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EN.605.202.87.SP18 Data Structures
Homework Assignment 7
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Assignment 7 – Trees

1. How many ancestors does a node at level n in a binary tree have? Provide justification.

The answer is n. Unlike humans, nodes in a binary tree have zero or one parents. The root node, at level 0, has 0. A child of the root node, in level 1, has 1. To complete an inductive proof one would need to show that a child node has exactly one more ancestor than its parent (if the parent exists).

Given a non-root node P at level n with ancestor set A(P) and that P has a child C. P is an ancestor of C that is not in the set A(P). We want to prove that A(C) is {A(P) union P}. Assume that there is some other ancestor of C that is not in the set {A(P) union P}, call it X. There would then be a path from C to the root through X and through A(P) which does not include X. That would imply the graph is cyclic which cannot be true for a binary tree.

2. Prove that a strictly binary tree (regular binary tree) with n leaves contains 2n-1 nodes. Provide justification.

The simplest regular binary tree available is a tree with one node (the root) and no children. It is a leaf node, there is 1, and the total number of nodes is 2(1) – 1 = 1. In any regular binary tree, a leaf node can have two children added and the tree is still regular. Both those children would be leaf nodes. So, if before it had n leaf nodes, now it has n less 1 (for the new parent) plus two (for the new children), or <u>n + 1</u>. The total number of nodes has increased by 2; if n increases by 1 the number of leaves increases by 2. So, it is true for n = 1. And if we assume it is true for n = k, it is also true for k + 1.

3. Explain in detail that if m pointer fields are set aside in each node of a general m-ary tree to point to a maximum of m child nodes, and if the number of nodes in the tree is n, the number of null child pointer fields is n*(m-1)+1.

Use 3-ary complete tree as an example.

```
      Tree         Level    Nodes in Level    Nodes Total
        r            0          1 = 3^0           1 = [3^(1+0) – 1]/(3-1)
    /   |   \
   /    |    \
  a     b     c      1          3 = 3^1           4 = [3^(1+1) – 1]/(3-1)
 /|\   /|\   /|\
d e f g h i j k l    2          9 = 3^2           13 = [3^(1+2) – 1]/(3-1)*
```

The total number of nodes in a <u>complete</u> m-ary tree with height k is
[m^(k + 1) – 1] / (m – 1)

The total number of leaves in the same m-ary tree is
m^k

The number of NULL pointers in this tree is

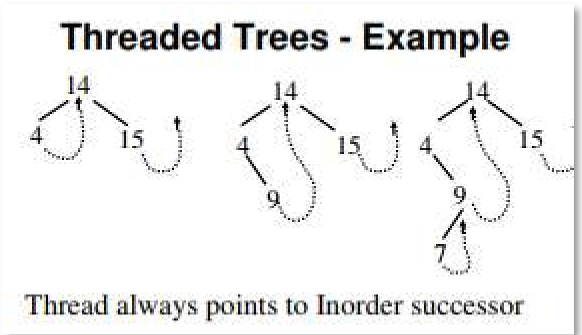
```
m^k * m = m^(k + 1)
        = m^(k + 1) – 1 + 1
        = [m^(k + 1) – 1]/(m – 1) * (m – 1) + 1
        = n * (m – 1) + 1
```

For every leaf node I remove, I remove it's m NULL pointers but add a NULL pointer for its parent. So, n goes down 1 and the number of NULL pointers goes down m – 1.

*I looked this up...

