

## Analysis and Comparison

```
optimized <- read.csv("optimized_l_stars.csv")
paired <- read.csv("paired_l_stars.csv")
randomized <- read.csv("randomized_l_stars.csv")
exact_scores <- read.csv("exact_scores.csv")
```

### Approximation ratios

calculate the approximation ratios and actual ratios for  $k = 3, 4, 5$

```
optimized.exact <- merge(optimized, exact_scores, by=c("k", "round")) %>%
  mutate(approx_ratio=2-1/k, actual_ratio=score/exact_score)
```

```
paired.exact <- merge(paired, exact_scores, by=c("k", "round")) %>%
  mutate(approx_ratio=2-1/k, actual_ratio=score/exact_score)
```

```
randomized.exact <- merge(randomized, exact_scores, by=c("k", "round")) %>%
  mutate(approx_ratio=2-1/(2*k), actual_ratio=score/exact_score)
```

check if actual ratio is smaller than the approximation ratio for all test cases

```
all(optimized.exact$actual_ratio <= optimized.exact$approx_ratio)
```

```
## [1] TRUE
```

```
all(paired.exact$actual_ratio <= paired.exact$approx_ratio)
```

```
## [1] TRUE
```

```
all(randomized.exact$actual_ratio <= randomized.exact$approx_ratio)
```

```
## [1] TRUE
```

### Optimized l-stars

Compute an average time and score for each distinct pair of  $k$  and  $l$ :

```
optimized <- optimized %>%
  group_by(k, l) %>%
  mutate(avg_time = mean(time), avg_score = mean(score)) %>%
  ungroup()
```

A table summarizes the average score for each pair of  $k$  and  $l$ :

```
optimized %>%
  group_by(k, l) %>%
  summarise(average_score=mean(score)) %>%
  spread(l, average_score) %>%
  kbl(caption = "Average Score") %>%
  kable_paper() %>%
  add_header_above(c(" " = 1, "l" = 3))
```

Table 1: Average Score

| k  | l      |        |        |
|----|--------|--------|--------|
|    | 2      | 3      | 4      |
| 3  | 84.6   | 81.0   |        |
| 4  |        |        | 162.3  |
| 5  | 299.8  | 291.3  |        |
| 7  | 650.7  | 630.4  | 618.1  |
| 9  | 1138.0 | 1142.7 |        |
| 10 |        |        | 1403.1 |
| 11 | 1793.0 | 1791.7 |        |
| 13 | 2509.6 | 2514.8 | 2508.1 |

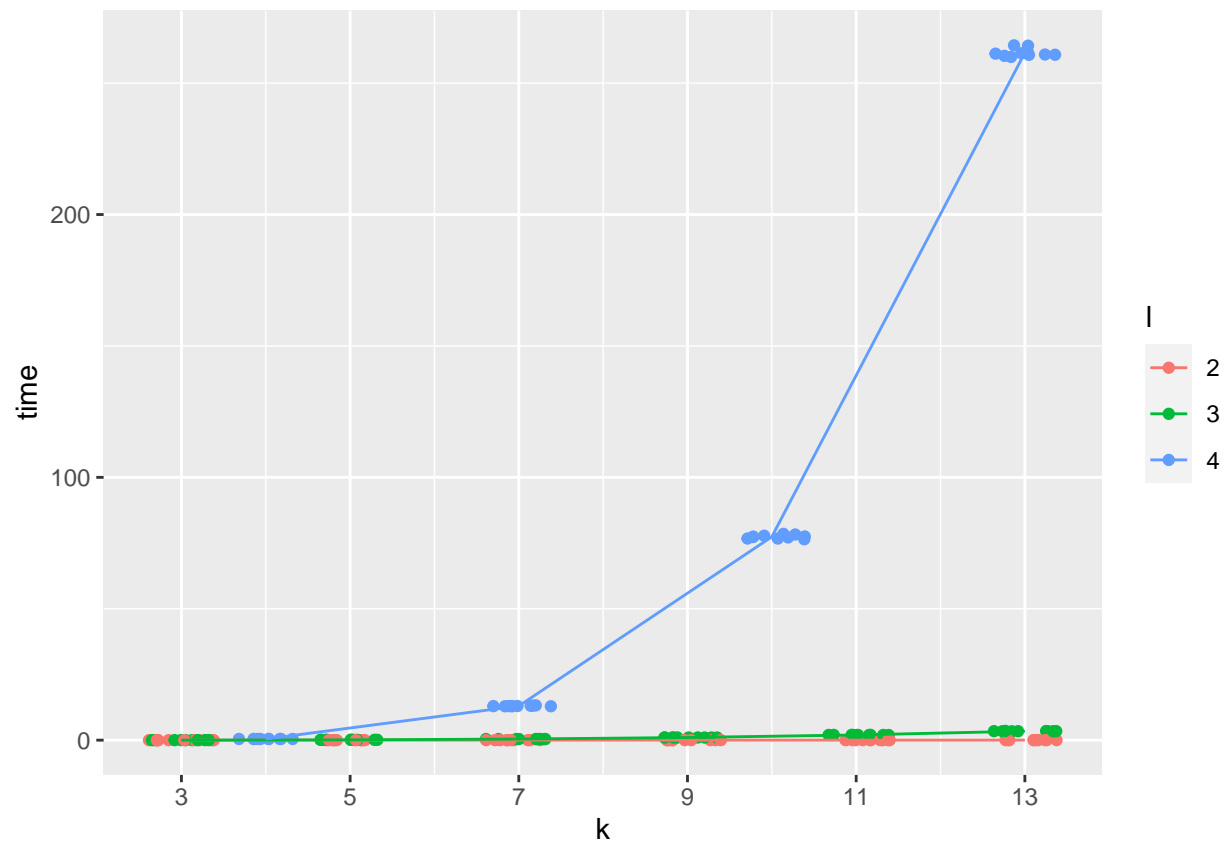
A table summarizes the average running time(s) for each pair of k and l:

```
optimized %>%
  group_by(k, l) %>%
  summarise(average_time=mean(time)) %>%
  spread(l, average_time) %>%
  kable()
```

| k  | 2         | 3         | 4           |
|----|-----------|-----------|-------------|
| 3  | 0.0014525 | 0.0160961 |             |
| 4  |           |           | 0.4570734   |
| 5  | 0.0024602 | 0.1281418 |             |
| 7  | 0.0050193 | 0.4288990 | 13.0049551  |
| 9  | 0.0079710 | 1.0116021 |             |
| 10 |           |           | 77.3848876  |
| 11 | 0.0117119 | 1.9815742 |             |
| 13 | 0.0158879 | 3.4252075 | 261.5766287 |

