$.

		Not Consider Endogeneity	Linear Control Function	Raw Lasso L = 1	Raw Lasso L = 5	Raw Lasso L = 10	Debiased Lasso L = 1	Debiased Lasso L = 5	Debiased Lasso L = 10
β_0	Mean Bias	-0.0202	0.0015	-0.1040	-0.1160	-0.1096	-0.0534	-0.0659	-0.0618
	Std	0.4732	0.1607	0.2049	0.2260	0.2136	0.1451	0.1433	0.1441
	Median Bias	0.0198	-0.0018	-0.1043	-0.1143	-0.1088	-0.0574	-0.0689	-0.0659
	Coverage	0.9660	0.9490	0.9280	0.9290	0.9290	0.9300	0.9250	0.9300
β_1	Mean Bias	-0.5455	-0.0591	-0.1972	-0.2002	-0.2002	-0.0682	-0.0833	-0.0868
	Std	0.2715	0.2367	0.0783	0.0775	0.0771	0.1956	0.1870	0.1917
	Median Bias	-0.5285	-0.0666	-0.1977	-0.1997	-0.2014	-0.0819	-0.0947	-0.0956
	Coverage	0.5090	0.9480	0.2500	0.2590	0.2460	0.9390	0.9280	0.9260
β_2	Mean Bias	0.1371	0.0180	0.1886	0.1933	0.1913	0.0473	0.0538	0.0522
	Std	0.1292	0.0844	0.0435	0.0431	0.0428	0.0754	0.0733	0.0747
	Median Bias	0.1444	0.0157	0.1914	0.1952	0.1936	0.0491	0.0539	0.0532
	Coverage	0.8290	0.9360	0.0150	0.0110	0.0100	0.9040	0.8820	0.8910
β_w	Mean Bias	0.2754	0.0271	-0.0511	-0.0894	-0.0677	-0.0149	-0.0296	-0.0195
	Std	0.1106	0.1068	0.0609	0.0626	0.0612	0.0992	0.0954	0.0977
	Median Bias	0.2758	0.0266	-0.0510	-0.0888	-0.0660	-0.0105	-0.0271	-0.0167
	Coverage	0.2800	0.9440	0.8730	0.6870	0.7920	0.9520	0.9470	0.9510

Table 2: w_i is determined by a simple linear regression, but we observe 100 more irrelevant candidates for instruments, n = 2280, true $\beta = (0.8, 0.2, -0.2, 0.7)'$, true $\tau = (3, 2, -0.5, 0.5, 0.5, 0, \cdots, 0)'$ for $(1, x_i, z_i)$.

		Not Consider Endogeneity	Control Function	Debiased L = 1	Raw L = 1	Debiased DNN L = 1	Raw DNN L = 1	Debiased DNN L = 5	Raw DNN L = 5	Debiased DNN L = 228	Raw DNN L = 228
β_0	Mean Bias	0.0057	-0.0072	0.0045	0.0028	0.0031	-0.0012	-0.0372	0.0025	-0.0196	-0.0015
	Std	0.4557	0.1546	0.1566	0.1437	0.1563	0.1450	0.1596	0.1464	0.1542	0.1438
	Median Bias	0.0389	-0.0090	0.0023	0.0080	0.0023	0.0023	-0.0389	0.0023	-0.0214	0.0023
	Coverage	0.9650	0.9440	0.9490	0.9450	0.9480	0.9460	0.9430	0.9410	0.9470	0.9440
β_1	Mean Bias	-0.2583	-0.0083	-0.0049	-0.0016	-0.0056	0.0068	-0.0411	0.0116	-0.0240	0.0054
	Std	0.2181	0.1709	0.1877	0.1693	0.1863	0.1728	0.2068	0.1660	0.1872	0.1687
	Median Bias	-0.2705	-0.0114	-0.0047	-0.0023	-0.0063	-0.0063	-0.0442	0.0069	-0.0235	0.0033
	Coverage	0.7760	0.9430	0.9440	0.9470	0.9440	0.9510	0.9590	0.9510	0.9420	0.9440
β_2	Mean Bias	0.0640	0.0076	0.0019	0.0025	0.0022	0.0035	0.0197	0.0021	0.0113	0.0038
	Std	0.1158	0.0640	0.0738	0.0634	0.0730	0.0653	0.0814	0.0635	0.0745	0.0637
	Median Bias	0.0622	0.0091	0.0018	0.0042	0.0014	0.0014	0.0224	0.0039	0.0115	0.0056
	Coverage	0.9290	0.9420	0.9390	0.9440	0.9350	0.9400	0.9450	0.9370	0.9370	0.9420
β_w	Mean Bias	0.2373	-0.0307	0.0004	-0.0057	-0.0013	-0.0113	-0.0655	-0.0020	-0.0356	-0.0105
	Std	0.1126	0.1378	0.1531	0.1367	0.1523	0.1423	0.1750	0.1370	0.1542	0.1394
	Median Bias	0.2412	-0.0268	0.0005	-0.0076	-0.0018	-0.0018	-0.0629	-0.0105	-0.0339	-0.0149
	Coverage	0.4410	0.9440	0.9520	0.9500	0.9510	0.9440	0.9490	0.9480	0.9470	0.9510

Table 3: Linear, but there is slightly misspecification in linear regression model. n = 2280. True $\beta = (0.5, 0.4, -0.2, 0.7)'$, and true $\tau = [1.5, 1, -0.25, 0.25, 0.25, 0.05, \cdots, 0.05, 0, \cdots, 0]'$ for $(1, x_i, z_{i,1}, z_{i,2}, z_{i,3})$

