

## Computer Science and Programming Homework November 16th, 2020

### Task 1 *Sorting Algorithm - Bubble Sort*

Please implement the bubble sort algorithm to sort a given list in ascending order. Compare all elements of a list pairwise, and switch them if they are in wrong order. What's the time complexity of bubble sort?

### Task 2 *Divide and conquer—integer partition*

An integer  $n$  can be written as  $n = m_1 + m_2 + \dots + m_k$ , where  $m_1 \leq m_2 \leq \dots \leq m_k$ ,  $1 \leq m_i \leq n$  and  $m_i \in \mathbb{Z}$ . Thus  $\{m_1, m_2, \dots, m_k\}$  is called a partition of  $n$ . Let the number of different partitions of an integer  $n$  is  $P(n)$ . Please write a python program to compute  $P(n)$ .

**Hint:** In this example, it is hard to find the recursion relation of  $P(n)$ , thus we consider adding another parameter  $t$ , and let  $q(n, t)$  to be the number of partitions whose maximum additive number is not greater than  $t$ . In other words,  $q(n, t)$  is the number of different partitions  $\{m_1, m_2, \dots, m_k\}$ , where  $m_k \leq t$ . It is obviously that  $P(n) = q(n, n)$ . In order to compute  $q(n, t)$ , we can divide all situations into these cases:

1. IF  $n == 0$ , then  $q(0, t) = 0$
2. If  $n == 1$ , then  $q(1, t) = 1$ . Because there is only one partition:  $1 = 1$
3. IF  $t == 1$ , then  $q(n, 1) = 1$ . Because there is only one partition:  $n = 1 + 1 + 1 + \dots + 1$
4. IF  $n < t$ , then  $q(n, t) = q(n, n)$
5. IF  $n \geq t$ , according to whether  $t$  is included in the partition, we can divide it into:
  - (a) IF  $t$  is included, then there are  $q(n - t, t)$  partitions.
  - (b) IF  $t$  is not included, then there are  $q(n, t - 1)$  partitions.

Taken together, if  $n \geq t$ ,  $q(n, t) = q(n - t, t) + q(n, t - 1)$