CPSC413 Problem Set 1

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1 Give a precise definition of the Stable Matching Problem as described on p. 3-4 of the textbook.

Precondition

1. A set M = m_1, m_2,...,m_n of n men.
2. A set W = w_1, w_2,...,w_n of n women.
3. Each man has a ranking of all women in W in order of preference, an ordered list.
4. Each woman has a ranking of all men in M in order of preference, an ordered list.

Postcondition

1. A set S of ordered pairs $(m, w) \in$ M x W such that $m \in M$ and $w \in W$ appear in exactly one pair in S
2. Within the set S of ordered pairs $(m, w) \in$ M x W, m and w will not prefer each other to their assigned partners in S.

2 True or false? In every instance of the Stable Matching Problem, there is a stable matching containing a pair (m, w) such that m is ranked first on the preference list of w and w is ranked first on the preference list of m.

False. There is a case such that a stable match is made containing the pair (m,w), but m is not ranked first on the preference list of w, or w is not ranked first on the preference list of m.
Take for example the stable matching of the sets $M = \{m_1, m_2\}$, $W = \{w_1, w_2\}$.
With the preference lists:
$m_1 : w_1, w_2; \quad m_2 : w_2, w_1$
$w_1 : m_2, m_1; \quad w_2 : m_1, m_2$

Suppose the matching $S = \{(m_1, w_2), (m_2, w_1)\}$. It's then a perfect matching because both women got their first preferences, and have no incentives to leave their current partners. The men however are both with their secondary preferences, and thus in this perfect matching there is no set such that w is ranked first in either preference lists of m.

3 True or false? Consider an instance of the Stable Matching Problem in which there exists a man m and a woman w such that m is ranked first on the preference list of w and w is ranked first on the preference list of m. Then in every stable matching S for this instance, the pair (m, w) belongs to S.

True. Because if they were paired with anyone lower on their preference list, they would both leave their current pairs to form a new set.

4 How many times will the while loop in the Stable Matching Algorithm be executed?

Best case run time will be when each man proposes to a different woman, in which the while loop will only run through each man once, giving a tight lower bound of n, or $\Omega(n)$.

Worst case run time is achieved when every man has the same preference list, and proposes to the same woman each time the while loop iterates through the set of M. The while loop would then have to iterate $n + (n-1) + (n-2) + ... + 1$ times, or $\sum_{i=1}^{n} i = n((1+n)/2)$, giving an upper bound of n^2, or $O(n^2)$

5 Stability of Television Networks

B. There will not always be a stable set of schedules.
example. Suppose the following set of TV shows and associated ratings for n = 2 prime time programming slots.
Network A: s_1, rated 3; s_2, rated 5.
Network B: s_3, rated 4, s_4, rated 2.

This set of shows cannot have a stable pair of matchings because Network A can win more prime-time slots by matching network B's show 3 with show 2, and show 4 with show 1.