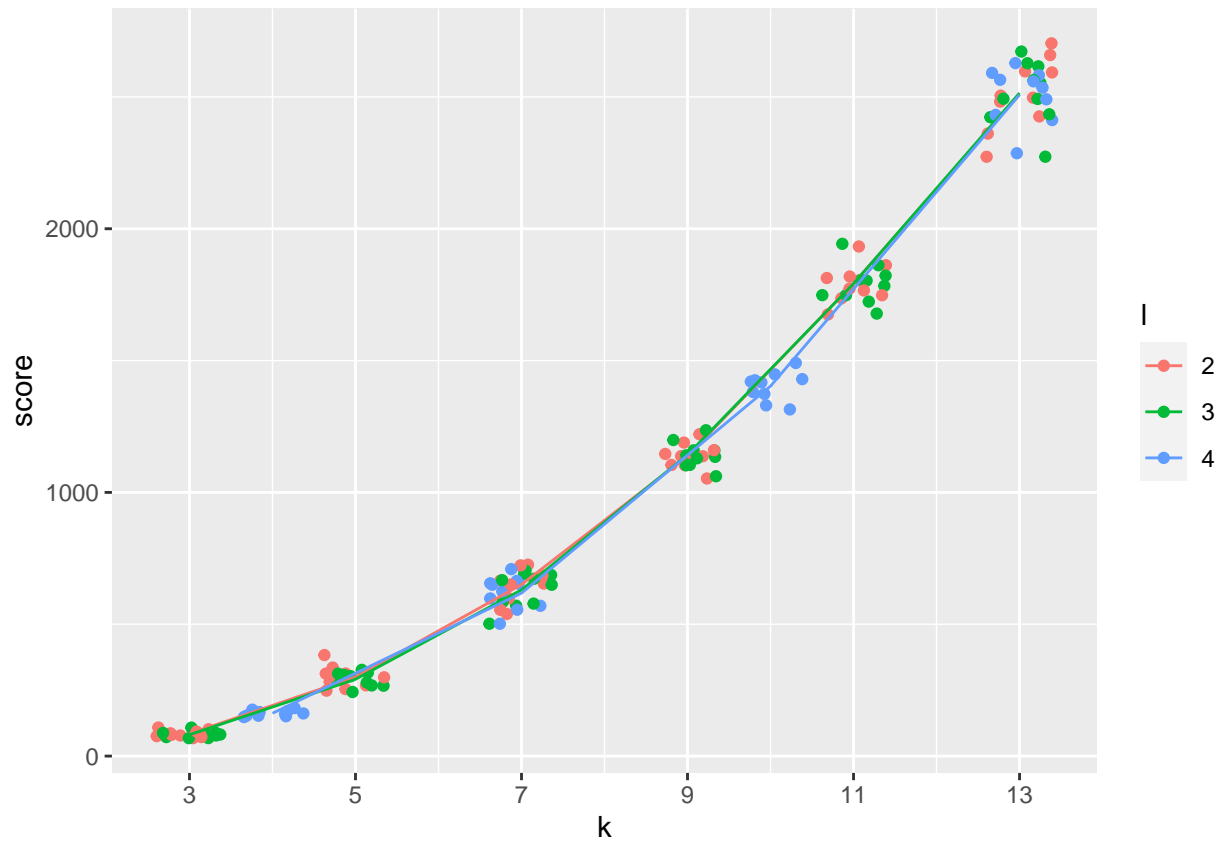
Running time against the number of sequences k

```
optimized %>%
  mutate(l=as.factor(l)) %>%
  ggplot(aes(k, time)) +
  geom_jitter(aes(color=l)) +
  geom_line(aes(k, avg_time, color=l)) +
  scale_x_continuous(breaks=c(3,5,7,9,11,13))
```



SP score against the number of sequences k

```
optimized %>%
  mutate(l=as.factor(l)) %>%
  ggplot(aes(k, score)) +
  geom_jitter(aes(color=l)) +
  geom_line(aes(k, avg_score, color=l)) +
  scale_x_continuous(breaks=c(3,5,7,9,11,13))
```



### (2l-1)-stars

Compute an average time and score for each distinct pair of k and l:

```
paired <- paired %>%
  group_by(k, l) %>%
  mutate(avg_time = mean(time), avg_score = mean(score)) %>%
  ungroup()
```

A table summarizes the average score for each pair of k and l:

```
paired %>%
  group_by(k, l) %>%
  summarise(average_score=mean(score)) %>%
  spread(l, average_score) %>%
  kable()
```

| k  | 2      | 3      |
|----|--------|--------|
| 3  | 84.6   |        |
| 5  | 299.8  | 289.9  |
| 7  | 650.7  |        |
| 9  | 1139.6 | 1139.9 |
| 11 | 1803.9 |        |
| 13 | 2517.8 | 2532.6 |