		Not Consider Endogeneity	Linear Control Function	Raw EN L = 1	Raw EN L = 5	Raw EN L = 10	Debiased EN L = 1	Debiased EN L = 5	Debiased EN L = 10
β_0	Mean Bias	-0.0103	0.0081	0.0189	0.0226	0.0215	-0.0019	0.0249	0.0213
	Std	0.4661	0.1832	0.2156	0.2176	0.2132	0.1904	0.2234	0.2192
	Median Bias	0.0347	0.0109	0.0179	0.0303	0.0270	0.0025	0.0245	0.0178
	Coverage	0.9660	0.9420	0.9580	0.9460	0.9470	0.9590	0.9600	0.9530
β_1	Mean Bias	-0.1161	-0.0679	-0.1551	-0.1616	-0.1568	-0.0560	-0.0580	-0.0584
	Std	0.1979	0.1108	0.1447	0.1315	0.1256	0.1303	0.1665	0.1599
	Median Bias	-0.1041	-0.0690	-0.1556	-0.1696	-0.1612	-0.0574	-0.0611	-0.0634
	Coverage	0.8900	0.9050	0.8780	0.7720	0.7580	0.9610	0.9570	0.9440
β_2	Mean Bias	0.0049	-0.0010	0.1039	0.1160	0.1086	0.0193	0.0191	0.0197
	Std	0.1161	0.0624	0.0717	0.0749	0.0738	0.0727	0.0827	0.0827
	Median Bias	0.0014	-0.0031	0.1038	0.1206	0.1128	0.0208	0.0256	0.0232
	Coverage	0.9450	0.9480	0.6750	0.6340	0.6580	0.9570	0.9610	0.9510
β_w	Mean Bias	0.2128	0.1447	-0.0120	-0.0297	-0.0163	0.0370	0.0105	0.0190
	Std	0.0849	0.0834	0.1278	0.1394	0.1350	0.1370	0.1518	0.1496
	Median Bias	0.2090	0.1412	-0.0101	-0.0245	-0.0119	0.0348	0.0065	0.0166
	Coverage	0.2990	0.5900	0.9580	0.9410	0.9540	0.9640	0.9540	0.9490

Table 4: Nonlinear Generalization of w_i. With GIC choice of λ. n = 2280. Polynomial degree = 2, and number of parameters in the machine learning first stage = 216. True $\beta = (-0.3, -0.4, -0.2, 0.7)'$.

		Not Consider Endogeneity	Linear Control Function	Raw EN L = 1	Raw EN L = 5	Raw EN L = 10	Debiased EN L = 1	Debiased EN L = 5	Debiased EN L = 10
β_0	Mean Bias	-0.0221	-0.0133	0.0506	0.0501	0.0563	0.0144	0.0313	0.0404
	Std	0.5192	0.3583	0.3059	0.3970	0.3235	0.0190	0.3625	0.2897
	Median Bias	-0.0203	-0.0078	0.0458	0.0638	0.0595	-0.2856	0.0450	0.0479
	Coverage	0.9480	0.9450	0.9440	0.9710	0.9480	0.9510	0.9760	0.9470
β_1	Mean Bias	-0.1690	-0.0753	-0.2436	-0.2659	-0.2517	-0.0728	-0.0785	-0.0740
	Std	0.2713	0.2323	0.1438	0.1736	0.1561	-0.0849	0.2116	0.2017
	Median Bias	-0.1918	-0.0777	-0.2556	-0.2809	-0.2662	0.3272	-0.0854	-0.0830
	Coverage	0.9260	0.9430	0.5680	0.6930	0.6070	0.9420	0.9660	0.9580
β_2	Mean Bias	0.0051	-0.0079	0.1710	0.1788	0.1749	0.0441	0.0465	0.0451
	Std	0.1660	0.1235	0.0674	0.0637	0.0688	0.0497	0.0971	0.1008
	Median Bias	0.0051	-0.0079	0.1815	0.1860	0.1835	-0.1559	0.0498	0.0492
	Coverage	0.9560	0.9530	0.1980	0.1420	0.1980	0.9190	0.9200	0.9240
β_w	Mean Bias	0.2390	0.1628	-0.1948	-0.2398	-0.2136	-0.0963	-0.1352	-0.1191
	Std	0.1633	0.1561	0.1453	0.1796	0.1652	-0.0911	0.1870	0.1728
	Median Bias	0.2365	0.1616	-0.1917	-0.2467	-0.2149	0.6037	-0.1376	-0.1133
	Coverage	0.6950	0.8290	0.7220	0.7490	0.7530	0.9030	0.9180	0.9050

Table 5: Nonlinear Generalization of w_i. With high-dimensional parameter space in the machine learning first stage, compared with the sample size n = 570.

		True Share	Drop 0	MLE No endogeneity	MLE with control function	Linear MLE first stage	Debiased Res-Logit Layer 1[1]	Raw Res-Logit Layer 1
β_0	Mean Bias	-0.0014	-0.1477	-0.8129	-0.0080	-0.0078	-0.0013	-0.0014
	Std	0.0357	0.0558	0.0322	0.0306	0.0375	0.0540	0.0378
	Median Bias	-0.0019	-0.1492	-0.8130	-0.0079	-0.0085	-0.0011	-0.0013
	Coverage	0.9460	0.2380	0.0000	0.9360	0.9470	0.9488	0.9488
β_1	Mean Bias	-0.0007	-0.1065	-0.8847	-0.0043	-0.0042	-0.0008	0.0005
	Std	0.0353	0.0519	0.0235	0.0268	0.0370	0.0497	0.0373
	Median Bias	-0.0026	-0.1097	-0.8855	-0.0037	-0.0044	-0.0022	-0.0012
	Coverage	0.9580	0.4420	0.0000	0.9510	0.9580	0.9488	0.9549
β_2	Mean Bias	-0.0004	0.0270	-0.0448	0.0010	0.0008	-0.0005	0.0001
	Std	0.0195	0.0211	0.0214	0.0192	0.0198	0.0243	0.0199
	Median Bias	0.0005	0.0278	-0.0449	0.0010	0.0009	-0.0008	-0.0001
	Coverage	0.9520	0.7540	0.4380	0.9460	0.9460	0.9448	0.9478
β_p	Mean Bias	0.0006	0.0646	0.4046	0.0030	0.0031	0.0011	0.0006
	Std	0.0148	0.0230	0.0087	0.0101	0.0156	0.0218	0.0157
	Median Bias	0.0011	0.0659	0.4046	0.0036	0.0036	0.0013	0.0011
	Coverage	0.9470	0.1970	0.0000	0.9380	0.9510	0.9559	0.9478

[1] Out of 1000 repetitions, there are three simulated samples that do not have convergence for the Res-Logit first stage. Those are excluded from the outcomes in the table.