4. Implement maketree, setleft, and setright for right in-threaded binary trees using the sequential array representation.

```
From lecture: thread always points to in-order successor
              in a right in-threaded tree
```




```
class TreeNode
  DataType  value   // null if the node is not being used
  int       thread  // the zero-indexed pos of the in-order successor

  method init
    value = null
    int = -1        // since value is null, in-order successor is undefined
                    // When value is not null, -1 will mean a null successor
  end-method
end-class

class Tree
  private int size, predetermined constant
  private array arr[] of TreeNodes, zero-indexed
  private boolean isMade = false

  // for zero-indexed
  // children of k are 2k + 1 and 2k + 2
  // parent of n is floor((n-1)/2)

  method init
    // populate array with null TreeNodes
    for i = 0 to size-1
      arr[i] = new TreeNode
    end-for
    isMade = false  // still
  end-method

  method MakeTree(DataType item)
    if isMade "throw error"
    else
      isMade = true
      arr[0].value = item
      arr[0].thread = -1
      here = 0
      phere = -1
  end-method

  method SetLeft(DataType item, int p)  // p the parent index
    // Allow overwriting
    if item is null "throw error: value cannot be null"
    if arr[p].value is null "throw error: parent doesn't exist"
    int c = 2*p + 1
    if !isMade "throw error: create root with MakeTree"
    if c >= size "throw error: array overflow"
```

```
    else
      arr[c].value = item
      arr[c].thread = GetSuccessor(c)
      return c
  end-method

  method SetRight(DataType item, int p)  // p the parent index
    if item is null "throw error: value cannot be null"
    if arr[p].value is null "throw error: parent doesn't exist"
    int c = 2*p + 2
    if !isMade "throw error: create root with MakeTree"
    if c >= size "throw error: array overflow"

    else
      arr[c].value = item
      arr[p].thread = c
      arr[c].thread = GetSuccessor(c)
      return c
  end-method

  private method GetSuccessor(int n)
    if arr[n].value = null "throw error: unused node"
    if n = 0 return null

    int cr = 2*n + 2 // right child
    if arr[cr].value != null return cr

    int p = floor((n-1) / 2)  // parent
    if n is odd (this is a left node) return p
    else return GetSuccessor(p)
  end-method

end-class
```

5. Implement inorder traversal for the right in-thread tree in the previous problem.

```
// Part of tree class, so private data available
method TraverseInorder(function F)
  boolean[] done = new boolean[size]
  for i = 0 to size - 1
    done[i] = false

  int here = 0  // start at root
  int cl = 1    // left child

  do while here > -1
    if arr[cl].value != null AND !done[cl]
      here = cl
      cl = 2*here + 1
      continue // jump to next iteration

    F(arr[here].value)  // DO STUFF
    done[here] = true
    here = arr[here].thread
    cl = 2*here + 1
  end-while
end-method
```

6. Define the Fibonacci binary tree of order n as follows: If n=0 or n=1, the tree consists of a single node. If n>1, the tree consists of a root, with the Fibonacci tree of order n-1 as the left subtree and the Fibonacci tree of order n-2 as the right subtree. Write a method that builds a Fibonacci binary tree of order n and returns a pointer to it.

```
class TreeNode
  DataType value
  TreeNode left = null
  TreeNode right = null
end-class

function FibTree(int n)
  if n < 0 "throw error and exit" else...

  TreeNode t = new TreeNode
  if n == 0 or n == 1
    return t  // right and left already init. to null

  else
    t.left = FibTree(n - 1)
    t.right = FibTree(n - 2)
    return t

end-function
```

7. Answer the following questions about Fibonacci binary tree defined in the previous problem.

a) Is such a tree strictly binary?

```
Discussion forum clarified this is to be read "is such a tree regular"?
Regular meaning that every node has 0 or 2 children.

Yes, FibTree(0) and FibTree(1) are leaves with no children.

Given a sub-tree S = FibTree(x) x > 1, the root will have two children and those will
be the root nodes of FibTree(x-1) and FibTree(x-2)
```

b) What is the number of leaves in the Fibonacci tree of order n?

```
n   Number of leaves
0     1
1     1
2     2

For 2, it is 2

  2
 / \
1   0

For 3, it is 3

    3
   / \
  2   1
 / \
1   0

For 4, it would be the number of leaves on the FibTree(3) subtree plus the number of
leaves on the FibTree(2) subtree, or 3 + 2 = 5.  The number of leaves will *always* be
the Fibonacci number for n.
```

c) What is the depth of the Fibonacci tree of order n?

```
The left-hand subtree of FibTree n is always FibTree(n - 1).  So, labelling the
vertices based on the n of the function call that defined them you get, for n = 5 for
example

        5
       / \
      4   3...
     / \
    3   2...
   / \
  2   1
 / \
1   0

This one has 5 levels and a height of 4. Because we are always "counting down" by one
on the left had side (and something lower on the right and its children) we will
always have n levels and a height of n-1.
```