2.3

By Bayes' rule,

$$p(c \mid x, \theta, \pi) = \frac{p(c, x \mid \theta, \pi)}{p(x \mid \theta, \pi)}$$

Now take log to $p(c \mid x, \theta, \pi)$, we have

$$\log p(c \mid x, \theta, \pi) = \log \frac{p(c, x \mid \theta, \pi)}{p(x \mid \theta, \pi)}$$

$$= \log p(c, x \mid \theta, \pi) - \log p(x \mid \theta, \pi)$$

$$= \log\left[p(c \mid \pi)\, p(x \mid c, \theta)\right] - \log p(x \mid \theta, \pi)$$

$$= \log p(c \mid \pi) + \log p(x \mid c, \theta) - \log p(x \mid \theta, \pi)$$

where $p(c \mid \pi) = \pi_c$,

$$\log p(x \mid c, \theta) = \prod_{j=1}^{784} p(x_j \mid c, \theta) = \sum_{j=1}^{784} \left[x_j \log \theta_{jc} + (1 - x_j) \log(1 - \theta_{jc})\right]$$

$$p(x \mid \theta, \pi) = \sum_{c=0}^{9} p(c \mid \pi)\, p(x \mid c, \theta)$$

For the following function log-likelihood:

π_c is given by MLE $\hat{\pi}_c$,

θ_{jc} is given by MAP estimator $\hat{\theta}_{MAP_{jc}}$

x_j is given by images