

2.3

By Bayes' rule,

$$p(c|x, \theta, \pi) = \frac{p(c, x | \theta, \pi)}{p(x | \theta, \pi)}$$

Now take log to  $p(c|x, \theta, \pi)$ , we have

$$\begin{aligned} \log p(c|x, \theta, \pi) &= \log \frac{p(c, x | \theta, \pi)}{p(x | \theta, \pi)} \\ &= \log p(c, x | \theta, \pi) - \log p(x | \theta, \pi) \\ &= \log [p(c | \pi) p(x | c, \theta)] - \log p(x | \theta, \pi) \\ &= \log p(c | \pi) + \log p(x | c, \theta) - \log p(x | \theta, \pi) \end{aligned}$$

where  $p(c | \pi) = \pi_c$ ,

$$p(x | c, \theta) = \prod_{j=1}^{784} p(x_j | c, \theta) = \prod_{j=1}^{784} [x_j \log \theta_{jc} + (1 - x_j) \log (1 - \theta_{jc})]$$

$$p(x | \theta, \pi) = \sum_{c=0}^9 p(c | \pi) p(x | c, \theta)$$

For the following function **log-likelihood** :

$\pi_c$  is given by MLE  $\hat{\pi}_c$ ,

$\theta_{jc}$  is given by MAP estimator  $\hat{\theta}_{\text{MAP}_{jc}}$

$x_j$  is given by images