2.6 we have x_E (observed), x_j (unobserved)

$$P(x_j \mid x_E, \theta, \pi) = \frac{P(x_j, x_E \mid \theta, \pi)}{P(x_E \mid \theta, \pi)}$$

$$= \frac{\sum_{c=0}^{9} P(x_j, x_E, c \mid \theta, \pi)}{\sum_{c=0}^{9} P(x_E, c \mid \theta, \pi)}$$ marginalize over c

$$= \frac{\sum_{c=0}^{9} P(x_j, x_E \mid c, \theta) P(c \mid \pi)}{\sum_{c=0}^{9} P(x_E \mid c, \theta) P(c \mid \pi)}$$ since $P(x, c \mid \theta, \pi) = P(c \mid \pi) P(x \mid c, \theta)$

$$= \frac{\sum_{c=0}^{9} P(x_j \mid c, \theta) P(x_E \mid c, \theta) P(c \mid \pi)}{\sum_{c=0}^{9} P(x_E \mid c, \theta) P(c \mid \pi)}$$ by conditional independence

where

$$P(c \mid \pi) = \pi_c$$

Since $x_E = \{x_p : \text{pixel } p \text{ is observed}\}$

we can express $P(x_E \mid c, \theta) = \prod P(x_e \mid c, \theta)$

$$= \prod \left(\theta_{ec}^{x_e} + (1 - \theta_{ec})^{1 - x_e} \right)$$

So $\log P(x_E \mid c, \theta) = \sum \left[x_e \log \theta_{ec} + (1 - x_e) \log (1 - \theta_{ec}) \right]$

But here for x_j, we are computing with $x_j = 1$

$$P(x_j = 1 \mid c, \theta) = \theta_{jc}$$

instead of the expression like $P(x_E \mid c, \theta)$