北京航空航天大學
BEIHANG UNIVERSITY

Introduction to Computer Science and Programming

Lecture 7

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Outline

- Recap
- More (and better) sorting: Mergesort
- Algorithm complexity analysis
- Debugging Python programs
- Review: Typical mistakes during lab classes

Recap

- What do you remember about last week's lecture?

The formal problem of sorting

- Sorting is a fundamental operation in CS

Input: A sequence of n numbers $\langle a_1, a_2, \ldots, a_n \rangle$.

Output: A permutation (reordering) $\langle a'_1, a'_2, \ldots, a'_n \rangle$ of the input sequence such that $a'_1 \leq a'_2 \leq \cdots \leq a'_n$.

Insertion sort: Python code

```python
def insertion_sort(A):
    for i in range(0,len(A)):
        cur_value = A[i]
        j = i - 1
        while j >= 0 and A[j] > cur_value:
            A[j+1] = A[j]
            j = j - 1
        A[j+1] = cur_value
```

Insertion sort: Python code

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def insertion_sort(A):
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            A[j+1] = A[j]
            j = j - 1
        A[j+1] = cur_value
```

Can we also do InsertSort to the right?

Big-O Notation

Formally speaking, let f, g: $N \rightarrow N$.

Then $\mathbf{f(n) = O(g(n))}$ **iff**

$$\exists n_0 \in N, c \in R.$$
$$\forall n \in N.$$
$$(n \geq n_0 \rightarrow f(n) \leq c \cdot g(n))$$

Intuitively, this means that f(n) is upper-bounded by g(n) aka f(n) is “at most as big as” g(n).

Bubble sort

- Compare all elements pairwise
  - Switch them if they are wrongly ordered

4	8	1	5	3	2	6	7

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- Compare all elements pairwise
  - Switch them if they are wrongly ordered

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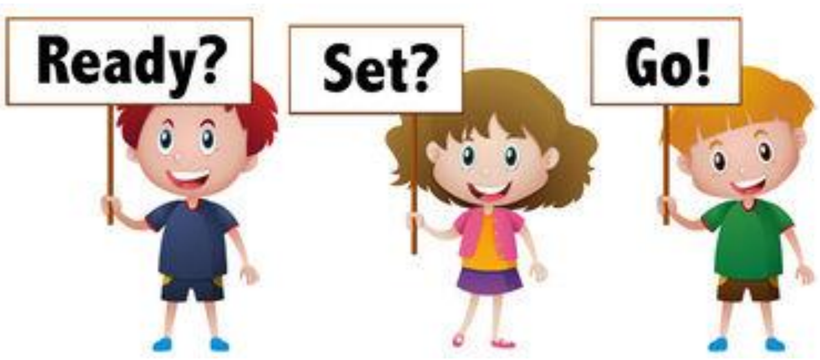
Let’s implement this in Python!

Best case vs. Worst case



1	2	3	4	5	...	n

n	...	5	4	3	2	1

9	7	n	...	1	4	2

Total work: $O(n)$ or $O(n^2)$ or $\Omega(n)$ or $\Omega(n^2)$?

Best case vs. Worst case

The worst-case runtime of insertion sort is $\Theta(n^2)$.

The best-case runtime of insertion sort is $\Theta(n)$.

Usually, we care more about the worst-case time.

Why?

Best case vs. Worst case

The worst-case runtime of insertion sort is $\Theta(n^2)$.

The best-case runtime of insertion sort is $\Theta(n)$.

Usually, we care more about the worst-case time.

We do not know the user's input at runtime, so we need to expect the worst-case.

It's acceptable, albeit not entirely precise, to say the runtime of insertion sort is $\Theta(n^2)$.

