

Assignment 10 – Sorting - Searching Ordered Data

Write pseudo-code not Java for problems requiring code. You are responsible for the appropriate level of detail. For all the questions in this set, assume you are working in arrays.

1. How many comparisons and interchanges (in terms of file size n) are performed by Simple insertion sort for the following files:

i) A sorted file

ii) A file that is sorted in reverse order (that is, from largest to smallest)

iii) A file in which $x[0]$, $x[2]$, $x[4]$... are the smallest elements in sorted order, and in which $x[1]$, $x[3]$, $x[5]$... are the largest elements in sorted order, e.g. [3 14 5 15 9 18 11 19].

i. $n-1$ comparisons, zero interchanges

The very first element is not compared to anything. Every subsequent element is just compared to the prev inserted one just to confirm it is bigger.

ii. Comparison and interchange count is the same: $(n^2 - n)/2$

For the first element, there are no comparisons or interchanges, it is the the first node in the list. For the i th element, it is compared to the $(i-1)$ elements and interchanges with all of them. So,
 $1 + 2 + 3 + \dots + n-1 = (n-1)*n/2 = (n^2 - n)/2$

iii.

Using the example, this is how the resulting list would be built

3	0 comparisons, 0 interchanges
3 14	1, 0
3 5 14	2, 1
3 5 14 15	1, 0
3 5 9 14 15	3, 2
3 5 9 14 15 18	1, 0
etc.	

Assume n is even. For $n = 2k$, the number of comparisons is
 $(1 + 2 + \dots + k) + (k - 1)$
 $= k(k + 1)/2 + k$
 $= n^2/8 + 3n/4$

The number of interchanges is

$$1 + 2 + \dots k-1$$
$$= (k-1) * k / 2$$
$$= n^2/8 - n/4$$

For odd n , the number of comparisons is just 1 more than for $(n-1)$ and the number of interchanges is the same.

2. How many comparisons and interchanges (in terms of file size n) are performed by Shell Sort using increments 2 and 1 for the following files:

i) A sorted file

ii) A file that is sorted in reverse order (that is, from largest to smallest)

iii) A file in which $x[0], x[2], x[4] \dots$ are the smallest elements in sorted order, and in which $x[1], x[3], x[5] \dots$ are the largest elements in sorted order, e.g. [3 14 5 15 9 18 11 19].

i. Assume $n = 2k$. First split the array in to two pieces

3 5 9 11 14 15 18 19

3 9 14 18
5 11 15 19

Using the result from 1a, the number of comparisons is
 $(k-1) + (k-1) + (n-1)$
 $= 2n - 3$

For odd n , $n = 2k + 1$, split into two pieces of size k and $k+1$
 $(k-1) + k + (n-1)$
 $= 2n - 3$

The number of interchanges is 0

ii. Assume n is even

ex. 19 18 15 14 11 9 5 3

Split to two arrays

19 15 11 5
18 14 9 3

3. Determine which of the following sorts is most efficient. Consider if the data is small and simple or larger and more complex.

a) simple insertion sort

b) straight selection sort

c) bubble sort

4. Determine the number of comparisons (as a function of n and m) that are performed in merging two ordered files a and b of sizes n and m , respectively, by the merge method presented in the lecture, on each of the following sets of ordered files:

a. $m=n$ and $a[i] < b[i] < a[i+1]$, e.g. $a=[6, 9, 12, 15, 29, 37]$ and $b=[8, 10, 14, 25, 33, 45]$

b. $m=n$ and $a[n] < b[1]$, e.g. $a=[2, 5, 9]$ and $b=[12, 14, 16]$

$a[i]$ refers the value in position i of file a , etc.

5. Determine the number of comparisons (as a function of n and m) that are performed in merging two ordered files a and b of sizes n and m , respectively, by the merge method presented in the lecture, on each of the following sets of ordered files:

a. $m=n$ and $a[n/2] < b[1] < b[m] < a[(n/2)+1]$,

e.g. $a = [2, 5, 7, 55, 61, 72]$ and $b = [9, 15, 17, 21, 29, 46]$

b. $m=1$ and $b[1] < a[1]$

c. $m=1$ and $a[n] < b[1]$

$a[i]$ refers the value in position i of file a , etc.

For questions 6 – 9, compare the efficiency of using sequential search on an ordered table of size n and an unordered table of the same size for the key key :

6. If no record with the key key is present

7. If one record with the key key is present and only one is sought.

8. If more than one record with the key key is present and it is desired to find only the first

9. If more than one record with the key key is present and it is desired to find them all.