

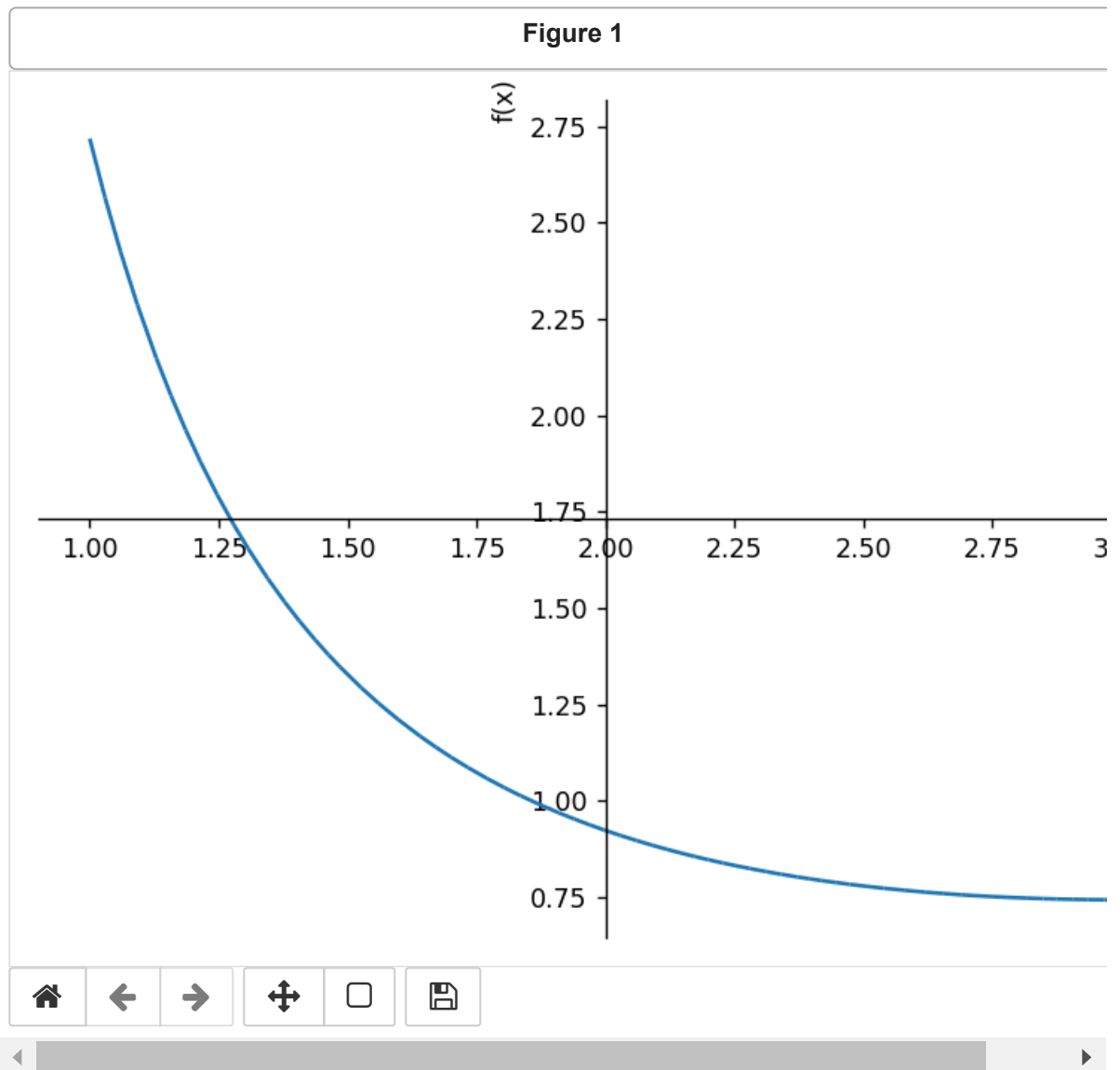
Double-click to insert team members' names here: Martin William Schramm, Harold Wu, Yida Zou

```
In [25]:  from numpy import *  
import sympy as sp
```