Best case vs. Worst case

The worst-case runtime of bubble sort is $\Theta(n^2)$.

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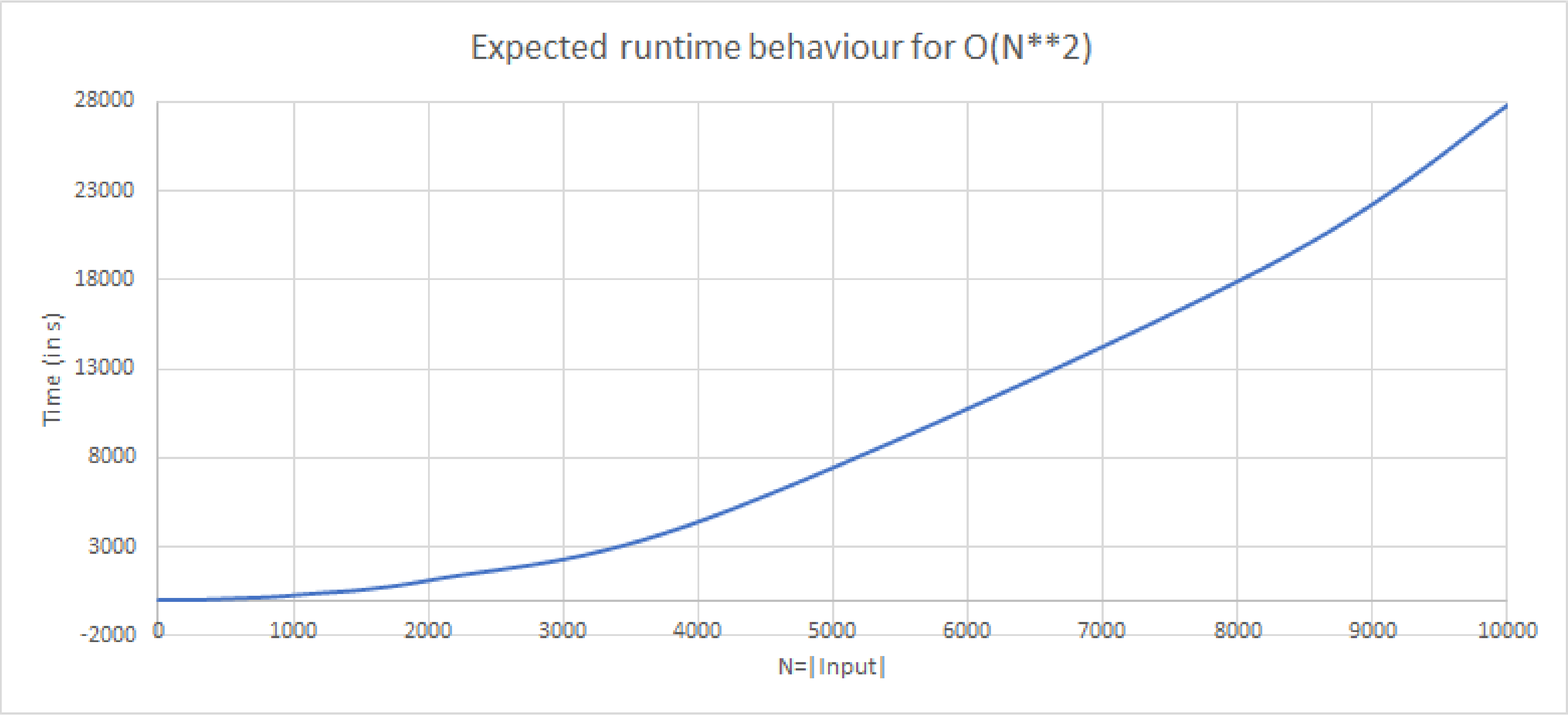


Is there any obvious problem with sorting so far from your side?

- All algorithms we know have a worst-case time complexity of O(N**2)
  - This is not good for large N …
    - If you double the size of the input, the runtime goes up by a factor of roughly four!
    - If you increase the size of the input by ten, the runtime goes up by a factor of roughly 100!
- This algorithm **does not scale** well for our problem

Can we do (significantly) better?

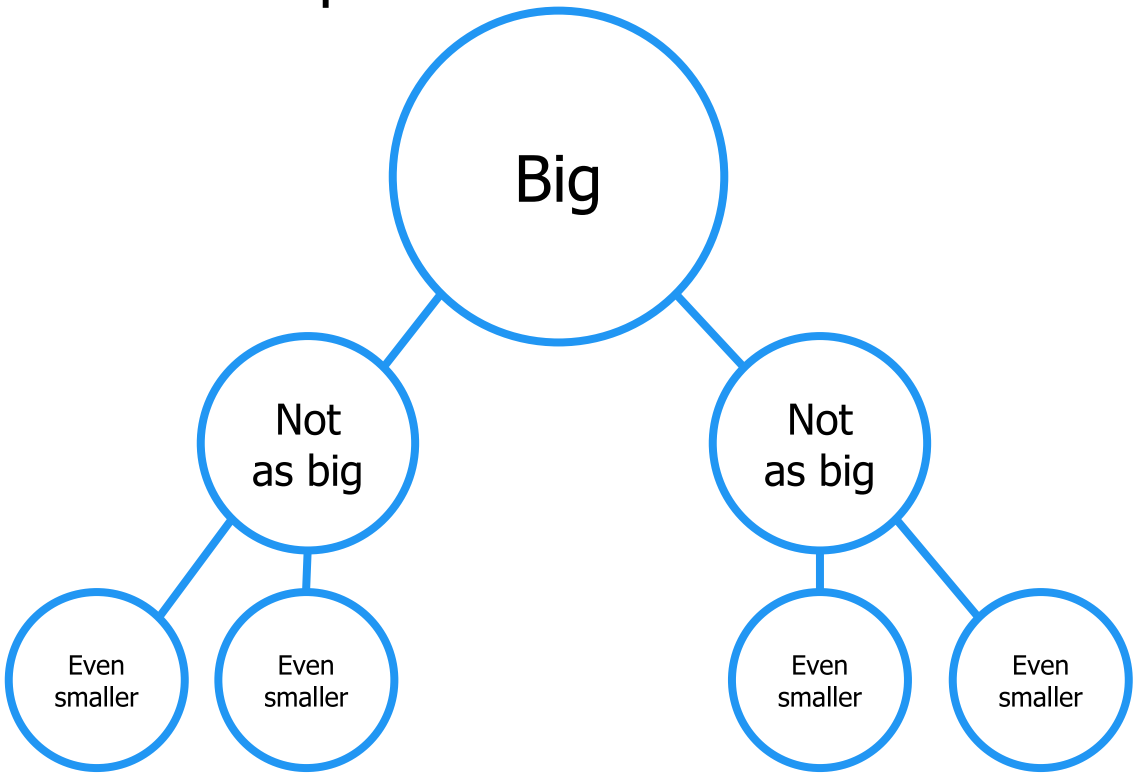
4	8	1	5	3	2	6	7

Divide and Conquer

Divide and Conquer

Divide: break current problem into smaller problems.

Conquer: solve the smaller problems and collate the results to solve the current problem.



Mergesort

Let’s use divide and conquer to improve upon insertion sort!

4	8	1	5	3	2	6	7

Let’s sort an unsorted list of numbers **`A`**.

1	4	5	8	2	3	6	7

Recursively sort each half, **`A[0:3]`** and **`A[4:7]`**, separately.

1	2	3	4	5	6	7	8

Merge the results from each half together.

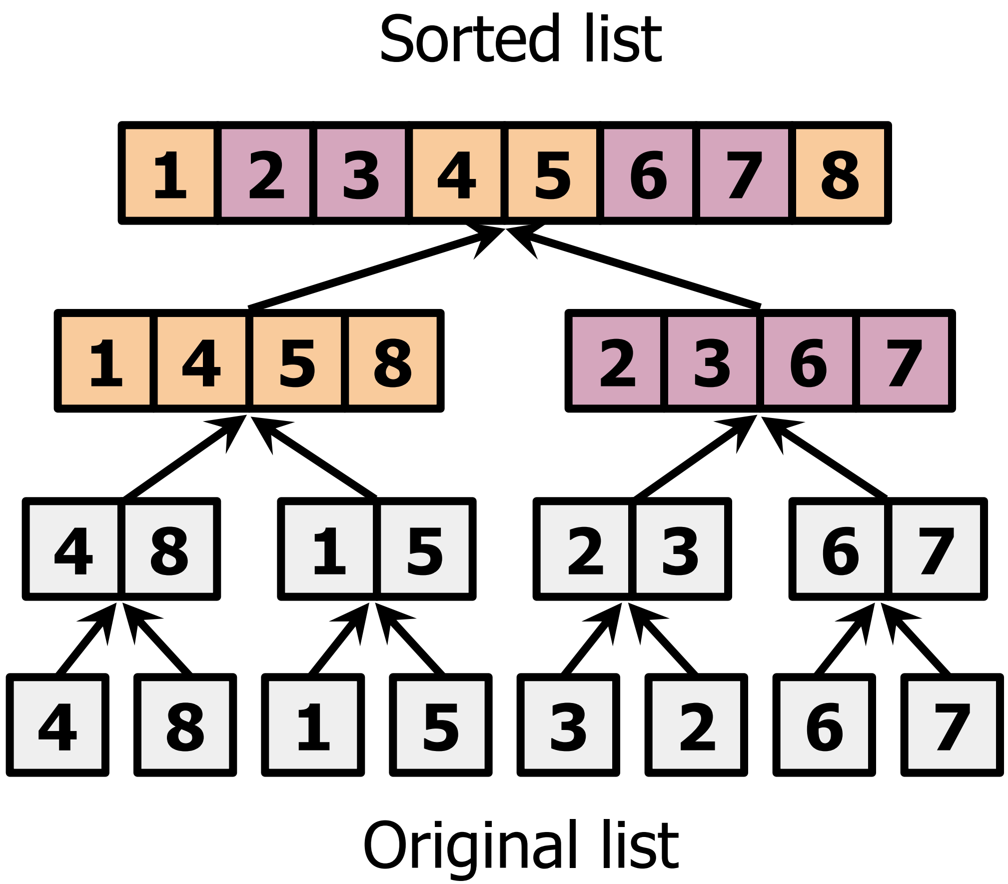
Mergesort: Pseudocode

