Beihang University
School of General Engineering

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Wei Lin and Yida Ding

Computer Science and Programming Homework Octorber 19nd, 2020

Task 1 ***Recursion and dictionaries***

The hanoi tower problem, which you have just learnt in the lecture, is an important example for you to understand recursion. In this problem, a number of disks are initially threaded to pole a, forming a tower with the largest disc at the bottom and the smallest disc at the top. One disc may be moved at a time, from the top of one pole to the top of another pole. A larger disc may not be moved on top of a smaller disc. The goal is to move a given number of disks from pole a to pole c with minimum steps.

1. Please implement a recursive function to print out the disk movement instructions for a given number of disks n. For instance, if $n = 2$, the instructions are as follows:

   - Move disk from a to b.
   - Move disk from a to c.
   - Move disk from b to c.

2. Prove in mathematics that the minimum number of steps to move n disks from a to c is given by

   $$F(n) = 2^n - 1 \tag{4}$$

   (The proving process is not required to be submitted.)

3. Now, we want to further explore the hanoi tower problem by adding one pole, namely four poles in all. The minimum number of steps to move n disks from a to d is expressed in Equation 5. The strategy is explained as follows:

   - Move x disks from the top of a to c, with the help of two empty poles, b and d. The number of steps in this procedure is denoted as $G(x)$.
   - Move the remaining $n - x$ disks of a to d, with the help of one empty pole b. The number of steps in this procedure is $2^{n-x} - 1$, which is obtained from Equation 4.
   - Move x disks from c to d, with the help of two empty poles, a and b. The number of steps in this procedure is also $G(x)$.

   Basically, $G(n)$ is determined by the parameter x. Please write a function to determine the minimum number of steps given the number of disks n.

   $$G(n) = 2G(x) + 2^{n-x} - 1 \quad (1 \leq\ x < n) \tag{5}$$

   Hint: When $n = 1$ and $n = 2$, $G(n)$ is easy to determined, just $G(1) = 1, G(2) = 3$.