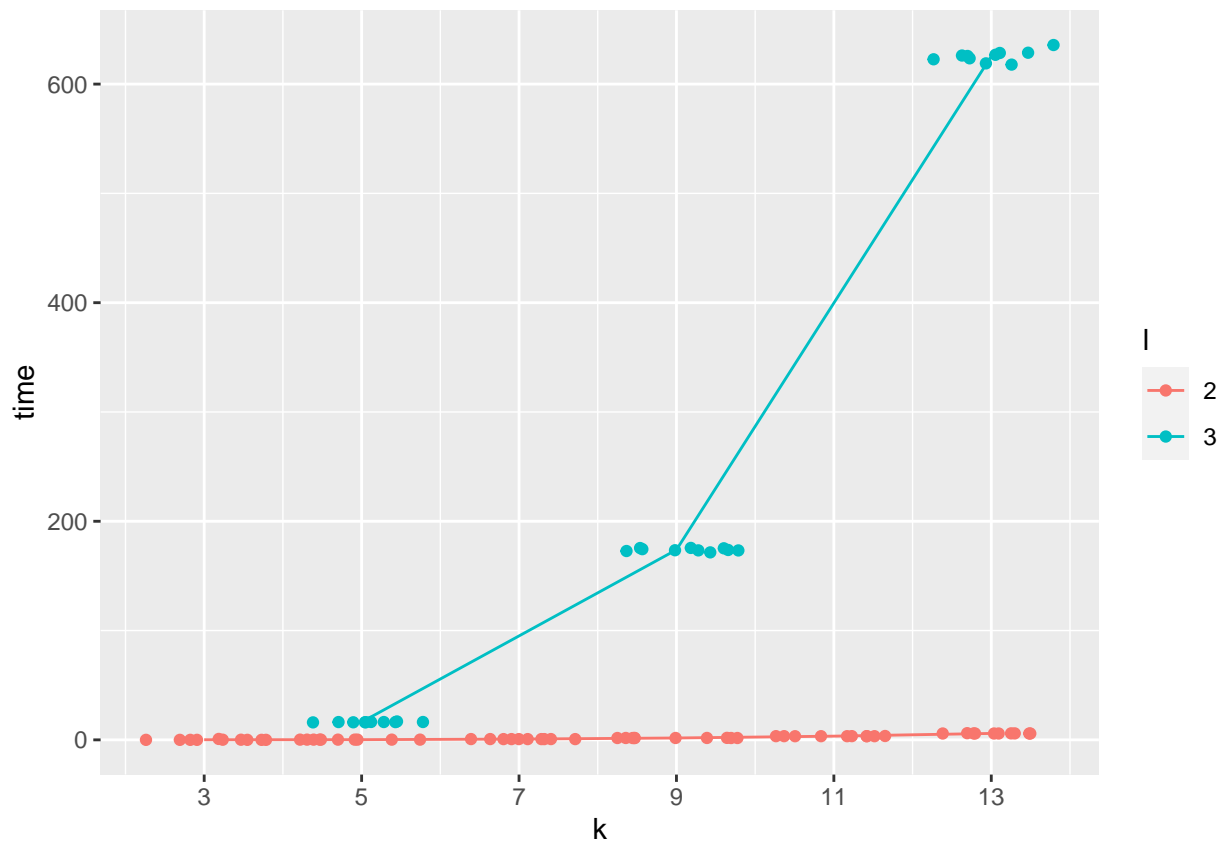
A table summarizes the average running time(s) for each pair of k and l:

```
paired %>%
  group_by(k, l) %>%
  summarise(average_time=mean(time)) %>%
  spread(l, average_time) %>%
  kable()
```

| k  | 2         | 3         |
|----|-----------|-----------|
| 3  | 0.1266516 |           |
| 5  | 0.2049488 | 16.27379  |
| 7  | 0.7079516 |           |
| 9  | 1.7148574 | 173.91189 |
| 11 | 3.3835768 |           |
| 13 | 5.8806980 | 625.46247 |

Running time against the number of sequences k

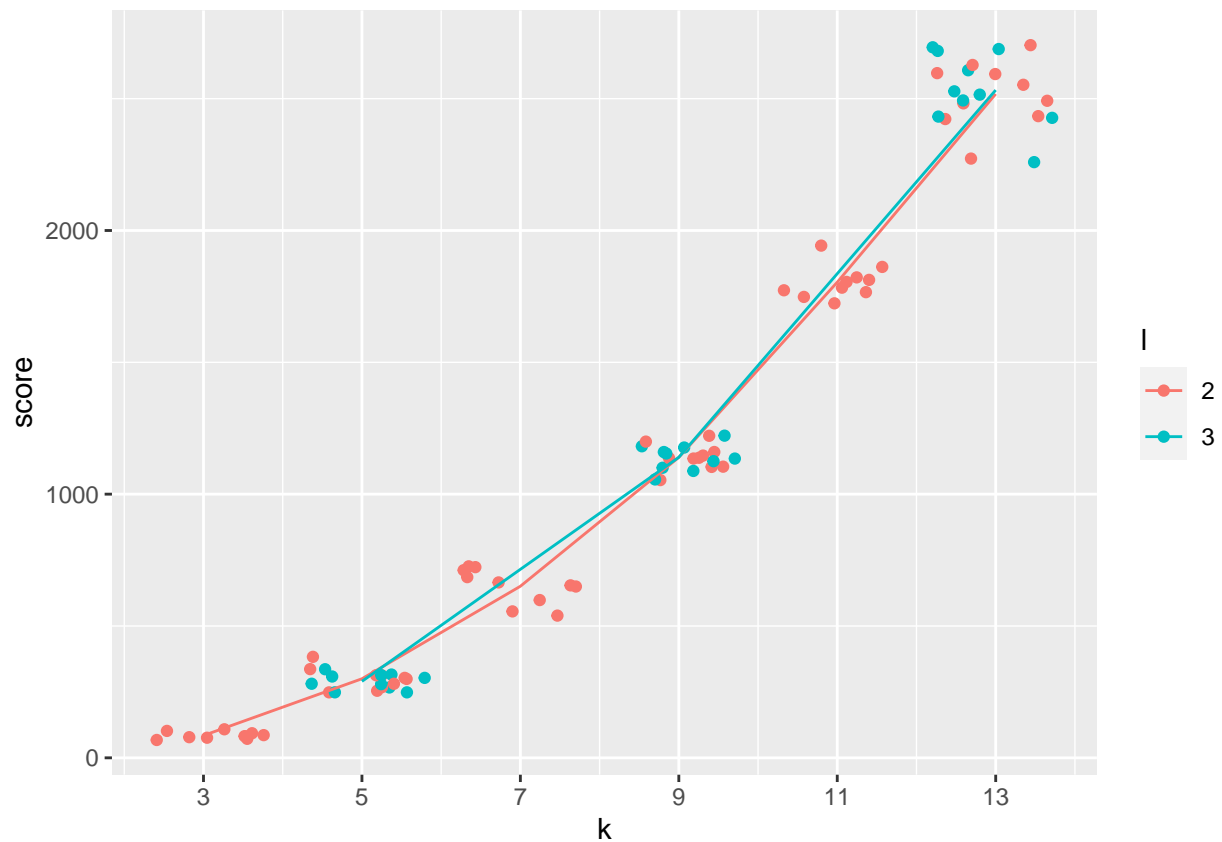
```
paired %>%
  mutate(l=as.factor(l)) %>%
  ggplot(aes(k, time)) +
  geom_jitter(aes(color=l)) +
  geom_line(aes(k, avg_time, color=l)) +
  scale_x_continuous(breaks=c(3,5,7,9,11,13))
```



