CS 512, Spring 2014

Assignment 5

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1 Problem 1

(a) $\phi = \mathbf{G}a$

  (i) the path $q_3 \rightarrow q_4 \rightarrow q_3 \rightarrow q_4 \rightarrow q_3$.... ($q_3$ and q_4 keep repeating) will satisfy Ga

  (ii) No because there are other paths do not satsify Ga

(b) $\phi = a\mathbf{U}b$

  (i) the path $q_3 \rightarrow q_2$.... (q_2 goes on forever) will statisfy $\phi = a\mathbf{U}b$

  (ii) No because there are other paths that do not satsify $\phi = a\mathbf{U}b$

(c) $\phi = a\mathbf{U}X(a \wedge \neg b)$

  (i) the path $q_3 \rightarrow q_4 \rightarrow q_3 \rightarrow q_4 \rightarrow q_3$.... ($q_3$ and q_4 keep repeating) will satisfy $\phi = a\mathbf{U}X(a \wedge \neg b)$

  (ii) No because not all of the other paths satsify $\phi = a\mathbf{U}X(a \wedge \neg b)$

(d) $\phi = \mathbf{X}\neg b \wedge \mathbf{G}(\neg a \vee \neg b)$

  (i) the path $q_3 \rightarrow q_1 \rightarrow q_2$..... will satisfy $\phi = \mathbf{X}\neg b \wedge \mathbf{G}(\neg a \vee \neg b)$

  (ii) No because all the other paths do not satsify $\phi = \mathbf{X}\neg b \wedge \mathbf{G}(\neg a \vee \neg b)$

(e) $\mathbf{X}(a \wedge b) \wedge F(\neg a \wedge \neg b)$

  (i) the path $q_3 \rightarrow q_4 \rightarrow q_3 \rightarrow q_1 \rightarrow q_2$...

(ii) No

(f) $a \wedge \mathbf{F}b$

(i) All possible paths satisfy $a \wedge \mathbf{F}b$.

(ii) Yes all paths satisfy $a \wedge \mathbf{F}b$

2 Problem 2

(a) $\mathbf{FG}\varphi$ and $\varphi \to \mathbf{X}\varphi$
The path $p \to \neg p \to \neg p \to p \to p \to p \to p...$ (p goes on forever) satisfies $\mathbf{FG}\varphi$ but not $\varphi \to \mathbf{X}\varphi$

(b) $\mathbf{FG}\varphi$ and $\neg\varphi\mathbf{UG}\varphi$
The path $\neg p \to \neg p \to p \to \neg p \to p \to p \to p \to p$...(p go on forever) satisfies $\mathbf{FG}$ but not $\neg\varphi\mathbf{UG}\varphi$

(c) $\mathbf{G}(\varphi \to \mathbf{X}\varphi)$ and $\neg\varphi\mathbf{UG}\varphi$ The path $p \to p \to p \to p...$ (p goes on forever), satsisfies $\mathbf{G}(\varphi \to \mathbf{X}\varphi)$ but not $\neg\varphi\mathbf{UG}\varphi$.

(d) $\mathbf{F}(\varphi \wedge \psi)$ and $(\mathbf{F}\varphi \wedge \mathbf{F}\psi)$ The path $p \to q \to p \to q \to p$...(have p, q alternate forever). This satisfies $(\mathbf{F}\varphi \wedge \mathbf{F}\psi)$ but not $\mathbf{F}(\varphi \wedge \psi)$

(e) (a) One implication has shown to be false in part a. To see why the implication is false going the other way, note that the path $\neg p \to \neg p \to \neg p...$ ($\neg p$ goes on forever) will satisfy $\varphi \to \mathbf{X}\varphi$ but not $\mathbf{FG}\varphi$.

(b) $\neg\varphi\mathbf{UG}\varphi$ implies $\mathbf{FG}\varphi$,
Because in the first case you wil have $\mathbf{G}\varphi$ after some $\neg\varphi$ which implies sometime in the future you will have globally φ.
The implication does not exist the other way from the counter example shown in part b.

(c) $\neg\varphi\mathbf{UG}\varphi$ implies $\mathbf{G}(\varphi \to \mathbf{X}\varphi)$
Because once you have globally φ, then you know that if you see φ the next state must also have φ.
The implication does not exist the other way from the counter example shown in part c.

(d) $\mathbf{F}(\varphi \wedge \psi)$ implies $(\mathbf{F}\varphi \wedge \mathbf{F}\psi)$

Since if we know $\varphi \wedge \psi$ will be true at some point in the future, at that point both φ and ψ have to be true seperately as well. (otherwise the $\wedge$ could not be true. Then we know sometime in the future $(\mathbf{F}\varphi \wedge \mathbf{F}\psi)$

3 Problem 3

$\varphi \mathbf{U} \psi \equiv \varphi \mathbf{W} \psi \wedge \mathbf{F} \psi$
proof.

