CS 512, Spring 2014

Assignment 7

Shan Sikdar

1 Problem 1.3.48 a,b,c,d

(a) $\mathbf{AF}q$
s_0: Yes: satisfies since q is true at s_0.
s_2: Yes: From s_2 you can only go to s_0 and s_3 where q is true

(b) $\mathbf{AG}(\mathbf{EF}(p \vee r))$
s_0: Yes: since in every state p or r is true
s_2: Yes: since in every state p or r is true

(c) $\mathbf{EX}(\mathbf{EX}r)$
s_0: Yes: look at the path $s_0 s_1 s_1$...
s_2: Yes: look at the path $s_2 s_0 s_1$...

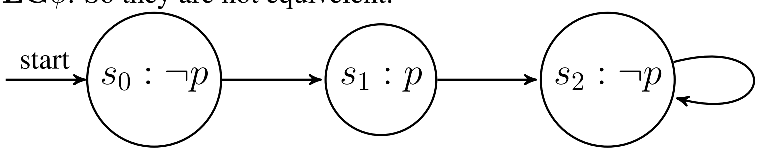
(d) $\mathbf{AG}(\mathbf{AF}q)$
s_0: No: since we can have the path $s_0 s_1 s_1$....
s_2: No: since we can have the path $s_2 s_0 s_1$...

2 Problem 3.4.9

new $\mathbf{AG}\phi$: $\mathbf{AX}(\mathbf{AG}\phi)$
new $\mathbf{EG}\phi$: $\mathbf{EX}(\mathbf{EG}\phi)$
new $\mathbf{AF}\phi$': $\mathbf{AXA}\phi$
new $\mathbf{EF}\phi$: $\mathbf{EXEF}\phi$
new $\mathbf{A}[\phi_1\mathbf{U}\phi_2]$: $\phi_1 \wedge \mathbf{AXA}[\phi_1\mathbf{U}\phi_2]$
new $\mathbf{A}[\phi_1\mathbf{U}\phi_2]$: $\phi_1 \wedge \mathbf{EXE}[\phi_1\mathbf{U}\phi_2]$

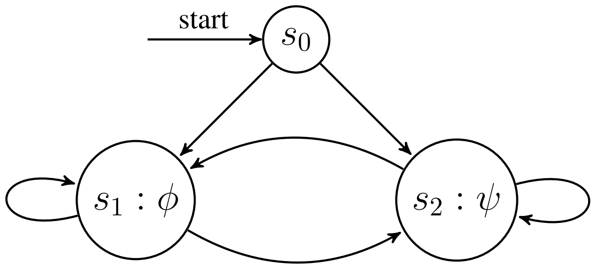
3 Problem 3.4.10 a,b,c,d,e

(a) $\mathbf{EF}\phi$ and $\mathbf{EG}\phi$
In the state transition system below note that: $\mathcal{M}, s_0 \models \mathbf{EF}\phi$ but not $\mathbf{EG}\phi$. So they are not equivelent.



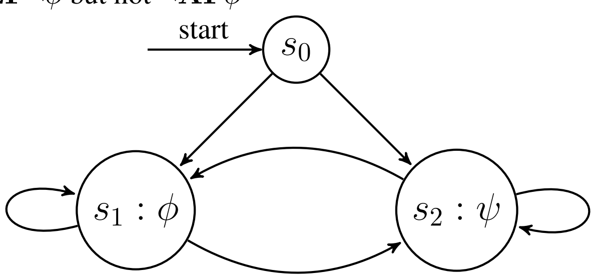
(b) $\mathbf{EF}\phi \vee \mathbf{EF}\psi$ and $\mathbf{EF}(\phi \vee \psi)$
They are equivelent.
1. First, assume that $s \models \mathbf{EF}\phi \vee \mathbf{EF}\psi$. Without loss of generality, assume that $s \models \mathbf{EF}\phi$. This means that there is a future state s_n, reachable from s, such that $s_n \models \phi$. But then $s_n \models \phi \vee \psi$ follows. But this means that there is a state rechable from s which satisfies $\phi \vee \psi$. Thus, $s \models \mathbf{EF}(\phi \vee \psi)$ follows.
2. Second, assume that $s \models \mathbf{EF}(\phi \vee \psi)$. Then there exists a state $s_m \models \phi \vee \psi$Withou loss of generality, we may assume that $s_m \models \psi$. Buth then we can conclude that $s \models \mathbf{EF}\psi$, as s_m is reachable from s. Therefore, we also have $s \models \mathbf{EF}\phi \vee \mathbf{EF}\psi$