SP score against the number of sequences k

```
paired %>%
  mutate(l=as.factor(l)) %>%
  ggplot(aes(k, score)) +
```

```
geom_jitter(aes(color=l)) +
geom_line(aes(k, avg_score, color=l)) +
scale_x_continuous(breaks=c(3,5,7,9,11,13))
```



### Randomized l-stars

Compute an average time and score for each distinct pair of k and l:

```
randomized <- randomized %>%
  group_by(k, l, eps) %>%
  mutate(avg_time = mean(time), avg_score = mean(score)) %>%
  ungroup()
```

A table summarizes the average score for each pair of k and l:

```
randomized %>%
  group_by(k, l) %>%
  summarise(average_score=mean(score)) %>%
  spread(l, average_score) %>%
  kable()
```

| k  | 2      | 3        | 4        |
|----|--------|----------|----------|
| 3  | 84.6   | 81.000   |          |
| 4  |        |          | 162.300  |
| 5  | 299.8  | 291.675  |          |
| 7  | 650.7  | 626.650  | 617.950  |
| 9  | 1138.0 | 1136.425 |          |
| 10 |        |          | 1396.750 |
| 11 | 1793.0 | 1794.850 |          |
| 13 | 2509.6 | 2514.825 | 2494.875 |

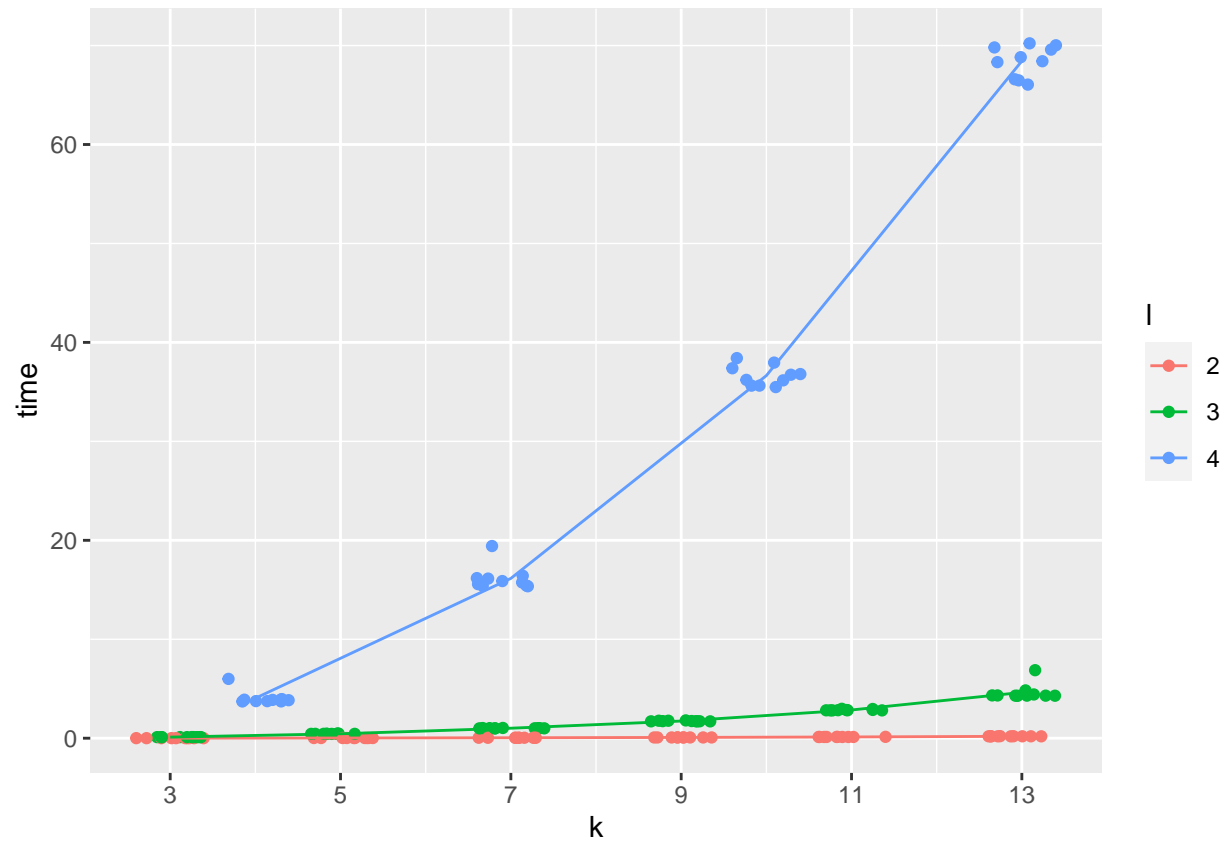
A table summarizes the average running time(s) for each pair of k and l:

```
randomized %>%
  group_by(k, l) %>%
  summarise(average_time=mean(time)) %>%
  spread(l, average_time) %>%
  kable()
```

| k  | 2         | 3         | 4         |
|----|-----------|-----------|-----------|
| 3  | 0.0037410 | 0.0691764 |           |
| 4  |           |           | 2.552521  |
| 5  | 0.0138204 | 0.2952708 |           |
| 7  | 0.0320227 | 0.6974272 | 10.968828 |
| 9  | 0.0557413 | 1.2328964 |           |
| 10 |           |           | 25.947399 |
| 11 | 0.0943120 | 2.0967800 |           |
| 13 | 0.1363006 | 3.1105206 | 47.361191 |

Running time against the number of sequences k (eps=0.1)