```
algorithm mergesort(list A):
  if length(A) ≤ 1:
    return A
  let left = first half of A
  let right = second half of A
  return merge(
    mergesort(left),
    mergesort(right)
  )
```

Total work: O(???)

Mergesort

Tracing the recursive calls …



Mergesort

Question 1 How do we prove this algorithm always sorts the input list?

Question 2 How efficiently does this algorithm sort the input list?

Proving Correctness

- Consider a list of length k.
  - If k is 0 or 1, `mergesort` correctly sorts the list since an empty or single-element list is already sorted (base case).
  - Now suppose `mergesort` correctly sorts lists of length 1 to k-1. Since `left` and `right` must have lengths 1 to k-1, `mergesort` correctly sorts these lists.
- By construction, `merge` joins the elements from the two sorted lists into a single sorted list of length k, which it returns.
  - Thus, `mergesort` returns the elements of the original list, but in sorted order (inductive case).
- In the top recursive call, `mergesort` sorts the original array of length n (conclusion).

Analyzing Runtime

Let $T(n)$ represent the runtime of `mergesort` on a list of length n.

$T(n/2)$ is the runtime of `mergesort` on a list of length n/2.

$T(6881441)$ is the runtime of `mergesort` on a list of length 6,881,441.
$T(\lceil n/17 \rceil)$ is the runtime of`mergesort` on a list of length $\lceil n/17 \rceil$.

Recall that `mergesort` on a list of length n calls `mergesort` once for `left` and once for `right`, which costs $\mathbf{T(\lceil n/2 \rceil) + T(\lfloor n/2 \rfloor)}$.

After that, it calls `merge` on the two sublists, which costs $\mathbf{\Theta(n)}$.

Here's our first **recurrence relation**:

$$T(0) = \Theta(1)$$

$$T(1) = \Theta(1)$$

$$T(n) = T(\lceil n/2 \rceil) + T(\lfloor n/2 \rfloor) + \Theta(n)$$

Analyzing Runtime

- A **recurrence relation** is a function or sequence whose values are defined in terms of earlier values.
- Here, we've written a recurrence relation for the runtime of mergesort. But we could have just as easily written one to describe something else recursive.
- For instance, the Fibonacci sequence can be defined by its recurrence relation $T(n) = T(n-1) + T(n-2)$, where $T(n)$ represents the n^{th} element of the sequence.

Analyzing Runtime