Table 6: Zero market share with linear pricing. True $\beta = (3, 2, -0.6, -1.2)'$, $\tau = (\tau_x', \tau_z')' = (1, 0.4, -0.3, 0.5, 1.2, 0.4)$.

		True Share	Drop 0	MLE No endogeneity	MLE with control function	Debiased MLE Lasso L = 1	Raw MLE Lasso L = 1	Debiased MLE Lasso L = 10	Raw MLE Lasso L = 10
β_0	Mean Bias	-0.0789	-0.5240	-0.7214	-0.0560	-0.0913	-0.1071	-0.0896	-0.1023
	Std	0.0324	0.0571	0.0306	0.0310	0.0533	0.0393	0.0534	0.0393
	Median Bias	-0.0794	-0.5180	-0.7224	-0.0558	-0.0868	-0.1063	-0.0845	-0.1015
	Coverage	0.3133	0.0000	0.0000	0.5767	0.6267	0.2067	0.6333	0.2400
β_1	Mean Bias	-0.0922	-0.4352	-0.8013	-0.0241	-0.0959	-0.1108	-0.0944	-0.1034
	Std	0.0347	0.0534	0.0236	0.0264	0.0509	0.0396	0.0511	0.0397
	Median Bias	-0.0932	-0.4345	-0.8015	-0.0231	-0.0948	-0.1085	-0.0936	-0.1005
	Coverage	0.2733	0.0000	0.0000	0.8500	0.5300	0.1933	0.5400	0.2933
β_2	Mean Bias	-0.0228	0.0534	-0.0730	0.0227	-0.0159	-0.0180	-0.0159	-0.0153
	Std	0.0173	0.0181	0.0216	0.0187	0.0231	0.0195	0.0232	0.0196
	Median Bias	-0.0234	0.0520	-0.0734	0.0227	-0.0167	-0.0179	-0.0172	-0.0158
	Coverage	0.7500	0.1567	0.0833	0.7500	0.9200	0.8433	0.9167	0.8767
β_p	Mean Bias	0.0439	0.1855	0.3722	0.0155	0.0490	0.0561	0.0485	0.0526
	Std	0.0156	0.0248	0.0086	0.0108	0.0234	0.0182	0.0235	0.0182
	Median Bias	0.0446	0.1851	0.3720	0.0145	0.0499	0.0550	0.0491	0.0515
	Coverage	0.2233	0.0000	0.0000	0.6933	0.4300	0.1567	0.4367	0.1833

Table 7: Zero market share with linear pricing, and with a bunch of irrelevant instruments. True $\beta = (3, 2, -0.6, -1.2)'$, $\theta = (\tau_x', \tau_z')' = (1, 0.4, -0.3, 0.5, 1.2, 0.4, 0, \cdots, 0)'$.

		True Share	Drop 0	MLE No endogeneity	MLE control function	Debiased Res-Logit Layer 1 L = 1	Raw Res-Logit Layer 1 L = 1
β_1	Mean Bias	-0.0110	-0.2283	-1.0394	-0.1627	-0.4302	-1.0975
	Std	0.1714	0.2214	0.0088	0.1111	0.1515	0.1007
	Median Bias	-0.0193	-0.2554	-1.0400	-0.1768	-0.4429	-1.1048
	Coverage	0.9467	0.8100	0.0000	0.6533	0.1967	0.0000
β_p	Mean Bias	0.0009	0.2569	1.2073	0.3906	0.5645	1.2334
	Std	0.1624	0.2201	0.0084	0.1044	0.1427	0.0954
	Median Bias	0.0094	0.2846	1.2075	0.4045	0.5778	1.2400
	Coverage	0.9467	0.7700	0.0000	0.0433	0.0300	0.0000
		Debiased Res-Logit Layer 5 L = 1	Raw Res-Logit Layer 5 L = 1	Debiased Res-Logit Layer 5 L = 10	Raw Res-Logit Layer 5 L = 10	Debiased Poly L = 1	Raw Poly L = 1
β_1	Mean Bias	-0.1498	-0.1923	-0.1648	-0.1712	-0.1776	-1.4436
	Std	0.1806	0.1456	0.1975	0.1469	0.3291	0.1359
	Median Bias	-0.1674	-0.1933	-0.1772	-0.1789	-0.2064	-1.4253
	Coverage	0.8733	0.7233	0.8896	0.7726	0.9367	0.0000
β_p	Mean Bias	0.2461	0.2911	0.2644	0.2688	0.2946	1.4768
	Std	0.1716	0.1389	0.1878	0.1401	0.3122	0.1300
	Median Bias	0.2608	0.2940	0.2759	0.2737	0.3153	1.4598
	Coverage	0.6900	0.4033	0.6890	0.4983	0.9000	0.0000

Table 8: Market share, pricing by Bertrand Competition. True $\beta = (1, -2)'$, and true $\tau = (\tau_x', \tau_z') = (1, 1, -0.5)'$.

	Tier 1 SPRB	Tier 2 SPRB	Other PRB	Other Western	Appalachia	Mid-continent
MMBtu/ton	17.6613	16.8586	16.9809	24.5660	18.5145	21.9418
	(0.1834)	(0.1767)	(0.6729)	(1.2782)	(3.6250)	(3.2108)
Sulfur % weight	0.2504	0.3058	0.3365	1.9655	0.6898	2.7152
	(0.0499)	(0.0208)	(0.0939)	(1.0480)	(0.2092)	(0.6864)
Ash % weight	4.9914	5.4268	5.0946	11.0948	12.4929	9.9531
	(0.4347)	(0.3428)	(1.1784)	(3.7780)	(5.2866)	(3.7480)
Cents MMBtu (Real)	34.0770	28.9759	28.0718	74.7183	37.4142	53.2300
	(7.2455)	(7.6250)	(8.2127)	(19.8008)	(22.3650)	(13.3230)
Tons(000)	138.2633	141.4755	113.2860	82.5121	187.2692	107.8122
	(147.3759)	(139.1950)	(130.9553)	(108.1117)	(195.8328)	(128.1342)
Proportion of Scrubber	0.4931	0.4796	0.5834	0.6453	0.7861	0.7404
	(0.5277)	(0.5005)	(0.5005)	(0.4888)	(0.5853)	(0.4233)
# of deliveries	6261	1713	3102	6117	2434	4814
Long-term	0.81	0.83	0.81	0.67	0.82	0.74
Spot	0.19	0.17	0.19	0.33	0.18	0.26

Standard errors in parentheses.