(c) $\mathbf{AF}\phi \vee \mathbf{AF}\psi$ and $\mathbf{AF}(\phi \vee \psi)$
In the state transition system below, look at paths $\pi_1 = s_0s_1s_1...$ and $\pi_2 = s_0s_2s_2.....$ They both statisy $\mathbf{AF}(\phi \vee \psi)$ but π_1 does not statisy $\mathbf{AF}\psi$ and π_2 does not statisfy $\mathbf{AF}\phi$ and therefore $\mathbf{AF}\phi \vee \mathbf{AF}\psi$ cannot hold



(d) $\mathbf{AF}\neg\phi$ and $\neg\mathbf{EG}\phi$
They are equivelent.
1. Assume $s \models \mathbf{EF}\neg\phi$ Then at some state along this path we have $s_m \models \neg\phi$ Then this path $s \not\models \mathbf{G}\phi$. Since was any arbitary path, then we know that $s \not\models EG\phi$. Then by definition we know $s \models \neg\mathbf{EG}\phi$.
2. Assume $s \models \neg\mathbf{EG}\phi$. Then we know at some point that $s_m \models \neg\phi$ So we know for all paths at some point will have $\neg\phi$. Therefore $s \models \mathbf{AF}\neg\phi$

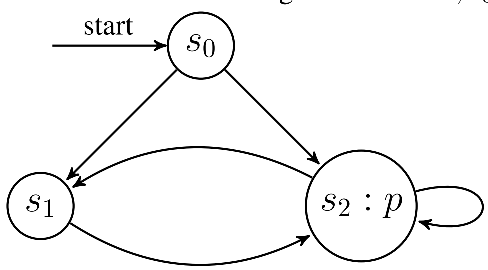
(e) $\mathbf{EF}\neg\phi$ and $\neg\mathbf{AF}\phi$
Look at the state transition diagram below. The path $s_0 s_1 s_1 ...$ satisfies $\mathbf{EF}\neg\phi$ but not $\neg\mathbf{AF}\phi$



4 3.5.6 a,b,c

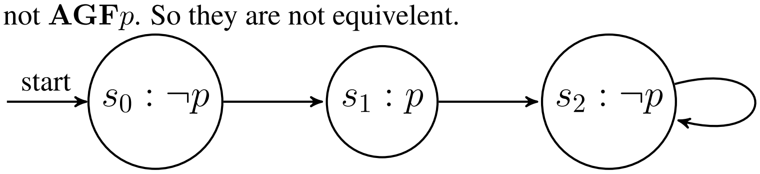
(a) $\mathbf{AFG}p$ and $\mathbf{AFAG}p$

Look at the transition digram below $\mathcal{M}, s_0 \models \mathbf{AFG}p$ but not $\mathbf{AFAG}p$.



(b) $\mathbf{AGF}p$ and $\mathbf{AGEF}p$

In the state transition system below note that: $\mathcal{M}, s_0 \models \mathbf{AGEF}p$ but not $\mathbf{AGF}p$. So they are not equivelent.



(c) $\mathbf{A}[(p\mathbf{U}r) \vee (q\mathbf{U}r)]$ and $\mathbf{A}[(p \vee q)\mathbf{U}r]$

Look at the transition diagram above. note that:

$\mathcal{M}, s_0 \models \mathbf{A}[(p \vee q)\mathbf{U}r]$ but not $\mathbf{A}[(p\mathbf{U}r) \vee (q\mathbf{U}r)]$. so they are not equivelent.