How do we solve our recurrence relation?

Assumption 1: First, it's helpful to assume that n is a power of two.

$$\cancel{T(0) = \Theta(1)}$$

$$T(1) = \Theta(1) = c_1$$

$$T(n) = T(\lceil n/2 \rceil) + T(\lfloor n/2 \rfloor) + \Theta(n)$$
$$= 2T(n/2) + c_2 n$$

Assumption 2: Let $c = \max\{c_1, c_2\}$

$$T(1) \leq c$$

$$T(n) \leq 2T(n/2) + cn$$

Analyzing Runtime

How do we solve our new recurrence relation?

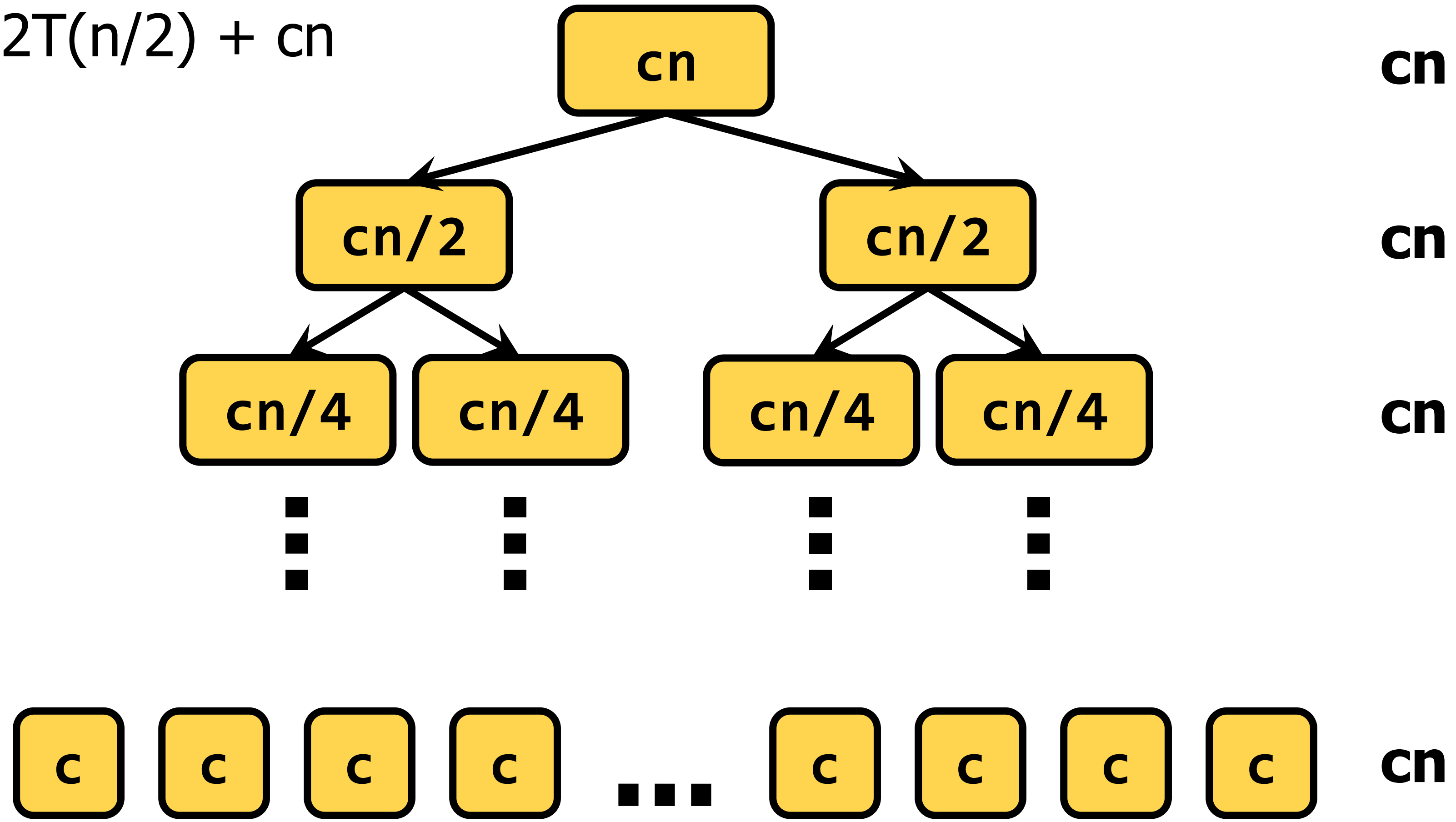
$$T(1) \leq c$$

$$T(n) \leq 2T(n/2) + cn$$

Recursion Tree Method

$T(1) \leq c$

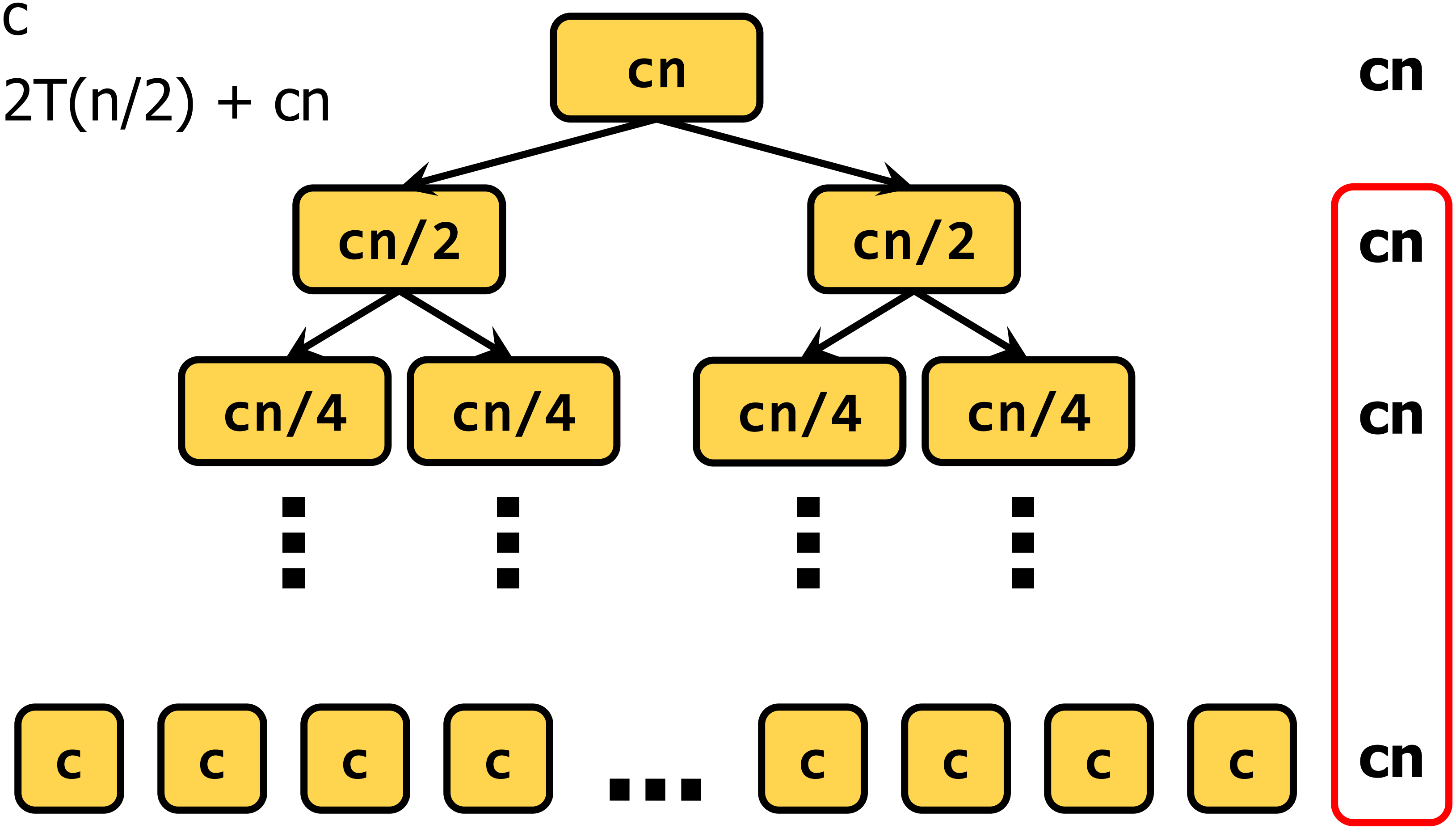
$T(n) \leq 2T(n/2) + cn$



Recursion Tree Method

$T(1) \leq c$

$T(n) \leq 2T(n/2) + cn$



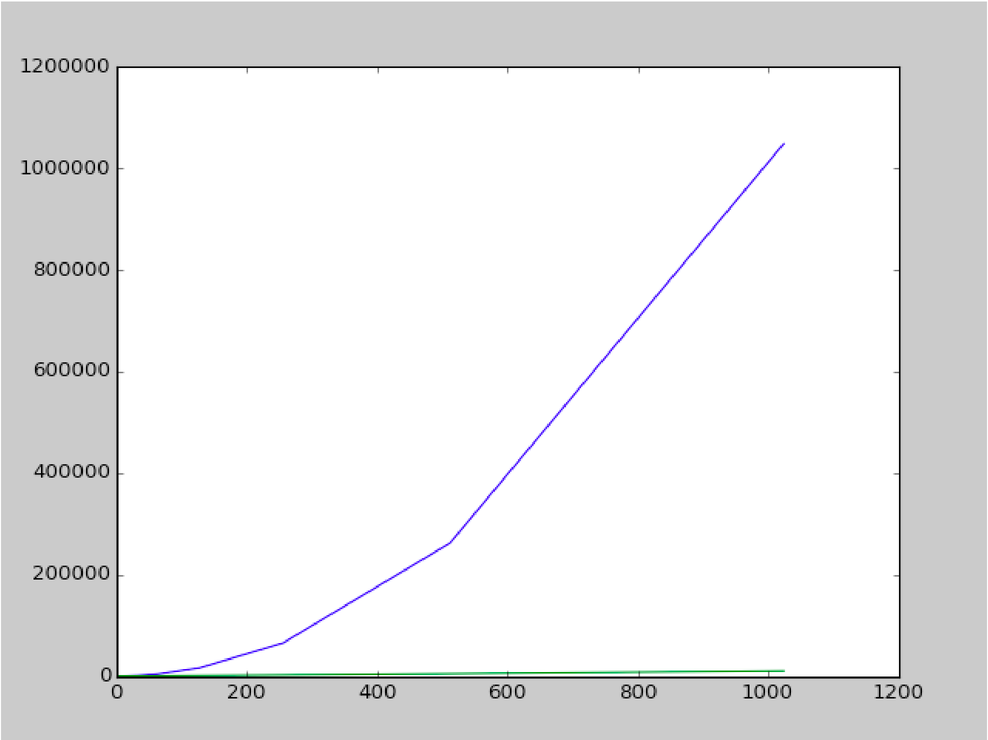
Total work: $cn \log_2 n + cn$

Analyzing runtime

The best and worst-case runtime of `mergesort` is $\Theta(n \log n)$.

The worst-case runtime of `insertion_sort` was $\Theta(n^2)$.

THIS IS A SIGNIFICANT (HUGE) IMPROVEMENT!!



Debugging
Python programs

Program for prime factors

- Break a number n into its prime factors
  - Iterate over each candidate p increasingly, and try to divide the current n (=x) by p

