ASTR513 Homework2

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1. From measurements to inferences

In class, we studied the problem of projectile motion in a vertical, uniform gravitational field $\vec{g}$ for which we want to infer the two components of the initial velocity vector of the projectile, u_x and u_y given a measurement of the height h and range R of the motion. Repeat this investigation in this homework problem with the additional complication that our knowledge of the magnitude of the gravitational field is not perfect.
In particular, assume that the uncertainties in our measurements of the height and range of the projectile are well approximated by Gaussian functions with means and standard deviations of ($h_0 = 1.0\,\mathrm{m}, \sigma_h = 0.2\,\mathrm{m}$) and ($R_0 = 10.0\,\mathrm{m}, \sigma_R = 0.2\,\mathrm{m}$), respectively. Also assume that the uncertainty in our knowledge of the magnitude of the gravitational field is also well described by a Gaussian with a mean and standard deviation of ($g_0 = 9.81\,\mathrm{ms}^{-2}, \sigma_g = 0.05\,\mathrm{ms}^{-2}$).

(a). follow the traditional approach to error propagation to infer the uncertainties in the inference of u_x and u_y, given the measurement uncertainties.

$$u_x = \sqrt{\frac{g}{2h}}\frac{R}{2} \tag{1}$$

$$u_y = \sqrt{2gh} \tag{2}$$

Using error propagation

$$\sigma_{u_x}^2 = \frac{g}{8h}\sigma_R^2 + \frac{R^2}{32gh}\sigma_g^2 + \frac{gR^2}{32h^3}\sigma_h^2 \tag{3}$$

$$\sigma_{u_y}^2 = \frac{h}{2g}\sigma_g^2\frac{g}{2h}\sigma_h^2 \tag{4}$$

Thus

$$\sigma_{u_x} = 1.13\,\mathrm{ms}^{-1} \tag{5}$$

$$\sigma_{u_y} = 0.043\,\mathrm{ms}^{-1} \tag{6}$$

(b). Following frequentist arguments, use Monte Carlo realizations of the three measurements (h, R, and g) based on the distribution of their uncertainties to investigate the correlation in the uncertainties of the inferred quantities (u_x and u_y). Use the same Monte Carlo realizations to generate the one-dimensional, marginalized distributions over ux and uy and compare them to the results of part (a).