# CS 512, Spring 2014

# Examples in Propositional Logic

Assaf Kfoury

## 1 Example: $P \vdash Q \rightarrow (P \wedge Q)$

| | | |
|---|---|---|
| 1 | $P$ | premise |
| 2 | $Q$ | assume |
| 3 | $P \wedge Q$ | $\wedge$i $1, 2$ |
| 4 | $Q \rightarrow (P \wedge Q)$ | $\rightarrow$i |

## 2 Example: $P \rightarrow (Q \rightarrow R) \vdash P \wedge Q \rightarrow R$

| | | |
|---|---|---|
| 1 | $P \rightarrow (Q \rightarrow R)$ | premise |
| 2 | $P \wedge Q$ | assume |
| 3 | $P$ | $\wedge$e$_1$ $2$ |
| 4 | $Q \rightarrow R$ | $\rightarrow$e $1,3$ |
| 5 | $Q$ | $\wedge$e$_2$ $2$ |
| 6 | $R$ | $\rightarrow$e $4,5$ |
| 7 | $P \wedge Q \rightarrow R$ | $\rightarrow$i |

## 3 Example: $P \wedge Q \to R \vdash P \to (Q \to R)$

| | | |
|---|---|---|
| 1 | $P \wedge Q \to R$ | premise |
| 2 | $P$ | assume |
| 3 | $Q$ | assume |
| 4 | $P \wedge Q$ | $\wedge$i 2, 3 |
| 5 | $R$ | $\to$e 1, 4 |
| 6 | $Q \to R$ | $\to$i |
| 7 | $P \to (Q \to R)$ | $\to$i |

## 4 Example: $P \to (Q \to R) \vdash (P \to Q) \to (P \to R)$

| | | |
|---|---|---|
| 1 | $P \to (Q \to R)$ | premise |
| 2 | $P \to Q$ | assume |
| 3 | $P$ | assume |
| 4 | $Q$ | $\to$e 2, 3 |
| 5 | $Q \to R$ | $\to$e1, 3 |
| 6 | $R$ | $\to$e 5, 4 |
| 7 | $P \to R$ | $\to$i |
| 8 | $(P \to Q) \to (P \to R)$ | $\to$i |

# 5 Example: $P \wedge \neg Q \to R, \; \neg R, \; P \; \vdash \; Q$

| | | |
|---|---|---|
| 1 | $P \wedge \neg Q \to R$ | premise |
| 2 | $\neg R$ | premise |
| 3 | $P$ | premise |
| 4 | $\neg Q$ | assume |
| 5 | $P \wedge \neg Q$ | $\wedge$i $3, 4$ |
| 6 | $R$ | $\to$e $1, 5$ |
| 7 | $\bot$ | $\neg$e $6, 2$ |
| 8 | $\neg\neg Q$ | $\neg$i |
| 9 | $Q$ | $\neg\neg$e $8$ |