(1) $\pi \models \varphi \mathbf{U} \psi$ iff
By definintion:

(2) $(\exists i \geq 1)[\pi^i \models \psi \wedge \pi^1 \models \varphi, ..., \pi^{i-1} \models \varphi]$ iff
since we know $(\exists i \geq 1)\pi^i \models \psi$ is true, and using properties of $\wedge$:

(3) $((\exists i \geq 1)\pi^i \models \psi) \wedge (\exists i \geq 1)[\pi^i \models \psi \wedge \pi^1 \models \varphi, ..., \pi^{i-1} \models \varphi]$ iff
Since we know that this statment is true, using properties of $\vee$:

(4) $((\exists i \geq 1)\pi^i \models \psi) \wedge (\exists i \geq 1)[\pi^i \models \psi \wedge \pi^1 \models \varphi, ..., \pi^{i-1} \models \varphi] \vee \forall k \pi^k \models \varphi$ iff
by definitions of $\mathbf{F}$ and $\mathbf{W}$:

(5) $\mathbf{F}\psi \wedge \varphi \mathbf{W} \psi$

(6) $\pi \models \mathbf{F}\psi \wedge \varphi \mathbf{W} \psi$

Since we have iff at every line we know that implications work both ways. So they are equivelent.

$\varphi \mathbf{W} \psi \equiv \varphi \mathbf{U} \psi \vee \mathbf{G} \varphi$
proof.

(1) $\pi \models \varphi \mathbf{W} \psi$ iff
By definition:

(2) $(\exists i \geq 1)[\pi^i \models \psi \wedge \pi^1 \models \varphi, ..., \pi^{i-1} \models \varphi] \vee \forall k \; \pi^k \models \varphi$ iff
By definition of $\mathbf{G}$:

(3) $(\exists i \geq 1)[\pi^i \models \psi \wedge \pi^1 \models \varphi, ..., \pi^{i-1} \models \varphi] \vee \mathbf{G}\varphi$ iff
by Definition of $\mathbf{U}$:

(4) $\varphi \mathbf{U} \psi \vee \mathbf{G}\varphi$

(5) $\pi \models \varphi \mathbf{U} \psi \vee \mathbf{G}\varphi$
Since we have iff at every line we know that implications work both ways. So they are equivelent.

4 Problem 4

function NNF(ϕ):

ϕ is a literal: return ϕ
ϕ is $\neg\neg\phi_1$: return $NNF(\phi_1)$
ϕ is $\phi_1 \wedge \phi_2$: return $NNF(\phi_1) \wedge NNF(\phi_2)$
ϕ is $\phi_1 \vee \phi_2$: return $NNF(\phi_1) \vee NNF(\phi_2)$
ϕ is $\neg(\phi_1 \wedge \phi_2)$: return $NNF(\neg\phi_1) \vee NNF(\neg\phi_2)$
ϕ is $\neg(\phi_1 \vee \phi_2)$: return $NNF(\neg\phi_1) \wedge NNF(\neg\phi_2)$

ϕ is $\mathbf{G}\phi_1$: return $\mathbf{G}NNF(\phi_1)$
ϕ is $\neg\mathbf{G}\phi_1$: return $\mathbf{F}NNF(\neg\phi_1)$

ϕ is $\mathbf{F}\phi_1$: return $\mathbf{F}NNF(\phi_1)$
ϕ is $\neg\mathbf{F}\phi_1$: return $\mathbf{G}NNF(\neg\phi_1)$

ϕ is $\mathbf{X}\phi_1$: return $\mathbf{X}NNF(\phi_1)$
ϕ is $\neg\mathbf{X}\phi_1$: return $\mathbf{X}NNF(\neg\phi_1)$

ϕ is ϕ_1 $\mathbf{U}$ ϕ_2: return $NNF(\phi_1)$ $\mathbf{U}$ $NNF(\phi_2)$
ϕ is $\neg(\phi_1\mathbf{U}\phi_2)$: return $NNF(\neg\phi_1)$ $\mathbf{R}$ $NNF(\neg\psi)$

ϕ is ϕ_1 $\mathbf{R}$ ϕ_2: return $NNF(\phi_1)$ $\mathbf{R}$ $NNF(\phi_2)$
ϕ is $\neg(\phi_1\mathbf{R}\phi_2)$: return $NNF(\neg\phi_1)$ $\mathbf{U}$ $NNF(\neg\psi)$

ϕ is ϕ_1 $\mathbf{W}$ ϕ_2: return $NNF(\phi_1)\mathbf{U}\ NNF(\phi_2) \vee G\ NNF(\phi)$
ϕ is $\neg(\phi_1\mathbf{W}\phi_2)$: return $NNF(\neg(\phi_1 U \phi_2)) \wedge NNF(\neg\mathbf{G}\phi_1)$

5 Problem 5

(a) No, take the path $s_2 \to s_4 \to s_2 \to s_4 \to s_2 \to s_4$... When you have s_2, r is true, when you have s_4 r is false. So this cannot satisfy $\mathbf{FG}r$

(b) Yes,
If you start from s_1 there are two ways to go s_3 and s_4. s_3 has an r so thats okay. If we then follow a path to s_4 we see that from s_4 any state that we move to will have an r. If you start from s_2, the next state is s_4 which like before any state you move to from there will have an r.

(c) Yes,
If $\mathbf{X}\neg r$ then you know the next state has to be s_4. Every state that you can move to from s_4 will have r as true. Then you know that in the next next state, r will be true. So $\mathbf{XX}r$

(d) No, take the path $s_2 \to s_4 \to s_2 \to s_4 \to s_2 \to s_4$.... When you have s_2, q is false, when you have s_4 q is true. So this cannot satisfy $\mathbf{G}q$

(e) Yes.
If you start from s_1 you have p and the next states contain a q or r. If you start from s_2 you start with r and then you have already staisfied the condition $q \vee r$ of the $\mathbf{U}$.

(f) No.
If you take the path $s_1 \to s_4 \to s_2$... then this doesn't work.