

2.6 we have x_E (observed) , x_j (unobserved)

$$\begin{aligned}
 P(x_j | x_E, \theta, \pi) &= \frac{P(x_j, x_E | \theta, \pi)}{P(x_E | \theta, \pi)} \\
 &= \frac{\sum_{c=0}^9 P(x_j, x_E, c | \theta, \pi)}{\sum_{c=0}^9 P(x_E, c | \theta, \pi)} && \text{marginalize over } c \\
 &= \frac{\sum_{c=0}^9 P(x_j, x_E | \theta, \pi) P(c | \pi)}{\sum_{c=0}^9 P(x_E | \theta, \pi) P(c | \pi)} && \text{since } P(x, c | \theta, \pi) = P(c | \pi) P(x | c, \theta) \\
 &= \frac{\sum_{c=0}^9 P(x_j | \theta, \pi) P(x_E | \theta, \pi) P(c | \pi)}{\sum_{c=0}^9 P(x_E | \theta, \pi) P(c | \pi)} && \text{by conditional independence}
 \end{aligned}$$

where

$$P(x_j | \theta, \pi) = \theta_{jc}^{x_j} (1 - \theta_{jc})^{1 - x_j}$$

$$P(c | \pi) = \pi_c$$

Since $x_E = \{x_p : \text{pixel } p \text{ is observed}\}$

$$\text{we can express } P(x_E | c, \theta) = \prod_{\text{all } p} P(x_p | c, \theta)$$

$$= \prod_{\text{all } p} \left(\theta_{pc}^{x_p} + (1 - \theta_{pc})^{1 - x_p} \right)$$