Astro540 Homework 1

Problem 1

- total proper motion μ

$$\mu = \sqrt{\mu_\delta^2 + \cos^2\delta\mu_\alpha^2}$$

- proper motion velocity component

$$v_\tau = \mu d = \frac{1}{0.376}\mathrm{pc}\cdot 1.21''\mathrm{a}^{-1} = 15.25\mathrm{km}\cdot\mathrm{s}^{-1}$$

- total velocity

$$v = \sqrt{v_t au^2 + v_r^2} = \sqrt{7.6^2 + 15.25^2}\mathrm{km}\cdot\mathrm{s}^{-1} = 17.04\mathrm{km}\cdot\mathrm{s}^{-1}$$

- when teh object is closest to sun, the direction of its motion is perpendicular to the line connected it and the sun. Thus the distance is caculated as

$$d_0 = d\cdot\sin\theta = \frac{1}{0.376}\mathrm{pc}\cdot\frac{v_\tau^2}{v} = 2.38\mathrm{pc}$$

- the proper motion

$$\mu = \frac{v}{d_0}1.51''\mathrm{a}^{-1}$$

Problem 2

thanks to the `astropy` coordinate module help to me do the coordinate conversion.

- velocity for solar motion relative to CMB:

$$v_{\odot-\mathrm{CMB}} = c \cdot \frac{\Delta T}{T} = 369.2\mathrm{km} \cdot \mathrm{s}^{-1}$$

to the direction

$$\alpha = 167.25^\circ$$
$$\delta = -7^\circ$$

- Since

$$\vec{v}_{\odot-\mathrm{CMB}} = \vec{v}_{\odot-\mathrm{MW}} + \vec{v}_{\mathrm{MW-CMB}}$$

we got

$$\vec{v}_{\mathrm{MW-CMB}} = \vec{v}_{\odot-\mathrm{CMB}} - \vec{v}_{\odot-\mathrm{MW}}$$

- in Celestial coordinate convert two motion in Cartesian representation

$$\vec{v}_{\odot-\mathrm{CMB}} = -357.44\vec{i} + 80.88\vec{j} + (-45.00)\vec{k}$$
$$\vec{v}_{\odot-\mathrm{MW}} = 108.70\vec{i} + (-97.862)\vec{j} + 119.34\vec{k}$$

- Thus in Celestial coordinate,

$$\vec{v}_{\mathrm{MW-CMB}} = 159.02\vec{i} + (-22.74)\vec{j} + 541.35k$$

- in spherical representation, the direction

$$ra = 159.02^\circ$$
$$dec = -22.74^\circ$$

with a velocity 541.3 $\mathrm{km} \cdot \mathrm{s}^{-1}$

- in galactic coordinate

$$w = -29.26$$
$$u = -466.20$$
$$v = 273.60$$

Problem 3

$$M_{B,disk} = M_{\odot} - 2.5\log(1.2\times10^{10}) = -19.72$$
$$M_{B,bulge} = M_{\odot} - 2.5\log(1.9\times10^{9}) = -17.71$$

- Luminosity density:

$$l = l_0\cdot\exp(-R_0/R_d) = 1280\cdot L_{\odot}\mathrm{pc}^{-2}\exp(-8/2.7) = 62.4L_{\odot}\mathrm{pc}^{-2}$$

- Since

$$3.8\mathrm{Magarcsec}^{-2} \sim 1L_{\odot}/(10AU)^{-2} = 4.25\times10^{8}L_{\odot}\mathrm{pc}^{-2}$$

$$\mu_k - 3.8 = -2.5\log(62.4/(4.25\times10^{8}))$$
$$\mu_k = 20.9\mathrm{mag}\cdot\mathrm{arcsec}^{-2}$$

Problem 4

a)

$$\frac{f_0}{f} = 10^{-(M_V-m_v)/2.5} = 3.63\times10^{7}$$

$$d = 10\mathrm{pc}\cdot\sqrt{\frac{f_0}{f}} = 60\mathrm{kpc}$$

b)

- number density of disk:

$$n_{\mathrm{disk}} = n_0\exp(-R/R_0)$$

- number density of halo star:

$$n_{\mathrm{halo}} = 0.1n_0\exp(-R/R_0)$$

- At far distance

$$\left.\frac{n_{\rm disk}}{n_{\rm disk}}\right|_{R=60{\rm kpc}} = 0.1 \cdot \frac{(60/3)^{-3.5}}{\exp(-60/3)} = 1356$$

$$P_{\rm halo} = 0.999$$

- At the solar neighborhood

$$\left.\frac{n_{\rm disk}}{n_{\rm disk}}\right|_{R=8{\rm kpc}} = 0.1 \cdot \frac{(8/3)^{-3.5}}{\exp(-8/3)} = 0.0002$$

$$P_{\rm halo} = 0.0002$$

c

$$A_v = 3.3E(B-V) = 1.65\frac{d}{\rm kpc}$$

For a $M_v = 5.1$ solar type star at the distance d kpc, the magnitude changes to

$$\begin{aligned} m_v &= M_v - 2.5\log\left[\frac{(0.01{\rm kpc})^2}{d^2}\right] \\ &= 5.1 - 2.5\log\frac{0.0001}{d^2} + 1.65d \end{aligned}$$

when $m_v = 24.0$, $d = 3.68$ kpc.