

CS 512, Spring 2014

Assignment 1

Shan Sikdar

Due Monday January 27th

1 (*) 2.1

Verify $\sum_{x=0}^1 p(x|\mu) = 1$

Proof. $\sum_{x=0}^1 p(x|\mu) = p(x=0|\mu) + p(x=1|\mu) = \mu + 1 - \mu = 1$

Verify: $\mathbb{E}[x] = \mu$

Proof. $\mathbb{E}[x] = \sum_x x p(x|\mu) = 1 * \mu + 0 * (1 - \mu) = \mu$

Verify $var[x] = \mu(1 - \mu)$

Proof.

$$\begin{aligned} var(x) &= \mathbb{E}[(x - E[x])^2] \\ &= (1 - \mu)(0 - \mu)^2 + \mu(1 - \mu)^2 \\ &= (1 - \mu)(\mu)^2 + \mu(1 - \mu)^2 \\ &= \mu - \mu^2 = \mu(1 - \mu) \end{aligned}$$

Show entropy of $H[x]$ of a Bernoulli distributed random variable x is given

by : $H[x] = -\mu \ln \mu - (1 - \mu) \ln(1 - \mu)$

By definition of entropy: $H[x] = -\sum_x p(x|\mu) \ln[p(x|\mu)]$

$$= -[(1 - \mu) \ln(1 - \mu) + (\mu \ln \mu)] = -\mu \ln \mu - (1 - \mu) \ln(1 - \mu)$$