

Naive Bayes model is built on the assumption of conditional independence.

If we have 2 pixels  $x_i$  and  $x_j$ ,  $i \neq j$ ,

$x_i$  and  $x_j$  are independent given  $C=c$ .

So I agree with  $p(x_i, x_j | c) = p(x_i | c) p(x_j | c)$

But  $x_i$  and  $x_j$  are marginal independent if  $p(x_i, x_j) = p(x_i) p(x_j)$

Now we marginalize over  $C$ ,  $C$  is the class from 0 to 9

$$p(x_i, x_j) = \sum_{c=0}^9 p(x_i, x_j, c)$$

$$= \sum_{c=0}^9 p(x_i, x_j | c) p(c)$$

$$= \sum_{c=0}^9 p(x_i | c) p(x_j | c) p(c) \quad \text{by conditional independence}$$

we notice that

$$p(x_i) p(x_j) = \sum_{c=0}^9 p(x_i, c) \sum_{c=0}^9 p(x_j, c)$$

$$= \sum_{c=0}^9 p(x_i | c) p(c) \sum_{c=0}^9 p(x_j | c) p(c) \quad \text{by Bayes' Rule}$$

$$\neq \sum_{c=0}^9 p(x_i | c) p(x_j | c) p(c) = p(x_i, x_j)$$

Therefore we conclude that

$$p(x_i, x_j) \neq p(x_i) p(x_j)$$

So  $x_i$   $x_j$  are not marginally independent