Table 9: Summary statistics for the coals originated from different regions.

	Model (1)	Model (2)	Model (3)	Model (4)
Constant	0.6776***	-0.1983***	0.2092	0.0348
	(0.0982)	(0.0105)	(0.1583)	(0.1857)
Heat	0.0925***	0.5969***	0.6913***	0.8420***
	(0.0091)	(0.0225)	(0.1247)	(0.1400)
Sulfur	-0.3489***	-0.2931***	-0.4633***	-0.4959***
	(0.0229)	(0.0225)	(0.0527)	(0.0610)
Ash	0.0029	0.0757***	0.0819***	0.1236***
	(0.0044)	(0.0039)	(0.0181)	(0.0209)
$\ln(p_{ijt})$	0.1177*	-6.2658***	-6.6400***	-8.5312***
	(0.0691)	(0.1621)	(1.4089)	(1.5853)
Sulfur × Scrubber	0.0409**	-0.0510***	-0.0596	-0.0402
	(0.0208)	(0.0162)	(0.0370)	(0.0465)
$\ln(Q_{i0t})$	-0.1143***	-0.0419***	-0.1133***	-0.1093***
	(0.0086)	(0.0090)	(0.0114)	(0.0133)
N	18149	18149	18141	18141

Standard errors in parentheses.

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 10: Coefficients estimated in the illustrative application.

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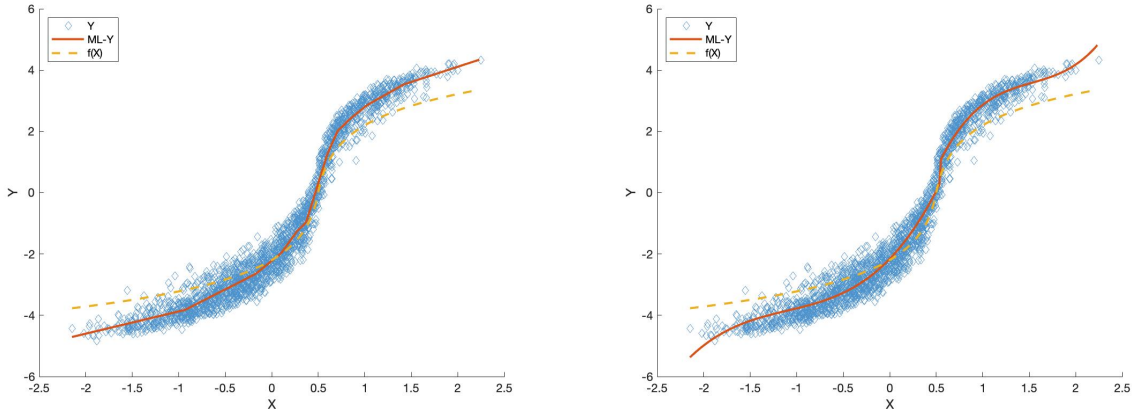
Appendix

Appendix A. A simple example showing the flexible machine learning method

Here we give two simple examples to demonstrate how flexible machine learning methods are in capturing the underlying data patterns. The first example is inherited from Bakhitov (2022). The DGP follows,

$$\xi_i = 2\,\text{Uniform}(0,1) - 1, \quad Z_i = N(0,1), \quad X_i = 0.5Z_i + \xi_i + N(0,0.1)$$
$$Y_i = \log(|16X_i - 8| + 1)\, sign(X - 0.5) + \xi_i + N(0,0.1)$$



(a) Estimated by (shallow) Neural Nets

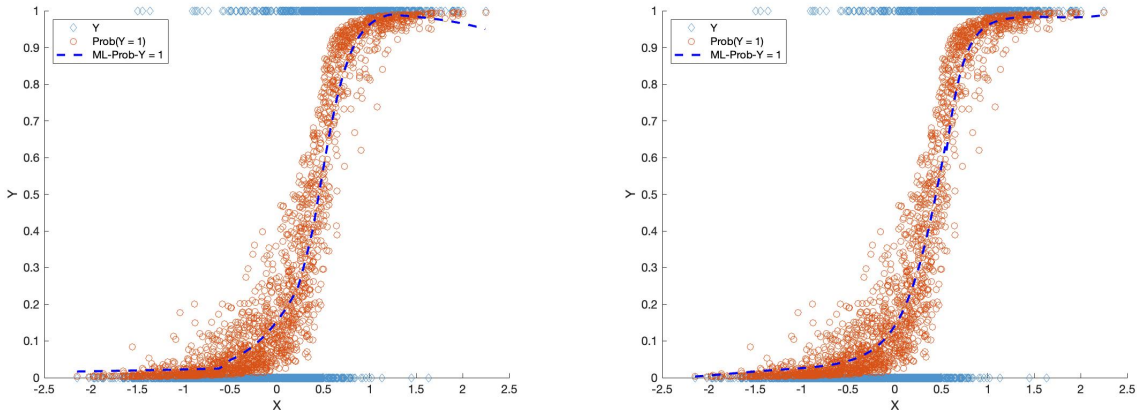
(b) Estimated by 3-degree B-spline

Figure 1: Flexible approximation of function value from simple machine learning methods.

From Figures (1) above, we can tell that the machine learning method would flexibly capture the influence of the unobserved part ξ_i. Machine learning methods may not be able to retrieve the underlying structural pattern, but they can provide a good approximation, as required for our first stage. Another example builds on the propensity score estimation with the DGP,

$$\xi_i = 2\,\text{Uniform}(0,1) - 1, \quad Z_i = N(0,1), \quad X_i = 0.5Z_i + \xi_i + N(0,0.1)$$
$$Y_i^* = \ln(|16X_i - 8| + 1)\, sign(X - 0.5) + \xi_i + N(0,1), \quad Y_i = \mathbf{1}\{Y_i^* > 0\}$$

In Figure 2, the red dots represent the probability of $Y_i = 1$, after accounting for the unobserved structural ξ_i. The pattern can be approximated well by some flexible machine learning methods, as Figure 2 suggests.



