

1

Attached separately.

2

(i)

proof:

$Ae^{i\theta}$ where A is complex.

A complex implies that $A = Be^{i\phi}$ where B and ϕ are real

so $Ae^{i\theta} = Be^{i\phi}e^{i\theta} = Be^{i(\theta+\phi)} = Be^{i(\beta)}$

Here B and β are real, so found a real representation.

(ii)

proof:

$Ae^{i\theta}$ where θ is complex.

θ complex means that $\theta = r(\cos(\phi) + i\sin(\phi))$ for some r and ϕ complex

So: $Ae^{i\theta} = Ae^{i r(\cos(\phi) + i\sin(\phi))} = Ae^{-r\sin(\phi)}e^{ircos(\phi)} = Ce^{i\alpha}$

Where C and α are real.

3

(A)

If we add to waves using the complex exponential, we add the real parts together. But then we are only adding the cosine parts of both waves so it represents the sum of two real cosine waves.

(B)

Square: $(Ae^{i2\pi ft})^2 = (A^2 e^{2(i2\pi ft)}) = (A^2 e^{i4\pi ft})$

Problem: Here the values of this result could still be negative. The problem with this is if we square the wave, all of its values should be greater than 0.

(C)

So let $\theta = 2\pi ft$.

The represent $A\cos(\theta) = A\left(\frac{e^{i\theta} + e^{-i\theta}}{2}\right)$

To show what happens with square show what happens when we square cosine:

$$\begin{aligned} \cos(\theta)^2 &= \left(\frac{e^{i\theta} + e^{-i\theta}}{2}\right)^2 = \left(\frac{e^{i\theta}}{2}\right)^2 + 2\left(\frac{e^{i\theta}}{2}\right)\left(\frac{e^{-i\theta}}{2}\right) + \left(\frac{e^{-i\theta}}{2}\right)^2 \\ &= \left(\frac{e^{i\theta}}{4}\right) + \frac{1}{2}(e^{i\theta-i\theta}) + \left(\frac{e^{-i\theta}}{4}\right) \\ &= \left(\frac{e^{i\theta}}{4}\right) + \frac{1}{2} + \left(\frac{e^{-i\theta}}{4}\right) \\ &= \frac{1}{2}\cos(2\theta) + \frac{1}{2} \end{aligned}$$