```python
def getPrimeFactors(n):
    result=[]
    x=n
    p=2
    while x!=1:
        if x%p==0:
            result.append(p)
            x=x/p
        else:
            p=p+1
    return(result)



L=getPrimeFactors(20)
print(L)
```

What is the shortest code for this task?

- Break a number n into its prime factors
- Use at most FIVE lines of code …

```
n=20
for x in range(2,n):
    while n%x==0:
        n=int(n/x)
        print(x)
```

Which version is better?

- Major goal is clarity of the code
  - Only afterwards, once you obtain working code, you start to *optimize*!
    - Speed, length, etc.
- Remember
  - “Programmers waste enormous amounts of time thinking about, or worrying about, the speed of noncritical parts of their programs, and these attempts at efficiency actually have a strong negative impact when debugging and maintenance are considered. We should forget about small efficiencies, say about 97% of the time: premature optimization is the root of all evil. Yet we should not pass up our opportunities in that critical 3%.”

Donald Knuth

Overview

- Types of Errors:
  - Syntax errors
  - Runtime Errors
  - Logical Errors

- The first two types lead to stack traces!

Understanding stack traces

- They tell you:
  - Where is the syntax/runtime error?
- Do not help for logical errors ☹

```
1  def getPrimeFactors(n):
2      result=[]
3      x=n
4      p=2
5      while x!=1:
6          if x%p==0:
7              result.add(p)
8              x=x/p
9          else:
10             p=p+1
11     return(result)
12
13
14
15 L=getPrimeFactors(20)
16 print(L)
```

```
In [9]: runfile('C:/Users/basti/Downloads/WinPython-64bit-3.6.2.0Qt5/settings/test.py', wdir='C:/Users/basti/Downloads/WinPython-64bit-3.6.2.0Qt5/settings')
Traceback (most recent call last):

  File "<ipython-input-9-181b47aba674>", line 1, in <module>
    runfile('C:/Users/basti/Downloads/WinPython-64bit-3.6.2.0Qt5/settings/test.py', wdir='C:/Users/basti/Downloads/WinPython-64bit-3.6.2.0Qt5/settings')

  File "C:\Users\basti\Downloads\WinPython-64bit-3.6.2.0Qt5\python-3.6.2.amd64\lib\site-packages\spyder\utils\site\sitecustomize.py", line 688, in runfile
    execfile(filename, namespace)

  File "C:\Users\basti\Downloads\WinPython-64bit-3.6.2.0Qt5\python-3.6.2.amd64\lib\site-packages\spyder\utils\site\sitecustomize.py", line 101, in execfile
    exec(compile(f.read(), filename, 'exec'), namespace)

  File "C:/Users/basti/Downloads/WinPython-64bit-3.6.2.0Qt5/settings/test.py", line 15, in <module>
    L=getPrimeFactors(20)

  File "C:/Users/basti/Downloads/WinPython-64bit-3.6.2.0Qt5/settings/test.py", line 7, in getPrimeFactors
    result.add(p)

AttributeError: 'list' object has no attribute 'add'
```

Understanding stack traces

- A stack trace is a nested list of function calls plus an error message
  - Usually, the inner/lowest function call causes the actual error
    - Note that you even get the line number!
  - The type of error is indicated by the error message

