CS 512, Spring 2014

Assignment 1

Shan Sikdar

Due Monday January 27th

1 Problem 1(a): $P \to Q, P \to \neg Q \vdash \neg P$

1	$P \to Q$	premise
2	$P \to \neg Q$	premise
3	P	assume
4	Q	$\to$e $1, 3$
5	$\neg Q$	$\to$e $2, 3$
6	$\bot$	$\neg$e $4, 5$
7	$\neg P$	$\neg$i

2 Problem 1(b): $P \to (Q \to R), P, \neg R \vdash \neg Q$

1	$P \to (Q \to R)$	premise
2	P	premise
3	$\neg R$	premise
4	Q	assume
5	$Q \to R$	$\to$e $1, 2$
6	R	$\to$e $4, 5$
7	$\bot$	$\neg$e $3, 6$
8	$\neg Q$	$\neg$i

3 Problem 2: 1.2.2(g): $p \wedge \neg p \vdash \neg(r \to q) \wedge (r \to q)$

1	$p \wedge \neg p$	premise
2	p	$\wedge e_1$ 1
3	$\neg p$	$\wedge e_1$ 1
4	$\bot$	$\neg$e $2, 3$
5	$\neg(r \to q) \wedge (r \to q)$	$\bot$e

4 Problem 2: 1.2.2(h): $p \to q, s \to t \vdash p \vee s \to q \wedge t$

Since in the truth table some valuations have the left evaluate to T while the result false, the sequent is not valid.

p	q	s	t	$p \to q$	$s \to t$	$p \vee s$	$q \wedge t$	$p \vee s \to q \wedge t$
T	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**
T	**F**	**T**	**T**	**F**	**T**	**T**	**F**	**F**
F	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**
F	**F**	**T**	**T**	**T**	**T**	**T**	**F**	**F**
T	**T**	**T**	**F**	**T**	**F**	**T**	**F**	**F**
T	**F**	**T**	**F**	**F**	**F**	**T**	**F**	**F**
F	**T**	**T**	**F**	**T**	**F**	**T**	**F**	**F**
F	**F**	**T**	**F**	**T**	**F**	**T**	**F**	**F**
T	**T**	**F**	**T**	**T**	**T**	**T**	**T**	**T**
T	**F**	**F**	**T**	**F**	**T**	**T**	**F**	**F**
F	**T**	**F**	**T**	**T**	**T**	**F**	**T**	**T**
F	**F**	**F**	**T**	**T**	**T**	**F**	**F**	**F**
T	**T**	**F**	**F**	**T**	**T**	**T**	**F**	**F**
T	**F**	**F**	**F**	**F**	**T**	**T**	**F**	**F**
F	**T**	**F**	**F**	**T**	**T**	**F**	**F**	**T**
F	**F**	**F**	**F**	**T**	**T**	**F**	**F**	**T**

5 Problem 2: 1.2.2(i): $\neg(\neg p \vee q) \vdash p$

1	$\neg(\neg p \vee q)$	premise
2	$\neg p$	assume
3	$\neg p \vee q$	$\vee$i 2
4	$\perp$	$\neg$e 1, 3
5	p	$\neg$i

6 Problem 3:

$S - 1 = C$

Justification: For every propostional atom you add to a well formed formula, you need a binary connective to join it to.

$$N \leq S + C$$

This inequality assumes that any series of negation signs can be reduced to one or no negation signs (e.g. $\neg\neg\neg q = \neg q$ or $\neg\neg\neg\neg q = q$) . Otherwise there is no upperbound for the value of N

7 Problem 4: 1.4.12(a):

If p is True, q is True, then $\neg p \vee (q \rightarrow p)$ is True while $\neg p \wedge q$ is False. So sequent not valid.

8 Problem 4: 1.4.12(b):

If r is True , q is False, and p is true, then $\neg r \rightarrow (p \vee q)$ is True, and $r \wedge \neg q$ is True. But $r \rightarrow q$ is False. So sequent not valid.

9 Problem 4: 1.4.12(c):

If p is True, q is False, and r is True, then $p \rightarrow (q \rightarrow r)$ is True. But $p \rightarrow (r \rightarrow q)$ is false. So sequent not valid.

10 problem 5:

Use induction based on the height of the parse tree formed by the WFF φ.

Base Case: Height = 1,
If Height = 1, then the parse tree must only conisist of propsitional atom, say p. Then $\varphi = p$ and $\varphi* = \neg p = \neg\varphi$.

Inductive Step: Assume true for all WFF's of height n. Prove for height n + 1.

Let φ be a WFF with height n+1. For φ to have this hieght one of the following cases must happen:
1. φ is made up of another WWF ϕ of parse tree height n with a $\neg$ in front of it: ($\varphi = \neg\phi$)
2. φ is made up of two WFF's ϕ, ρ of parse tree height n connected by $\wedge$: ($\varphi = \phi \wedge \rho$)
3. φ is made up of two WFF's ϕ, ρ of parse tree height n connected by $\vee$: ($\varphi = \phi \vee \rho$)

Case 1: $\varphi = \neg\phi$
Using the inductive hypothesis we know $\phi*$ is tuautologically equivelent to $\neg\phi$. So $\varphi = \phi* = \neg\phi$
Then:
$\varphi* = \neg(\neg\phi)$
$\varphi* = \neg(\phi*)$
$\varphi* = \neg(\varphi)$
$\varphi* = \neg\varphi$

Case 2: $\varphi = \phi \wedge \rho$. Using the inductive hypothesis we know $\neg\phi = \phi*$ and $\neg\rho = \rho*$
Then because:
$\neg\varphi = \neg(\phi \wedge \rho)$ and $\varphi* = \neg\phi \vee \neg\rho = \phi * \vee \rho*$.

I can use a truth table to show $\varphi*$ and $\neg\varphi$ tautological equivlence:

ϕ	ρ	$\varphi = \phi \wedge \rho$	$\neg\varphi$	$\neg\phi$	$\neg\rho$	$\varphi* = \neg\phi \vee \neg\rho$
T	**T**	**T**	**F**	**F**	**F**	**F**
T	**F**	**F**	**T**	**F**	**T**	**T**
F	**T**	**F**	**T**	**T**	**F**	**T**
F	**F**	**F**	**T**	**T**	**T**	**T**

The $\varphi*$ column has the same values as the $\neg\varphi$ column. So they are tautologically equivelent

Case 3: $\varphi = \phi \vee \rho$. Again using the inductive hypothesis, $\neg\phi = \phi*$ and $\neg\rho = \rho*$
Then because:
$\neg\varphi = \neg(\phi \vee \rho)$ and $\varphi* = \neg\phi \wedge \neg\rho = \phi * \wedge \rho*$.
I can use a truth table to show $\varphi*$ and $\neg\varphi$ tautological equivlence:

ϕ	ρ	$\varphi = \phi \vee \rho$	$\neg\varphi$	$\neg\phi$	$\neg\rho$	$\varphi* = \neg\phi \wedge \neg\rho$
T	**T**	**T**	**F**	**F**	**F**	**F**
T	**F**	**T**	**F**	**F**	**T**	**F**
F	**T**	**T**	**F**	**T**	**F**	**F**
F	**F**	**F**	**T**	**T**	**T**	**T**

The $\varphi*$ column has the same values as the $\neg\varphi$ column. So they are tautologically equivelent