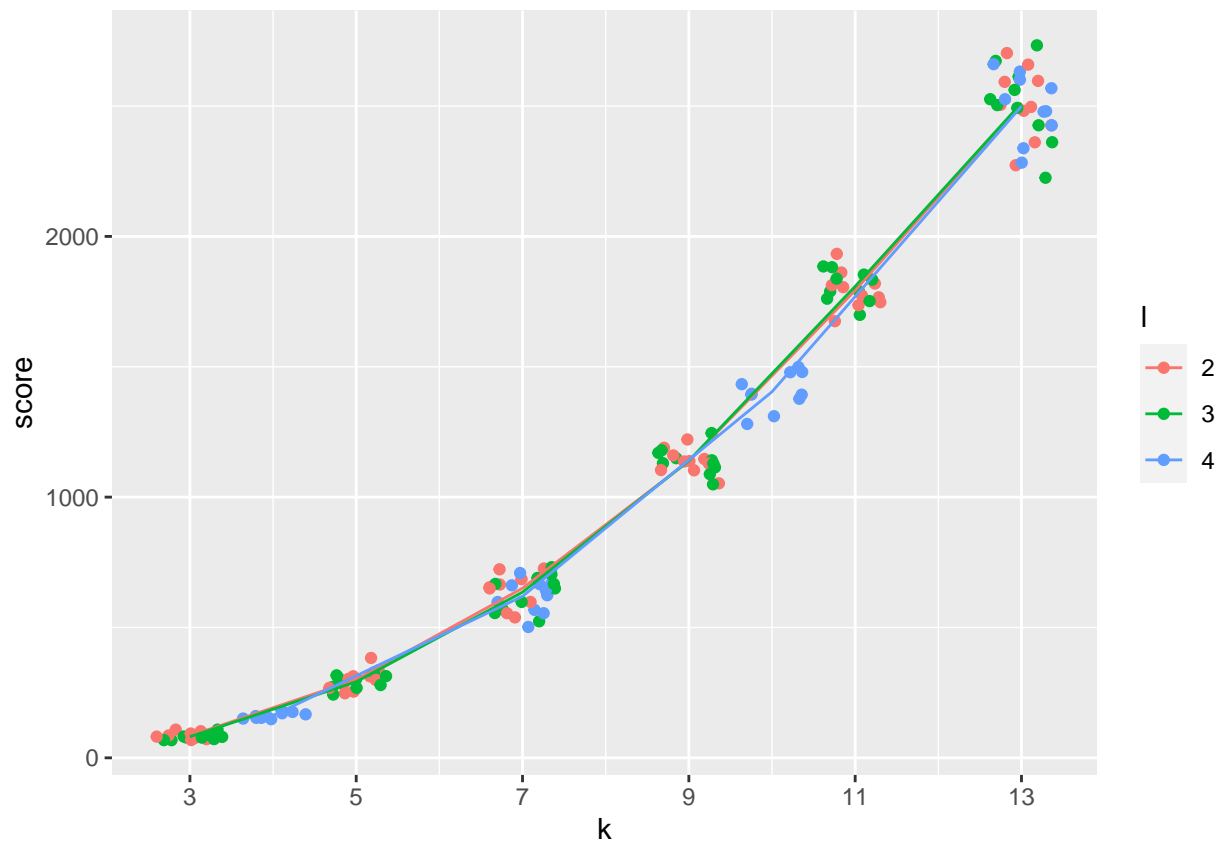
```
randomized %>%
  filter(eps==0.1) %>%
  mutate(l=as.factor(l)) %>%
  ggplot(aes(k, time)) +
  geom_jitter(aes(color=l)) +
  geom_line(aes(k, avg_time, color=l)) +
  scale_x_continuous(breaks=c(3,5,7,9,11,13))
```



SP score against the number of sequences k

```
randomized %>%
  filter(eps==0.1) %>%
  mutate(l=as.factor(1)) %>%
  ggplot(aes(k, score)) +
  geom_jitter(aes(color=l)) +
  geom_line(aes(k, avg_score, color=l)) +
  scale_x_continuous(breaks=c(3,5,7,9,11,13))
```





When  $l = 4$ , a table summarizes the average score for each pair of  $k$  and epsilon:

```
randomized %>%
  filter(l==4) %>%
  group_by(k, eps) %>%
  summarise(average_score=mean(score)) %>%
  spread(eps, average_score) %>%
  kable()
```

| k  | 0.1    | 0.3    | 0.6    | 0.9    |
|----|--------|--------|--------|--------|
| 4  | 162.3  | 162.3  | 162.3  | 162.3  |
| 7  | 619.5  | 616.6  | 616.9  | 618.8  |
| 10 | 1404.1 | 1391.0 | 1405.4 | 1386.5 |
| 13 | 2499.4 | 2500.0 | 2494.3 | 2485.8 |

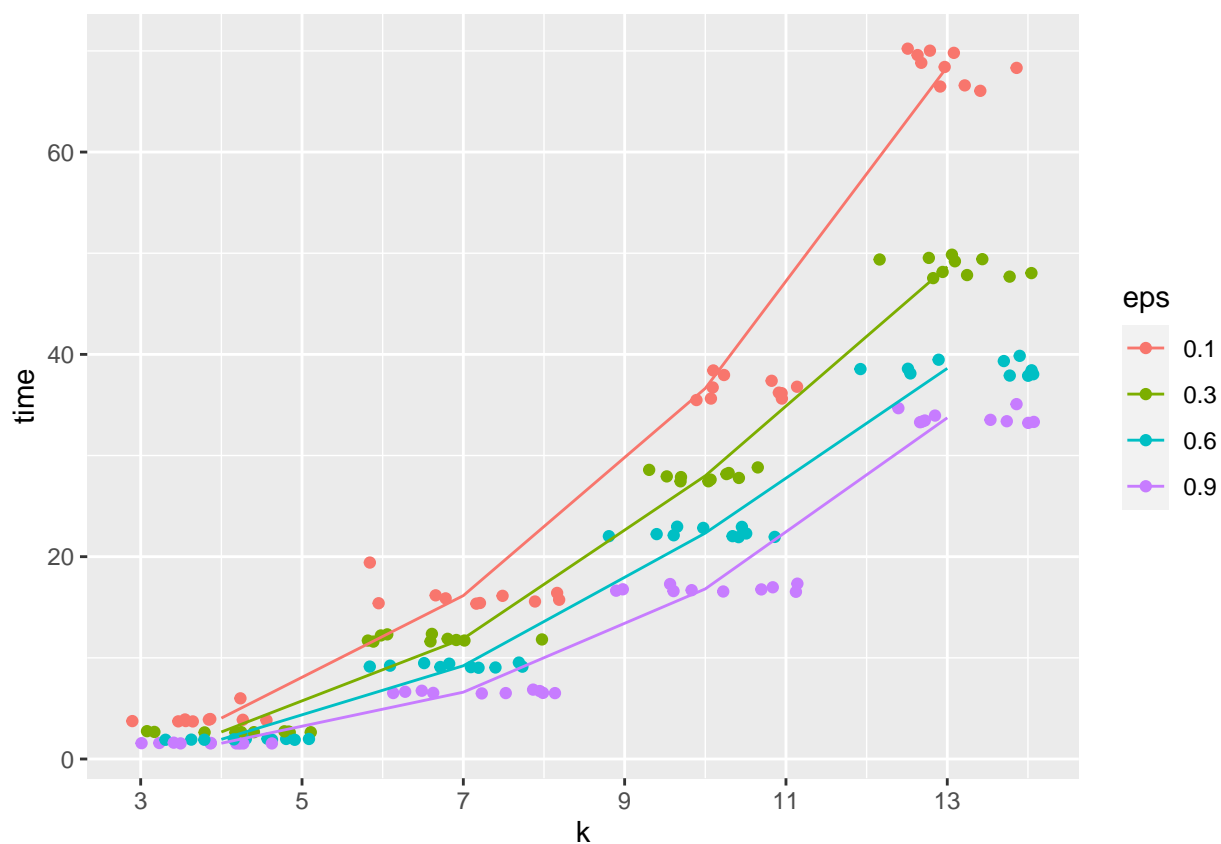
When  $l = 4$ , a table summarizes the average time for each pair of  $k$  and epsilon:

```
randomized %>%
  filter(l==4) %>%
  group_by(k, eps) %>%
  summarise(average_time=mean(time)) %>%
  spread(eps, average_time) %>%
  kable()
```