(a) Estimated by (shallow) Neural Nets

(b) Estimated by 3-degree B-spline

Figure 2: Flexible approximation of choice probability from simple machine learning methods.

Appendix B. Proofs of Results

Proof of Theorem 2

Proof. Here we verify that the assumptions in Chernozhukov et al. (2022a) are satisfied in our setting. Appendix F in Chernozhukov et al. (2022a) offers the conditions sufficient for the consistency of β. Conditions for the consistency and asymptotic normality are overlapping, so here we try to verify them together. First, Our Assumptions 3 are exactly the same as Theorem A3 i), ii), and iii) in Chernozhukov et al. (2022a), Appendix F, while Theorem A3 v) is a sufficient condition to our Assumption 3(*iv*). Assumptions 4(*v*) leads to $E[||\phi(W, \gamma_0, \alpha_0, \theta_0)||] < \infty$.

Theorem 1 gives the orthogonal moment condition based on ψ. Next, we want to show that our assumptions are sufficient to guarantee assumption 1 in Chernozhukov et al. (2022a). For simplicity, we write $\hat{\alpha}_\ell(W_i) = \hat{\alpha}_\ell(W_i, \hat{\pi}_\ell)$, $\hat{\rho}_\ell(W_i) = \hat{\rho}_\ell(W_i, \hat{\pi}_\ell)$, and $\hat{\gamma}_\ell(W_i) = \hat{\gamma}_\ell(W_i, \hat{\pi}_\ell)$ for individual data point. Similar notations apply to α_0, γ_0 and ρ_0.

By Assumptions 4(*i*), and γ enters into the original moment condition m linearly,

$$\int_W ||m(W_i, \hat{\gamma}_\ell, \beta) - m(W_i, \gamma_0, \beta))||^2 F_0(dW) \leq C||\hat{\rho}_\ell - \rho_0||^2 \xrightarrow{p} 0$$

The last inequality follows from the definition of ρ such that $||\hat{\rho}_\ell - \rho_0|| = ||y_\ell - \hat{\pi}_\ell - y_\ell + \pi_0|| = ||\pi_0 - \hat{\pi}_\ell||$. Hence, the norm convergence defined in π is the same as in ρ. By Jensen's inequality, $0 \leq (\int_W ||m(W, \hat{\gamma}_\ell, \beta) - m(W, \gamma_0, \beta)||)^2 \leq \int_W ||m(W, \hat{\gamma}_\ell, \beta) - m(W, \gamma_0, \beta)||^2 \xrightarrow{p} 0$ – the Assumption iv) in Appendix F, Chernozhukov et al. (2022a) is satisfied.

Then by Assumptions 4(*ii*), each element inside $\alpha_0(W)$ is bounded.

$$\begin{aligned}\int_W ||\phi(W_i, \hat{\gamma}_\ell, \alpha_0) - \phi(W_i, \gamma_0, \alpha_0)||^2 F_0(dW) &= \int_W ||\alpha_0(W)||^2 [\hat{\rho}_\ell(W_i) - \rho_0(W_i)]^2 F_0(dW) \\ \leq C \int_W [\hat{\rho}_\ell(W_i) - \hat{\rho}_\ell(W_i)]^2 F_0(dW) &= C||\hat{\rho}_\ell - \rho_0||^2 \xrightarrow{p} 0\end{aligned}$$

Also by Assumption 4(*ii*), 4(*iii*), the law of iterated expectation gives,

$$\begin{aligned}&\int_W ||\phi(W_i, \gamma_0, \hat{\alpha}_\ell) - \phi(W_i, \gamma_0, \alpha_0)||^2 F_0(dW) = \int_W ||\hat{\alpha}_\ell(W_i) - \alpha_0(W_i)||^2 \rho_0(W_i)^2 F_0(dW) \\ &= \int_W ||\hat{\alpha}_\ell(W_i) - \alpha_0(W_i)||^2 E[\rho(W_i)^2|W] F_0(dW) \leq C \int_W ||\hat{\alpha}_\ell(W_i) - \alpha_0(W_i)||^2 F_0(dW) \xrightarrow{p} 0\end{aligned}$$

Therefore, the Assumption 1 (i), (ii), and (iii) of Chernozhukov et al. (2022a) are satisfied. As for Assumption 2 in Chernozhukov et al. (2022a) and Assumption vi) in Appendix F of Chernozhukov et al. (2022a) for consistency, the interaction term $\hat{\Delta}_\ell(W) = \phi(W_i, \hat{\gamma}_\ell, \hat{\alpha}_\ell) - \phi(W_i, \gamma_0, \hat{\alpha}_\ell) - \phi(W_i, \hat{\gamma}_\ell, \alpha_0) + \phi(W_i, \gamma_0, \alpha_0) = [\hat{\alpha}_\ell(W_i) - \alpha_0(W_i)][\hat{\rho}_\ell(W_i) - \rho_0(W_i)]$. Note that because we have multiple moment conditions, $\hat{\Delta}_\ell(W_i)$ is a vector. For the j-th moment condition, by Assumptions 4(*iii*), 4(*iv*) and Cauchy-Schwarz inequality, there are,

$$\begin{aligned}\int_W \hat{\Delta}_{j,\ell}(W_i) F_0(dW) &= \int_W [\hat{\alpha}_{j,\ell}(W_i) - \alpha_{j,0}(W_i)] [\hat{\rho}_\ell(W_i) - \rho_0(W_i)] F_0(dW) \\ &\leq ||\hat{\alpha}_{j,\ell} - \alpha_{j,0}|| \times ||\hat{\rho}_\ell - \rho_0|| \leq C||\hat{\pi}_\ell - \pi_0|| \times ||\hat{\pi}_\ell - \pi_0|| = o_p(1/\sqrt{N})\end{aligned}$$

$$\begin{aligned}\int_W ||\hat{\Delta}_\ell(W_i)||^2 F_0(dW) &= \int_W ||\hat{\alpha}_\ell(W_i) - \alpha_0(W_i)||^2 [\hat{\rho}_\ell(W_i) - \rho_0(W_i)]^2 F_0(dW) \\ &\leq C \int_W [\hat{\rho}_\ell(W_i) - \rho_0(W_i)]^2 F_0(dW) = C||\hat{\rho}_\ell - \rho_0||^2 \xrightarrow{p} 0\end{aligned}$$