```
In [9]: runfile('C:/Users/basti/Downloads/WinPython-64bit-3.6.2.0Qt5/settings/test.py', wdir='C:/Users/basti/Downloads/WinPython-64bit-3.6.2.0Qt5/settings')
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AttributeError: 'list' object has no attribute 'add'
```

What errors/exceptions exist?

List of all exceptions (errors):

http://docs.python.org/3/library/exceptions.html#bltin-exceptions

Two other resources, with more details about a few of the errors:

http://inventwithpython.com/appendixd.html

http://www.cs.arizona.edu/people/mccann/errors-python

Common errors

- IndexError
  - Raised when a sequence subscript is out of range.
- KeyError
  - Raised when a mapping (dictionary) key is not found in the set of existing keys.
- NameError
  - Raised when a local or global name is not found.
- SyntaxError
  - Raised when the parser encounters a syntax error.
- IndentationError
  - Base class for syntax errors related to incorrect indentation.
- TypeError
  - Raised when an operation or function is applied to an object of inappropriate type.
- KeyboardInterrupt
  - Raised when the user hits the interrupt key (normally Control-C or Delete).

Where is the bug? Divide and conquer

Where is the defect (or “bug”)?

- Your goal is to find the one place that it is
- Finding a defect is often harder than fixing it

- Initially, the defect might be anywhere in your program
  - It is impractical to find it if you have to look everywhere
- Idea: **bit by bit reduce the scope of your search**
- Eventually, the defect is localized to a few lines or one line
  - Then you can understand and fix it

- Four ways to divide and conquer:
  1. In the program code
  2. During the program execution
  3. In test cases
  4. During the development history

Divide and conquer in the program code

- Localize the defect to part of the program
- Code that isn't executed cannot contain the defect

Three approaches:

- Use print statements to see which parts are still executed
- Test one function at a time
- Split complex expressions into simpler ones

Example: Failure in

```
result = print(f(list(g(input()))))
```

Change it to

```
user_input=input()
gresult=g(user_input)
print("Result of g:",gresult)
L=list(gresult)
fresult=f(L)
print(fresult)
```

Divide and conquer in the program code

- Localize the defect to part of the program
- Code that isn't executed cannot contain the defect

Three approaches:

- Use print statements to see which parts are still executed
- Test one function at a time
-

Important hint for many of you:

Deactivate user input during debugging!

Change it to

```
user_input=60 #input()
gresult=g(user_input)
print("Result of g:",gresult)
L=list(gresult)
fresult=f(L)
print(fresult)
```

Divide and conquer in execution time via print (or "logging") statements

- A sequence of **`print`** statements is a record of the execution of your program
- The **`print`** statements let you see and search multiple moments in time
- Print statements are a useful technique, in moderation
- Be disciplined
  - Too much output is overwhelming rather than informative
  - Remember the scientific method: have a reason (a hypothesis to be tested) for each print statement
  - Don't *only* use print statements

Divide and conquer in test cases

- Your program fails when run on some large input
  - It's hard to comprehend the error message
  - The log of print statement output is overwhelming
- Try a smaller input
  - Choose an input with some but not all characteristics of the large input
  - Example: duplicates, zeroes in data, …
  - Find a Minimum Working Example (MWE)
    - Everything needed to reproduce the bug should be there, but nothing else

Divide and conquer in development history

- The code used to work (for some test case)
- The code now fails
- The defect is related to some line you changed

- This is useful only if you kept a version of the code that worked (use good names!)
- This is most useful if you have made few changes
- Moral: test often!
  - Fewer lines to compare
  - You remember what you were thinking/doing recently

A metaphor about debugging

If your code doesn't work as expected, then by definition you don't understand what is going on.

- You're lost in the woods.
- You're behind enemy lines.
- All bets are off.
- Don't trust anyone or anything.

Don't press on into unexplored territory -- go back the way you came!
(and leave breadcrumbs!)

Time-Saving Trick: Make Sure you're Debugging the Right Problem

- The game is to go from "working to working"
- When something doesn't work, STOP!
  - It's wild out there!
- FIRST: go back to the last situation that worked properly.
  - Rollback your recent changes and verify that everything still works as expected.
  - Don't make assumptions – by definition, you don't understand the code when something goes wrong, so you can't trust your assumptions.
  - You may find that even what previously worked now doesn't
  - Perhaps you forgot to consider some "innocent" or unintentional change, and now even tested code is broken

A bad timeline

- A works, so celebrate a little
- Now try B
- B doesn’t work
- Change B and try again
- Change B and try again
- Change B and try again

…

A better timeline

- A works, so celebrate a little
- Now try B
- B doesn't work
- *Rollback to A*
- Does A still work?
  - Yes: Find A' that is somewhere between A and B
  - No: You have *unintentionally changed something else*, and there's no point futzing with B at all!

These "innocent" and unnoticed changes happen more than you would think!

- You add a comment, and the indentation changes.
- You add a print statement, and a function is evaluated twice.
- You move a file, and the wrong one is being read
- You're on a different computer, and the library is a different version

Thank you very much!