| k  | 0.1       | 0.3       | 0.6       | 0.9       |
|----|-----------|-----------|-----------|-----------|
| 4  | 4.039749  | 2.670902  | 1.940969  | 1.558463  |
| 7  | 16.152144 | 11.900376 | 9.216258  | 6.606534  |
| 10 | 36.642168 | 27.996650 | 22.334511 | 16.816268 |
| 13 | 68.434450 | 48.664306 | 38.616601 | 33.729408 |

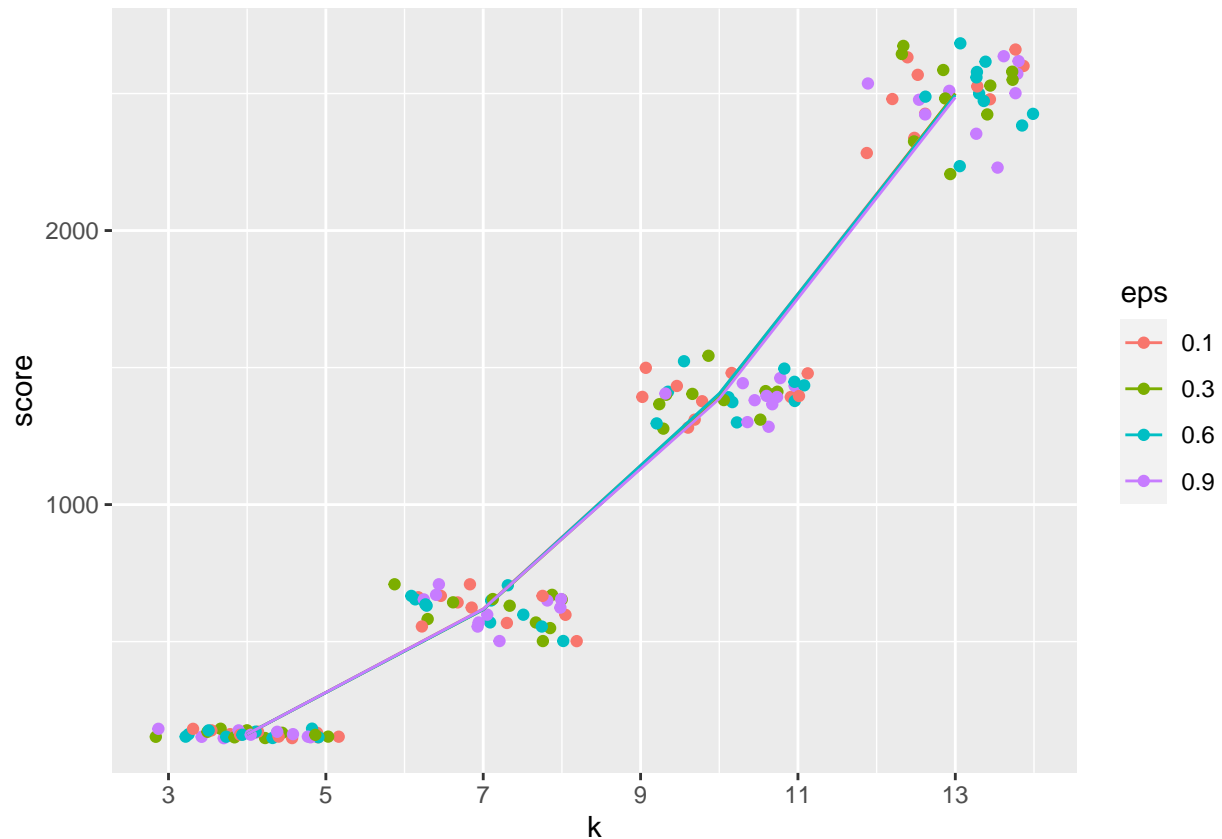
When  $l = 4$ , running time against the number of sequences  $k$  for different epsilon

```
randomized %>%
  filter(l==4) %>%
  mutate(eps=as.factor(eps)) %>%
  ggplot(aes(k, time)) +
  geom_jitter(aes(color=eps)) +
  geom_line(aes(k, avg_time, color=eps)) +
  scale_x_continuous(breaks=c(3,5,7,9,11,13))
```



When  $l = 4$ , score against the number of sequences  $k$  for different epsilon

```
randomized %>%
  filter(l==4) %>%
  mutate(eps=as.factor(eps)) %>%
  ggplot(aes(k, score)) +
  geom_jitter(aes(color=eps)) +
  geom_line(aes(k, avg_score, color=eps)) +
  scale_x_continuous(breaks=c(3,5,7,9,11,13))
```