Moreover, Assumption 3 and 4 in Chernozhukov et al. (2022a) is exactly the same with Assumption 4(*iv*), 4(*v*) and 4(*vi*). We do not explicitly account for Assumption 5 of Chernozhukov et al. (2022a)

since our moment condition $m(W, \gamma, \beta)$ is linear in β. We have $m(W_i, \gamma, \beta)$ is differentiable with respect to $\beta \in \mathcal{B}$, and $\partial m(W, \gamma, \beta)/\partial \beta_j - \partial m(W, \gamma, \beta_0)/\partial \beta_j = 0$ by the linear moment construction, so that ii) and iii) of Assumption 5 in Chernozhukov et al. (2022a) are satisfied.

With this, we have all the assumptions required in Chernozhukov et al. (2022a) for the consistency and root-N asymptotic normality of $\hat{\beta}$, which leads directly to the main conclusion proposed in Theorem 2. □

Appendix C. Numerical Implementation

C.1 ResLogit Algorithm from Wong and Farooq (2021)

Wong and Farooq (2021) proposed a partial linear multinomial Logit regression with the nonlinear part approximated by an underlying multi-layer neural network. Following the notations in Wong and Farooq (2021), the probability of choosing a good j in market t for a group of homogeneous consumers can be calculated with a linear part, $V_{jt} = x'_{jt}\beta_x + w'_{jt}\beta_w = W'_{jt}\beta$ and a nonlinear part g_{jt}, such that, $Pr_{jt} = \exp(V_{jt} + g_{jt})/\sum_{\ell \in J} \exp(V_{\ell t} + g_{\ell t})$, where $\boldsymbol{g}_t = (g_{0t}, g_{1t}, \cdots, g_{Jt})'$ is defined by an M-layer neural nets,

$$\boldsymbol{g}_t = -\sum_{m=1}^{M} \ln(1 + \exp(\theta^{(m)} h_t^{m-1})) \tag{13}$$

θ is a large matrix with $M \times J$ rows and J columns. The first J rows denote as $\theta^{(1)}$, and other $\theta^{(m)}$ defined following the similar way. For each layer, we have $J \times J$ parameters to estimate. The function h is defined recursively by,

$$\begin{aligned} h_t^{(0)} &= \boldsymbol{V}_t \\ h_t^{m} &= h_t^{m} - \sum_{m'=1}^{m} \ln(1 + \exp(\theta^{(m')} h_t^{m'-1})) \end{aligned}$$

For instance, if we want to have M = 3, then consequently, equation (13) becomes $\boldsymbol{g}_t = -\big[\ln(1 + \exp(\theta^{(1)} h^{(0)})) + \ln(1 + \exp(\theta^{(2)} h^{(1)})) + \ln(1 + \exp(\theta^{(3)} h^{(2)}))\big]$, with h

$$\begin{aligned} h^{(0)} &= \boldsymbol{V}_t \\ h^{(1)} &= h^{(0)} - \ln(1 + \exp(\theta^{(1)} h^{(0)})) \\ h^{(2)} &= h^{(1)} - \big[\ln(1 + \exp(\theta^{(2)} h^{(1)})) + \ln(1 + \exp(\theta^{(1)} h^{(0)}))\big] \end{aligned}$$

Wong and Farooq (2021) studies this basically for approximating a nested Logit structure when the underlying nesting structure is unknown. Although a large number of parameters, which grows with both the number of layers and the number of goods J inside a market, are needed to construct $\boldsymbol{g}_t$, unlike the usual exponentially-expanding sieve-based method, it provides a flexible approximation with only polynomially growth. It even implicitly allows different functions for different goods. In this case, even though the asymptotic properties still require further research, we choose the ResLogit as one of the alternatives because it provides a manageable estimation of the interaction behaviors across different goods inside a market, which is principal in industrial organization.

Below we provide the analytical derivatives for Pr_{jt} regarding both β and θ.

$$\frac{\partial Pr_{jt}}{\partial \beta} = \frac{\exp(V_{jt}+g_{jt})\{[\frac{\partial V_{jt}}{\partial \beta}+\frac{\partial g_{jt}}{\partial \beta}]\times[\sum_{\ell=1}^{J}\exp(V_{jt}+g_{jt})]-\sum_{\ell=1}^{J}\exp(V_{\ell t}+g_{\ell t})\times[\frac{\partial V_{\ell t}}{\partial \beta}+\frac{\partial g_{\ell t}}{\partial \beta}]\}}{\left(\sum_{\ell=1}^{J}\exp(V_{jt}+g_{jt})\right)^2}$$

$$\frac{\partial g_{jt}}{\partial \beta} = -\sum_{m=1}^{M}\frac{1}{1+\exp(\theta^{(m)}h^{(m-1)})}\times\exp(\theta^{(m)}h^{(m-1)})\times\theta^{(m)}\times\frac{\partial h^{m-1}}{\partial \beta},\quad \frac{\partial h_{jt}^{(0)}}{\partial \beta}=\frac{\partial V_{jt}}{\partial \beta}=W_{jt}$$

$$\frac{\partial h_{jt}^{(m)}}{\partial \beta} = \frac{\partial h_{jt}^{(m-1)}}{\partial \beta}-\left\{\sum_{m'=0}^{m-1}\frac{1}{1+\exp(\theta_j^{(m'+1)}h_t^{(m')})}\times\exp(\theta_j^{(m'+1)}h_t^{(m')})\times\theta_j^{(m'+1)}\times\frac{\partial h_t^{(m')}}{\partial \beta}\right\}$$

$$\frac{\partial Pr_{jt}}{\partial \theta_j^{(m)}} = \frac{\exp(V_{jt}+g_{jt})\{\frac{\partial g_{jt}}{\partial \theta_j^{(m)}}\times[\sum_{\ell=1}^{J}\exp(V_{jt}+g_{jt})]-\sum_{\ell=1}^{J}\exp(V_{\ell t+g_{\ell t}})\times\frac{\partial g_{\ell t}}{\partial \beta}\}}{\left(\sum_{\ell=1}^{J}\exp(V_{jt}+g_{jt})\right)^2}$$

