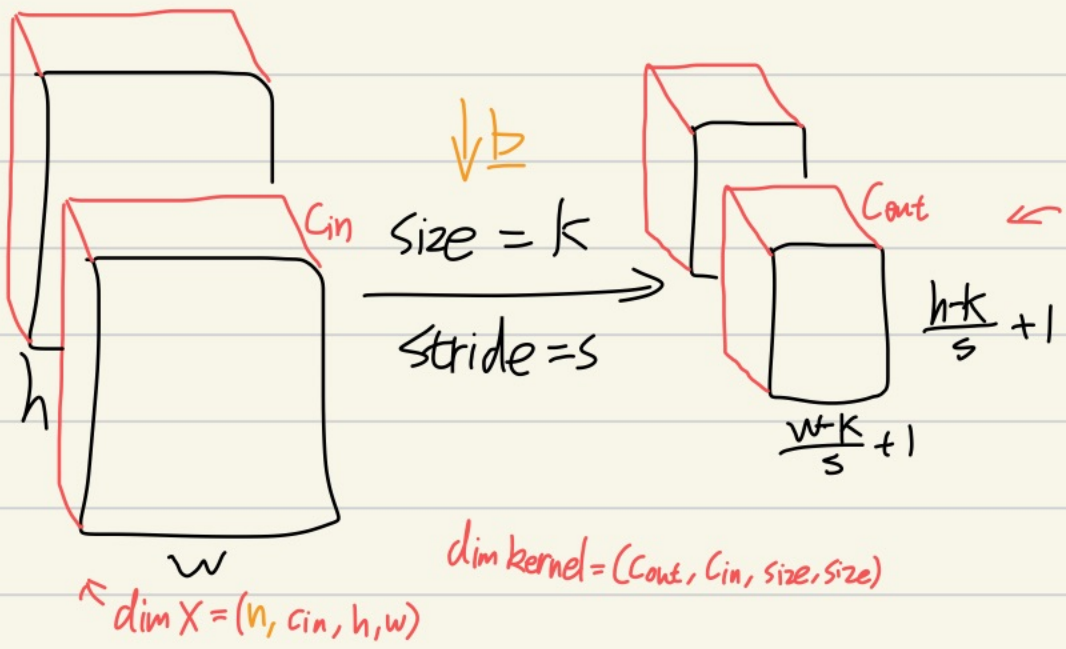
Img2col & Col2img [used by Keras]



dim X = (n, cin, h, w)

dim kernel = (Cout, Cin, size, size)

dim Y = $(n, C_{out}, \frac{h-k}{s}+1, \frac{w-k}{s}+1)$

Suppose $\frac{w-k}{s}$ & $\frac{h-k}{s}$ are integers and $s < k$

img2col

X —img2col→ colX

n, cin, h, w → $n \times (\frac{h-k}{s}+1) \times (\frac{w-k}{s}+1),\ K^2 \cdot C_{in}$

Kernel —reshape→ colKernel

C_{out}, C_{in}, K, k → $K^2 \cdot C_{in}, C_{out}$

bias —reshape→ colbias

$C_{out}, 1, 1$ → $1, C_{out}$

forward

colX · colKernel + colbias = colres

$n \times (\frac{h-k}{s}+1) \times (\frac{w-k}{s}+1), C_{out}$

colres —reshape→ tmpres

$n, \frac{h-k}{s}+1, \frac{w-k}{s}+1, C_{out}$

tmpres —exchange dimension→ Y

$n, C_{out}, \frac{h-k}{s}+1, \frac{w-k}{s}+1$

backward

grad Y $\xrightarrow{\text{back exchange}}$ grad tmpres ($n, \frac{h-k}{s}+1, \frac{w-k}{s}+1, C_{out}$) $\xrightarrow{\text{back reshape}}$ grad colres ($n\times(\frac{h-k}{s}+1)\times(\frac{w-k}{s}+1), C_{out}$)

grad colbias = sum(grad colres, axis=0)
$1, C_{out}$

grad colKernel = colX^T · grad colres
$K^2 \cdot C_{in}, C_{out}$ — colX^T: $K^2 C_{in}, n\times(\frac{h-k}{s}+1)\times(\frac{w-k}{s}+1)$; grad colres: $n\times(\frac{h-k}{s}+1)\times(\frac{w-k}{s}+1), C_{out}$

grad colX = grad colres · colKernel^T
$n\times(\frac{h-k}{s}+1)\times(\frac{w-k}{s}+1), K^2 C_{in}$ — grad colres: $C_{out}, K^2 C_{in}$

Same as ANN

col2img

grad colbias $\xrightarrow{\text{back reshape}}$ grad bias
$1, C_{out}$ — $C_{out}, 1, 1$

grad colkernel $\xrightarrow{\text{exchange dimension}}$ $C_{out}, K^2 \cdot C_{in}$ $\xrightarrow{\text{back reshape}}$ grad Kernel
$K^2 \cdot C_{in}, C_{out}$ — C_{out}, C_{in}, K, K

grad colX $\xrightarrow{\text{col2img}}$ grad X
$n\times(\frac{h-k}{s}+1)\times(\frac{w-k}{s}+1), K^2 C_{in}$ — n, C_{in}, h, w