### comparison

**fixed approximation ratio** For a fixed theoretical approximation ratio =  $2-l/k$  - optimized:  $l=2$  - (2l-1)-stars:  $l=2$  - randomized:  $l=4$ ,  $\text{eps}=0.1$

```
optimized.2 <- optimized %>%
  filter(l==2) %>%
  mutate(algo="optimized")

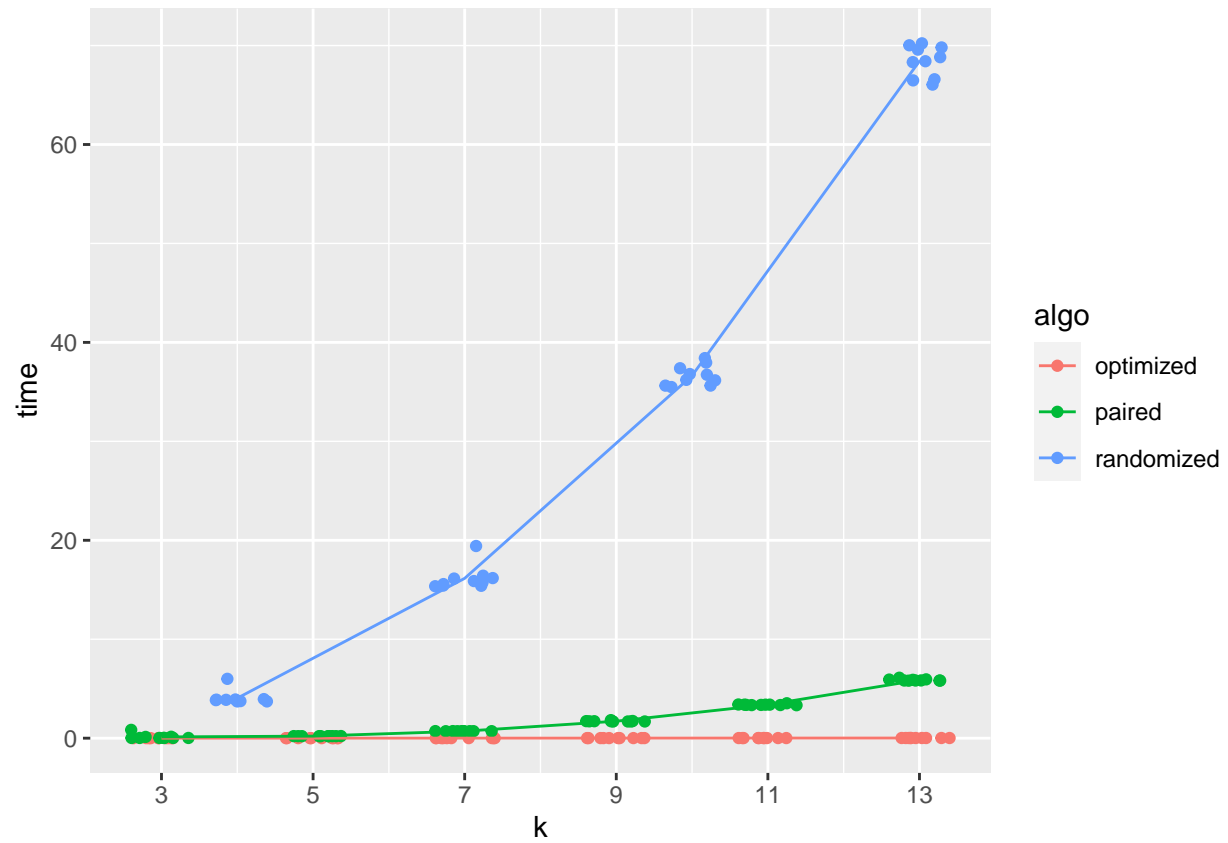
paired.2 <- paired %>%
  filter(l==2) %>%
  mutate(algo="paired")

randomized.2 <- randomized %>%
  filter(l==4, eps==0.1) %>%
  select(-eps) %>%
  mutate(algo="randomized")

df <- rbind(optimized.2, paired.2, randomized.2)
```

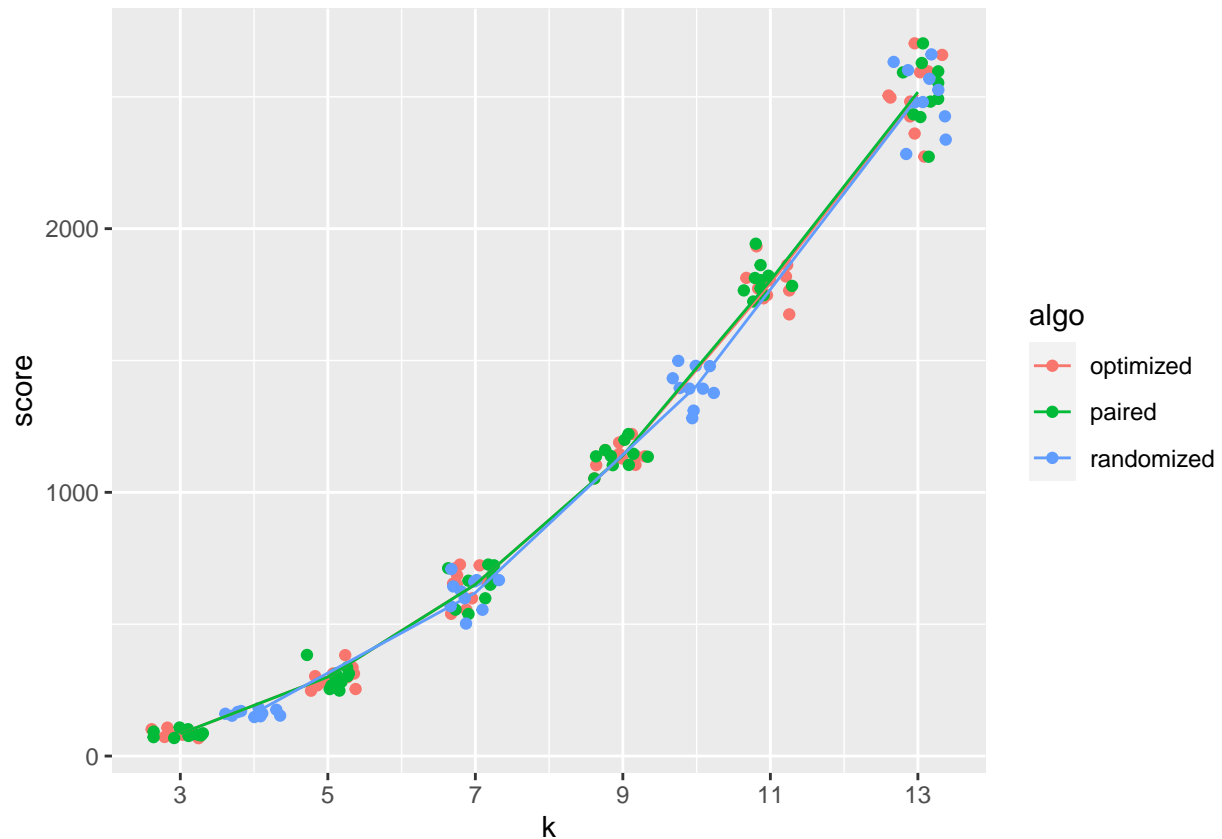
Running time against the number of sequences k

```
df %>%
  ggplot(aes(k, time)) +
  geom_jitter(aes(color=algo)) +
  geom_line(aes(k, avg_time, color=algo)) +
  scale_x_continuous(breaks=c(3,5,7,9,11,13))
```