$\partial g_t/\partial\theta_j^{(m')}$ is a matrix with the element in jth row, j'th column equals to

$$\frac{\partial g_{jt}}{\partial \theta_{jj'}^{(m)}} = -\left\{\frac{\exp(\theta_j^{(m)}h^{(m-1)})\times h_{j'}^{(m-1)}}{1+\exp(\theta_j^{(m)})h^{(m-1)}}+\sum_{m+1}^{M}\frac{\exp(\theta_j^{(m')}h^{(m'-1)})\times\theta_{jj'}^{m'}\times\partial h_{j'}^{(m'-1)}/\partial\theta_{jj'}^{(m)}}{1+\exp(\theta_j^{(m')}h^{(m'-1)})}\right\}$$

$$\frac{\partial h^{(m')}}{\partial \theta_{jj'}^{(m)}} = \begin{cases}[0,\cdots,0]', & \text{if } m'<m\\ \dfrac{\exp(\theta_j^{(m)}h^{(m-1)})\otimes[0,\cdots,h_{j'}^{m-1},\cdots,0]'}{1+\exp(\theta_j^{(m)}h^{(m-1)})}, & \text{if } m'=m\\ \dfrac{\partial h^{(m')}}{\partial \theta_{jj'}^{(m)}}=\dfrac{\partial h^{(m'-1)}}{\partial \theta_{jj'}^{(m)}}-\sum_{\tilde{m}=m}^{m'-1}\left[\dfrac{\exp(\theta_j^{(\tilde{m}+1)}h^{(\tilde{m})})\times[\theta_j^{(\tilde{m}+1)}\otimes\partial h^{(\tilde{m})}/\partial\theta_{jj'}^{(m)}]}{1+\exp(\theta_j^{(\tilde{m}+1)}h^{(\tilde{m})})}\right]-\dfrac{\partial h^{(m)}}{\partial \theta_{jj'}^{(m)}}. & \text{if } m'>m\end{cases}$$

C.2 Other Machine Learning methods

We use B-spline for low-dimensional first-stage approximation. Suppose we have a vector X. We normalize it by $\tilde{X}=(X-\min(X))/(\max(X)+1-\min(X))$. Then the B-spline for X_i is defined

recursively following Dung and Tjahjowidodo (2017), with knots ϱ as $[0,0,0,0.5,0.5,0.5,1,1,1]'$. A d-degree B-spline is given by $S(\tilde{X}_i) = \sum_{i=1}^{m-d-1} N_{i,d}(\tilde{X}_i)a_i$, where $N_{i,d}(\tilde{X}_i)$ is the ith basis function for d-degree B-splines. With knot points $\varrho_0 = \varrho_1 = \cdots \varrho_d \leq \varrho_{d+1} \leq \varrho_{d+2} \leq \cdots \leq \varrho_{m-d-1} < \varrho_{m-p+1} = \cdots = \varrho_m$, basis function are then defined recursively,

$$N_{i,0}(\tilde{X}_i) = \begin{cases} 1, & \text{if } \varrho_i \leq \tilde{X}_i < \varrho_{i+1} \\ 0, & \text{otherwise} \end{cases}, \quad N_{i,j}(\tilde{X}_i) = \frac{\tilde{X}_i - \varrho_i}{\varrho_{i+j} - \varrho_i} N_{i,j-1}(\tilde{X}_i) + \frac{\varrho_{i+j+1} - \tilde{X}_i}{\varrho_{i+j+1} - \varrho_i} N_{i+1,j-1}(\tilde{X}_i).$$

Overall, the number of basis function for $\tilde{X}_i$ equals to $m - d - 1$. So here, for degree = 2, there would be totally 9 - 2 - 1 = 6 basis for a single $\tilde{X}_i$. The entire $\tilde{X}$ comes by taking the tensor product across each element. The total number of parameters would grow exponentially with the dimension of X included.

Appendix D. Other Monte Carlo Simulations

This section contains several other Monte Carlo simulations. For the binary choice model, suppose, instead of a strong instrumental variable that enters linearly into the system of determining w_i, we may have a weak instrument z_{i2},

$$\begin{aligned} w_i =& 0.1 \times z_{i2} \times \xi_i + 0.2\xi_i + 0.04N(0,1) + 2 \\ &+ 2 \times \Phi\left[\frac{z_{i2}^2}{2} - 1 + \xi_i + \sqrt{0.5 + 0.05z_{i2}^4 + \ln((1+z_{i2}^4)/10)} \times 2\right] \end{aligned} \tag{14}$$

Table 11 and Table 12 contain the Monte Carlo results for this weak-instrument setting for sample sizes = 4560 and 28800 respectively. Although the estimation of β_w is contaminated by the near-zero correlation between z_{i2} and w_i, leading to overwhelming variance, the performance of the moments in the debiased method is still reasonable. This demonstrates the robustness of our method. The absolute bias of β_w decreases from 0.5399 in the no-endogeneity case to around 0.05 in our machine learning method. On the other hand, the randomness in estimation increases owing to the high standard error, and the differences among those machine-learning algorithms become indistinguishable.

Another simulation in Table 13 have w_i as a discrete-valued response variable with $w_i = 0.4z_{i1} + \mathbf{1}(0.7 + 0.3x_{i1} + z_{i2} \geq \xi_i)$, where z_{i1} takes either 0 or 1, and the overall w_i can take four values, $\{0, 0.4, 1, 1.4\}$. Single discrete endogenous variables usually cannot be handled with the control

function. Research like Lin and Wooldridge (2017) uses the control function to deal with the case where they both simultaneously have a discrete and a continuous endogenous variable. The system is not invertible to obtain ξ_i because the inequalities lead to a range than a point identification. So there is misspecification for the polynomial expansions based on (x_{i1}, z_{i2})[29]. Table 13 shows the basic simulation results.

