CS 542, Spring 2014

Assignment 5

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1 8.3

Show by direct evaluation that this ditribution has the property that $p(a,b) \neq p(a)p(b)$ but that $p(a,b|c) = p(a|c)p(b|c)$ for both c = 0 and c = 1.
By the summation rule from chapter 1: $p(a,b) = \sum_{c \in 0,1} p(a,b,c)$
So for example $p(0,0) = \sum_{c \in 0,1} p(0,0,c) = .192 + .144 = .136$.
Doing similar calculations gives the results:

a	b	$p(a,b)$
0	0	.336
0	1	.264
1	0	.256
1	1	.144

Using the summation rule again we can obtain $p(a) = \sum_{b \in 0,1} \sum_{c \in 0,1} p(a,b,c)$ and $p(b) = \sum_{a \in 0,1} \sum_{c \in 0,1} p(a,b,c)$
So for example:
$p(a=0) * p(b=0) = (\sum_{b \in 0,1} \sum_{c \in 0,1} p(a,b,c) \sum_{a \in 0,1} \sum_{c \in 0,1} p(a,b,c)) = (.192 + .144 + .048 + .216) * (.192 + .144 + .192 + .064) = (.6) * (.592) = .3352$
Doing similar calculations gives the results:

a	b	$p(a)p(b)$
0	0	.3552
0	1	.2448
1	0	.2368
1	1	.1632

So we can see that $p(a,b) \neq p(a)p(b)$
For conditional probability applying the product rule from chapter 1, we have
$p(c,a,b) = p(a,b|c)p(c)$ which leads to $p(a,b|c) = \frac{p(a,b,c)}{\sum_a \sum_b p(a,b,c)}$
Using a similar technique you can derive:
$p(a|c) = \frac{\sum_b p(a,b,c)}{\sum_a \sum_b p(a,b,c)}$ and
$p(b|c) = \frac{\sum_a p(a,b,c)}{\sum_a \sum_b p(a,b,c)}$

Solving and plugging in the values gives the following results:

a	b	c	$p(a,b\|c)$	$p(a\|c) * p(b\|c)$
0	0	0	.400	.400
0	1	0	.100	.100
1	0	0	.400	.400
1	1	0	.100	.100
0	0	1	.277	.277
0	1	1	.415	.415
1	0	1	.123	.123
1	1	1	.185	.185

Since the last two columns are the same $p(a,b|c) = p(a|c)p(b|c)$ and so theres is conditional independence.

2 8.4

Evaluate the distributions $p(a)$ $p(b|c)$ and $p(c|a)$ correspnding to the joint distribution given in table 8.2 Hence show by direct evaluation that $p(a,b,c) = p(a)p(c|a)p(b|c)$.

$p(a)$ and $p(b|c)$ were computed in the last problem with the following values:

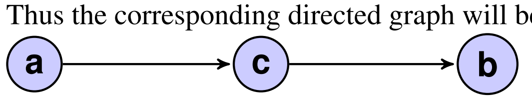
a	$p(a)$
0	.6
0	.4

b	c	$p(b|c)$
0	0	.8
0	1	.2
1	0	.4
1	1	.6

Similar to $p(b|c)$ and $p(a|c)$, we can use product rule so
$p(a|c) = \frac{\sum_b p(a,b,c)}{\sum_b \sum_c p(a,b,c)}$
Calculating the values results in :

b	c	$p(c|a)$
0	0	.4
0	1	.6
1	0	.6
1	1	.4

(sanity check .8 *.6 *.4 = .192) So using all these obtained values and multiplying them together, we can get the values of p(a,b,c) found in table 2 on page 419. Therefore $p(a,b,c) = p(a)p(c|a)p(b|c)$
Thus the corresponding directed graph will be:



3 8.11

Given $P(D = 1|G = 1) = .9$ and $P(D = 0|G = 0)$
Show (1) $P(F = 0|D = 0)$ and (2) $P(F = 0|D = 0, B = 0)$
Note: Not actually putting the calculations into the writeup or it would take forever to type.
Using Bayes Theorem, and marginalizing, and then evaluating
(1) $P(F = 0|D = 0)$
$= \frac{P(D=0|F=0)P(F=0)}{P(D=0)}$
$= \frac{(\sum_{B,G\in\{0,1\}} P(D=0|G)P(G|B,F)P(B))P(F=0)}{\sum_{B,G,F\in\{0,1\}} P(D=0|G)P(G|B,F)P(B)P(F))} = .213$
(2)For the second probability do not marginalize over B and keep it fixed at its observed value: (1) $P(F = 0|D = 0, B = 0)$
$= \frac{P(D=0|F=0)P(F=0)}{P(D=0)}$
$= \frac{(\sum_{G\in\{0,1\}} P(D=0|G)P(G|B,F)P(B))P(F=0)}{\sum_{G,F\in\{0,1\}} P(D=0|G)P(G|B,F)P(B)P(F))} = .110$
Since thr probablilites are lower than the values calcualted in the example it shows that the driver is not very reliable. Also the probabilites are lower because these observations provide alternate explanations as to why the guage should read zero.

4 8.14

The energy function (8.42) is $E(x, y) = h\sum_i x_i - \beta\sum_{ij} x_i x_j - \nu\sum x_i y_i$. $(n, h, \beta \geq 0$ and $x_i, y_i \in \{-1, 1\}$
Setting $h = \beta = 0$ gives $E(x, y) = -N\sum x_i y_i$. The most probable configuration is when the energy is lowest.
This happens when the negative sign in from of N stays, which can only happen if the values of x_i, y_i are either both 1 or both negative 1. Therefore $x_i = y_i$ for all i.