SP score against the number of sequences k

```
df %>%
  ggplot(aes(k, score)) +
  geom_jitter(aes(color=algo)) +
  geom_line(aes(k, avg_score, color=algo)) +
  scale_x_continuous(breaks=c(3,5,7,9,11,13))
```



**fixed clique size** For a fixed clique size  $l$  - optimized:  $l=3$  - (2l-1)-stars:  $l=2$  - randomized:  $l=3$ ,  $\text{eps}=0.1$

```
optimized.3 <- optimized %>%
  filter(l==3) %>%
  mutate(algo="optimized")

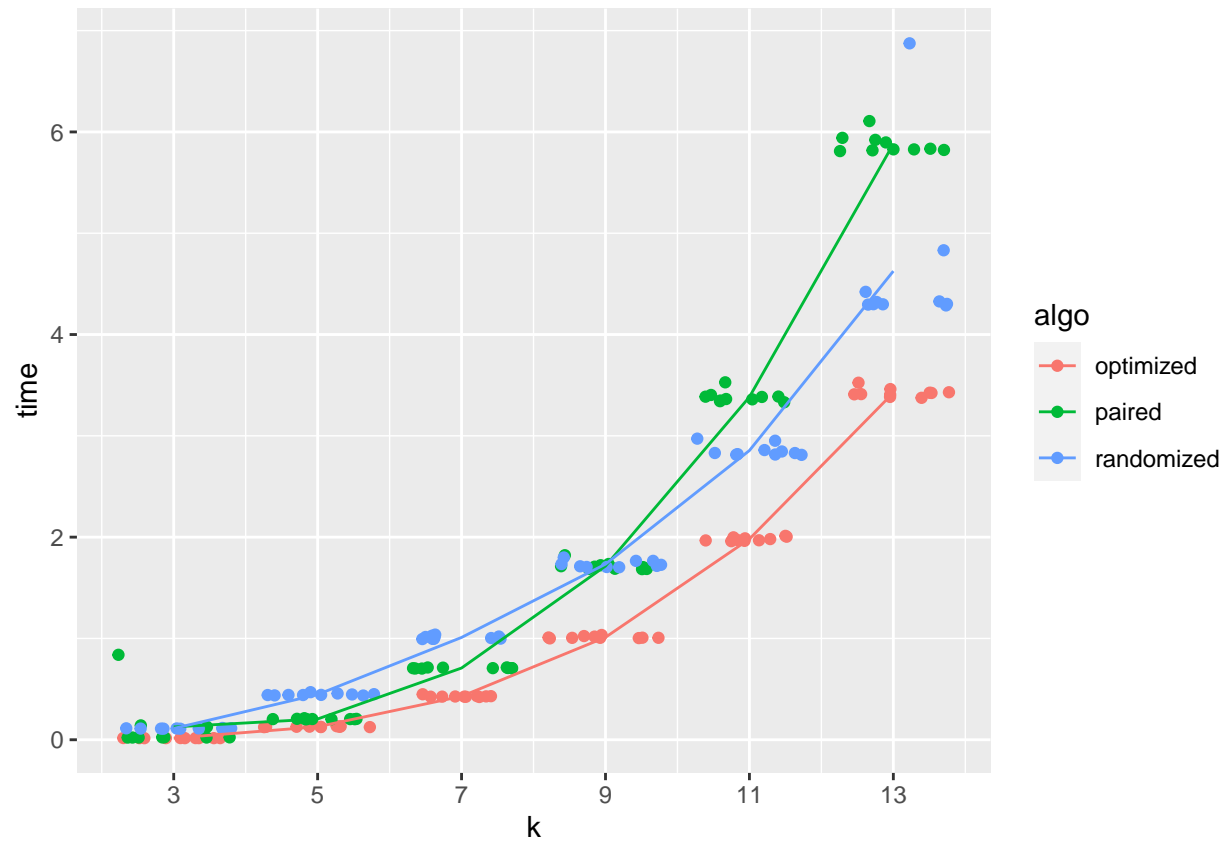
paired.3 <- paired %>%
  filter(l==2) %>%
  mutate(algo="paired")

randomized.3 <- randomized %>%
  filter(l==3, eps==0.1) %>%
  select(-eps) %>%
  mutate(algo="randomized")

df.3 <- rbind(optimized.3, paired.3, randomized.3)
```

Running time against the number of sequences  $k$

```
df.3 %>%
  ggplot(aes(k, time)) +
  geom_jitter(aes(color=algo)) +
  geom_line(aes(k, avg_time, color=algo)) +
  scale_x_continuous(breaks=c(3,5,7,9,11,13))
```



SP score against the number of sequences k

```
df.3 %>%
  ggplot(aes(k, score)) +
  geom_jitter(aes(color=algo)) +
  geom_line(aes(k, avg_score, color=algo)) +
  scale_x_continuous(breaks=c(3,5,7,9,11,13))
```