#1a Plot function

```
In [26]:  matplotlib notebook
```

```
In [27]:  x=sp.symbols('x')  
F=sp.exp(x)/x**3  
Fdef=sp.integrate(F,(x,1,3))  
sp.plot(F,(x,1,3))
```



```
Out[27]: <sympy.plotting.plot.Plot at 0x24cd7f00160>
```

#1b Left Endpoint Approximation

```
In [28]: ▶ x=sp.symbols('x')
d_x=(3-1)/100
x_i=arange(1,3,d_x)
y_i=exp(x_i)/x_i**3
LI=sum(y_i)*d_x
print("The approximation with 100 left endpoint rectangles is",LI)
```

The approximation with 100 left endpoint rectangles is 2.294111486034061

## #2 Right Endpoint Approximation

```
In [29]: ▶ x=sp.symbols('x')
d_x=(3-1)/100
x_i=arange(1+d_x,3+d_x,d_x)
y_i=exp(x_i)/x_i**3
RI=sum(y_i)*d_x
print("The approximation with 100 right endpoint rectangles is",RI)
```

The approximation with 100 right endpoint rectangles is 2.254624024963537

## #3 Midpoint Approximation

```
In [30]: ▶ x=sp.symbols('x')
dx=(3-1)/100
x_i=arange(1+dx/2,3-dx/2,dx)
y_i=exp(x_i)/x_i**3
midSum100 = sum(y_i)*dx
print("The approximation with 100 midpoint rectangles is",midSum100)

#PART B
avg = (LI+RI)/2
print("The average of the left and right endpoint approximations is",avg)
print("This is approximately equal to the midpoint approximation")
```

The approximation with 100 midpoint rectangles is 2.2740959634770928

The average of the left and right endpoint approximations is 2.2743677554987993

This is approximately equal to the midpoint approximation

## #4ab Trapezoid Approximation

```
In [31]: ❏ from scipy.integrate import trapz
x=sp.symbols('x')
dx=(3-1)/100
x_i=arange(1,3+dx,dx)
y_i=exp(x_i)/x_i**3
trapSum100 = sum(y_i)*dx
print("The approximation with 100 trapezoid rectangles is",trapSum100)
#PART B
avg = (LI+RI)/2
print("The average of the left and right endpoint approximations is",avg)
print("This is not as equal to the trapezoid approximations")
```

