CS 512, Spring 2014

Assignment 2

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Due Monday January 27th

1 Problem 1

Algebraic Structure: $\tau = (T, Lt, \mathbb{N}, Rt, height)$. Define the size operation $||$: $\tau \to \mathbb{N}$ and the height operation height: $\tau \to \mathbb{N}$ in the style of the definitions for the operations Node Lt, and Rt. If $t \in T$ is represented graphically by a finite binary tree, then $|t|$ should return should return the total number of leaf nodes in t + the total number of internal nodes in t, and the hieght(t) should return the length of the longest path in t.

$$(1) \qquad |t| = \begin{cases} |t_1| + |t_2|, & \text{if } t =< t_1 \ \ t_2 > . \\ 1, & \text{otherwise} \end{cases}$$

$$(2) \qquad height(x) = \begin{cases} max(|t_1| + |t_2|) + 1, & \text{if } t =< t_1 \ \ t_2 > . \\ 0, & \text{otherwise} \end{cases}$$

2 Problem 2

Algebraic Structure: $A = (\mathbb{N} - 0, lcm, gcd, \trianglelefteq)$ where for all $m, n \in \mathbb{N} - 0$, we have that: $m \trianglelefteq n$ iff "m divides n".
(a) Show that A is a lattice, where lcm playes the role of $\vee$ and gcd plays the role of $\wedge$.
(b) Is A a distributice lattice? Justify your answer carefully, based on the distributivity axioms that A must satisfy.

(a)
Need to Show:
(1) Show $(A, \trianglelefteq)$ is a poset.

For all $a, b, c \in \mathbb{N} - 0$
reflexive: "a divides a" therefore $a \trianglelefteq a$
anit-symmetric:
Assume $a \trianglelefteq b$ and $b \trianglelefteq a$. Then "a divides b" and "b divides a".
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transitive:
Assume $a \trianglelefteq b$ and $b \trianglelefteq c$. Then "a divides b" and 'b divdes c".
If "a divides b" then b must be some multiple of a ($b = m \cdot a, m \in \mathbb{N} - 0$).
If "b divides c" then c must be some multiple of b ($c = n \cdot b, m \in \mathbb{N} - 0$).
So $c = n \cdot b = n \cdot (m \cdot a)$ where $m, n \in \mathbb{N} - 0$
But then c is some multiple of a. So "a divides c".
Therefore $a \trianglelefteq c$.

(2) For all $m, n \in A$ the least upper bound of a and b in the odering $\trianglelefteq$ exists, is unique m and is the result of the operation $m \vee n$
(2) For all $m, n \in A$ the greatest lower bound of a and b in the odering $\trianglelefteq$ exists, is unique m and is the result of the operation $m \wedge n$