Table 13 shows that even when w_i is discrete, not accounting for the possible endogeneity would give very biased estimations. The debiased machine learning resolves the majority of endogeneity, despite the misspecification. On the other hand, one can see the necessity of the debiased process. The estimator without the orthogonal moment generates a much higher bias than the case that ignores the endogeneity.

29. Due to this misspecification, there is no guarantee that the first-stage machine learning would be a consistent estimator. One could construct some counterexamples resulting in catastrophic estimations.

		Not Consider Endogeneity	Linear Control Function	Raw Lasso L = 1	Raw Lasso L = 5	Raw Lasso L = 10	Debiased Lasso L = 1	Debiased Lasso L = 5	Debiased Lasso L = 10
β_0	Mean Bias	0.0078	-0.0304	-0.0581	-0.0712	-0.0640	-0.0159	-0.0230	-0.0201
	Std	0.4424	0.1115	0.2679	0.2977	0.2830	0.1820	0.1806	0.1807
	Median Bias	0.0419	-0.0312	-0.0535	-0.0648	-0.0585	-0.0192	-0.0226	-0.0220
	Coverage	0.9710	0.9450	0.9510	0.9490	0.9490	0.9540	0.9510	0.9520
β_1	Mean Bias	0.0140	0.0023	-0.1509	-0.1628	-0.1567	-0.0144	-0.0177	-0.0159
	Std	0.1801	0.1012	0.1053	0.1115	0.1078	0.0922	0.0897	0.0907
	Median Bias	0.0137	0.0048	-0.1559	-0.1657	-0.1614	-0.0128	-0.0158	-0.0140
	Coverage	0.9470	0.9640	0.7240	0.7340	0.7340	0.9450	0.9480	0.9440
β_2	Mean Bias	-0.0117	0.0001	0.1099	0.1296	0.1191	0.0090	0.0131	0.0109
	Std	0.1087	0.0550	0.0737	0.0780	0.0756	0.0618	0.0602	0.0607
	Median Bias	-0.0157	-0.0011	0.1111	0.1310	0.1207	0.0093	0.0133	0.0111
	Coverage	0.9480	0.9550	0.7030	0.6640	0.6640	0.9660	0.9630	0.9640
β_w	Mean Bias	0.5399	-0.1925	-0.0628	-0.0350	-0.0516	0.0544	0.0575	0.0530
	Std	0.0516	1.9560	2.4934	2.7787	2.6331	1.8235	1.7987	1.8003
	Median Bias	0.5398	-0.1196	-0.1589	-0.1698	-0.1723	0.0633	0.0565	0.0624
	Coverage	0	0.9650	0.9520	0.9530	0.9530	0.9690	0.9650	0.9680

Table 11: Nonlinear generation of w_i, by B-spline polynomial approximation, with degree of B-spline = 2, n = 4560. True $\beta = (0.8, 0.2, -0.2, 0.7)'$.

		Not Consider Endogeneity	Linear Control Function	Raw Lasso L = 1	Raw Lasso L = 5	Raw Lasso L = 10	Debiased Lasso L = 1	Debiased Lasso L = 5	Debiased Lasso L = 10
β_0	Mean Bias	-0.0182	-0.0356	0.0126	0.0084	0.0140	-0.0040	-0.0022	-0.0016
	Std	0.4300	0.0517	0.0916	0.1035	0.1014	0.0828	0.0892	0.0904
	Median Bias	0.0210	-0.0350	0.0064	0.0042	0.0070	-0.0070	-0.0080	-0.0086
	Coverage	0.9720	0.8880	0.9520	0.9530	0.9530	0.9590	0.9630	0.9630
β_1	Mean Bias	0.0264	0.0050	-0.0644	-0.0796	-0.0718	-0.0023	-0.0042	-0.0034
	Std	0.1570	0.0359	0.0433	0.0471	0.0462	0.0412	0.0418	0.0438
	Median Bias	0.0274	0.0045	-0.0629	-0.0780	-0.0695	-0.0031	-0.0052	-0.0036
	Coverage	0.9450	0.9560	0.7030	0.6230	0.6820	0.9490	0.9490	0.9570
β_2	Mean Bias	-0.0086	0.0000	0.0412	0.0510	0.0452	0.0019	0.0026	0.0020
	Std	0.0970	0.0227	0.0281	0.0310	0.0295	0.0245	0.0252	0.0253
	Median Bias	-0.0092	-0.0013	0.0384	0.0478	0.0417	0.0017	0.0024	0.0021
	Coverage	0.9450	0.9490	0.7300	0.6770	0.7070	0.9470	0.9470	0.9480
β_w	Mean Bias	0.5114	-0.2344	-0.5004	-0.5095	-0.5363	-0.0036	-0.0432	-0.0488
	Std	0.0278	0.5731	0.7246	0.7908	0.8181	0.8860	0.9339	0.9867
	Median Bias	0.5111	-0.2178	-0.4742	-0.4625	-0.4704	0.0111	-0.0090	0.0022
	Coverage	0	0.9370	0.9090	0.9140	0.9240	0.9640	0.9580	0.9610

Table 12: Nonlinear generation of w_i, by B-spline polynomial approximation, with degree of B-spline = 2, n = 28800. True $\beta = (0.8, 0.2, -0.2, 0.7)'$.

		Not Endogeneity	Raw Lasso L = 1	Deabised Lasso L = 1
β_0	Mean Bias	0.2599	0.4097	0.0475
	Std	0.4858	0.3641	0.2317
	Median Bias	0.2879	0.3793	0.0408
	Coverage	0.9390	0.8160	0.9370
β_1	Mean Bias	0.0275	-0.1975	-0.0291
	Std	0.1855	0.1312	0.0890
	Median Bias	0.0283	-0.2023	-0.0295
	Coverage	0.9420	0.6590	0.9280
β_2	Mean Bias	0.0027	0.1262	0.0155
	Std	0.1071	0.0691	0.0484
	Median Bias	0.0010	0.1338	0.0164
	Coverage	0.9590	0.5040	0.9340
β_w	Mean Bias	-0.3043	-0.4815	-0.0778
	Std	0.1894	0.2879	0.2192
	Median Bias	-0.3149	-0.4842	-0.0819
	Coverage	0.6300	0.6140	0.9280

Table 13: Misspecification with binary endogenous variable. With true $\beta = ()'$