The approximation with 100 trapezoid rectangles is 2.3089896615327183

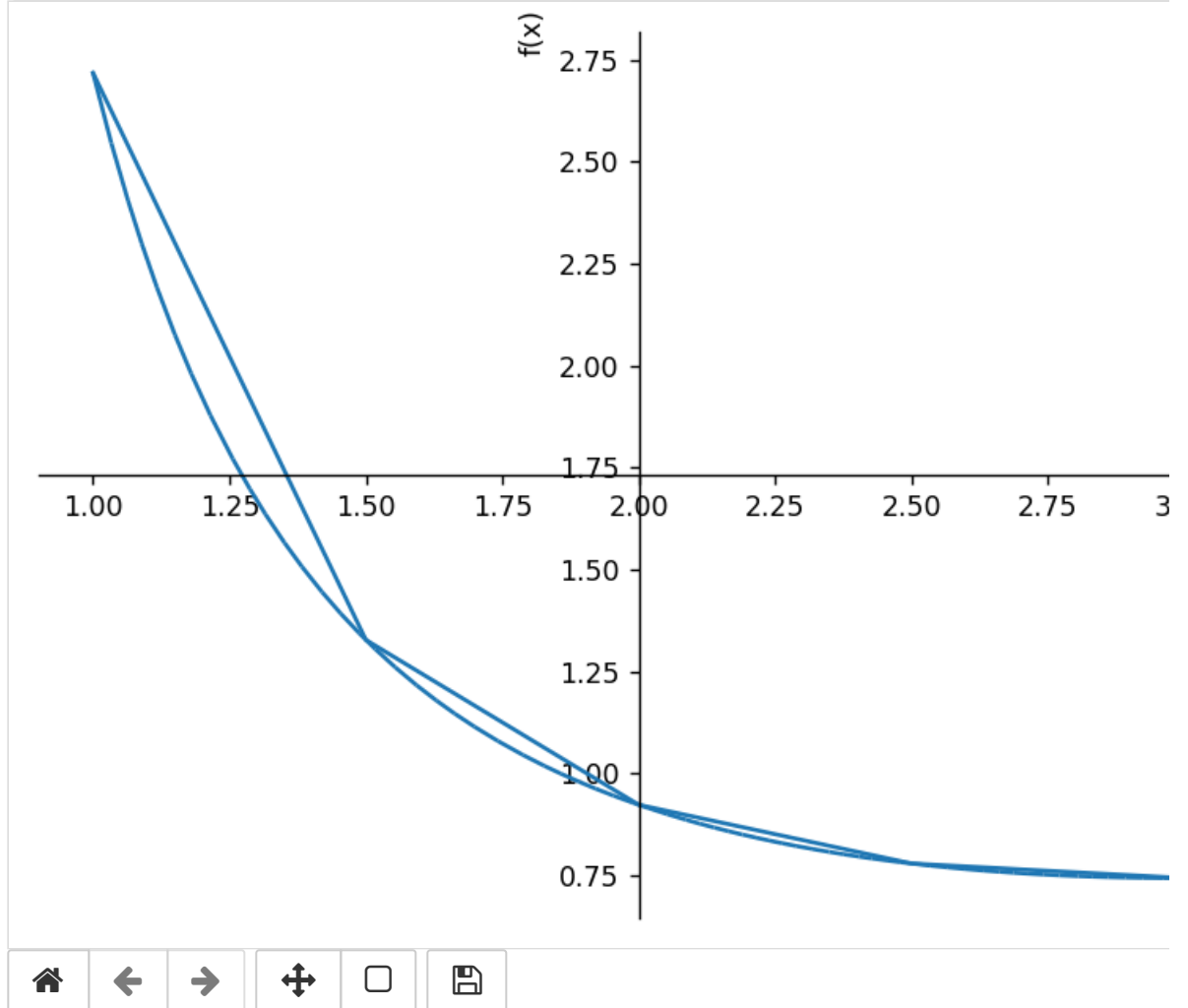
The average of the left and right endpoint approximations is 2.2743677554987993

This is not as equal to the trapezoid approximations

#4c Illustrate Trapezoid Approximation

```
In [32]: x=sp.symbols('x')
f=sp.exp(x)/x**3
sp.plot(f,(x,1,3))
xp=[1,1.5,2,2.5,3]
yp=[f.subs({x:i}) for i in xp]
import matplotlib.pyplot as plt
plt.plot(xp,yp)
```

Figure 2



Out[32]: [<matplotlib.lines.Line2D at 0x24cd95d4850>]

#### #5a Simpsons Rule

```
In [33]:  from scipy.integrate import simps
          dx = (3-1)/100
          xi = arange(1, 3+dx, dx)
          yi = exp(x_i)/x_i**3
          simpsum = simps(yi, xi)
          print("The Simpson method approximation is "+str(simpsum)+".")
```

The Simpson method approximation is 2.274186633030346.

#### #5b Errors in Approximations

```
In [34]:  a = 2.27418655599653 #actual
          print("Left Approximation:", fabs(a-LI))
          print("Right Approximation:", fabs(a-RI))
          print("Midpoint Approximation:", fabs(midSum100-LI))
          print("Trapezoid Approximation:", fabs(trapSum100-LI))
          print("Simpsons Approximation:", fabs(simpsum-LI))
```

Left Approximation: 0.01992493003753104  
Right Approximation: 0.01956253103299277  
Midpoint Approximation: 0.020015522556968257  
Trapezoid Approximation: 0.014878175498657331  
Simpsons Approximation: 0.019924853003714826