Pendant drop method for measuring the surface tension of water droplets using a smartphone and Matlab at home

LU Yi-fan, JIN Zi-cheng, REN Wen-yi

(College of Science, Northwest A&F University, Yangling, Shaanxi 712100, China)

Abstract: An experiment of measuring the surface tension of water droplets based on the pendant drop method is developed by using a smartphone and Matlab software that can be used at home. The surface tension coefficient is calculated by photographing water droplets with a smartphone and then using Matlab to recognize and calculate the edges. For various clarity cases, appropriate analysis and processing methods are presented, and testing results show that the method is stable and trustworthy. It is a typical case of post-epidemic experimental university physics teaching at home.

Keywords: surface tension; smartphone; home physics experiment; Matlab edge detection; pendant drop method

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The arrival of the COVID-19 pandemic has disrupted traditional educational paradigms and led to the emergence of diversified teaching methods such as online classrooms. Under home conditions, the limitations on experimental facilities pose new challenges for remote teaching of university physics experiments. Various forms of teaching methods, such as virtual simulation experiments, computer simulation experiments, and home experiments, provide diverse solutions for remote experimental teaching. In order to cultivate students' hands-on ability, independent thinking, and spirit of scientific exploration, while maintaining simplicity and feasibility, the design of home-based physics experiments has been increasing. At the same time, with the rapid development of smartphones, their portability, professionalism, and popularity have led to an increasing number of physics experiments being conducted with the combination of smartphones in recent years. In this context, this paper proposes a home-based experiment using a smartphone to measure the surface tension coefficient of liquids.

The common measurement methods of liquid surface tension include the pendant drop method [1], Wilhelmy plate method, ring method, capillary rise method [2], maximum bubble pressure method, drop weight method, capillary probe method [3], Laplace pressure method [4], etc. One of the simpler measurement methods is the pendant drop method, which is based on the principle that when the test droplet is suspended in a static state at the mouth of the capillary, its shape is mainly determined by the balance between gravity and surface tension. Therefore, by measuring certain key geometric parameters of the suspended droplet (such as the maximum diameter of the droplet), the surface tension coefficient of the liquid can be calculated.

There have been extensive studies on the experimental method of measuring liquid surface tension using the pendant drop method. Zhao et al. conducted an experiment by combining CCD cameras, injection pumps, LED light sources, and computers to extract the edge of the droplet [5]. This is a common experimental approach in pendant drop measurements. Wan et al. calculated the image magnification by measuring the droplet's pipette diameter [6]. Although this is a standard method, it has two obvious drawbacks. First, in-home experiments, the pipette diameter cannot be accurately obtained due to uneven thickness, easy deformation, lack of measurement tools, etc. Second, when the pipette diameter is a variable in the experiment, the measurement of its diameter needs to be repeated every time when the capillary tube is replaced. Goy et al. used mobile phones to capture droplet images [7], greatly simplifying the experimental setup and operation. However, the droplet edge obtained from the photos is not easily distinguishable by previous computer algorithms, making precise extraction difficult. Using a ruler to measure length parameters on the screen is convenient but introduces significant errors.

Therefore, this paper proposes an experimental approach for measuring the surface tension coefficient of water droplets using the pendant drop method with a smartphone based on a reference object. The width of the reference object is known, and the use of computer vision methods combined with Matlab software and edge detection algorithms greatly enhances the simplicity of the experiment, making it more suitable for home use.

1 Experimental principle

1.1 Pendant Drop Method for Measuring Surface Tension

When measuring the surface tension of a liquid using the pendant drop method, the first step is to obtain a stationary droplet suspended at the mouth of a tube. In this paper, water (the liquid to be tested) is dripped into an ordinary glass tube or a pipette to achieve this. At this point, the droplet exhibits an ellipsoidal contour due to the balance between gravity and surface tension. The surface tension coefficient can be obtained by measuring the edge profile. The schematic diagram of the pendant drop method shown in Figure 1 has the following relationship [8].

$$\frac{1}{r/b}+\frac{\sin\phi}{x/b}=2+\beta(z/b) \tag{1}$$

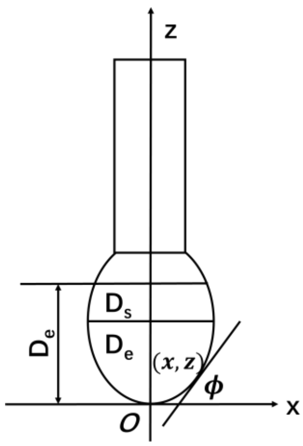


Fig. 1 Pendant drop method schematic diagram.

Among them, at point (x,z) on the contour of the water droplet, r represents the radius of curvature, ϕ represents the angle between the tangent line at that point and the positive horizontal direction. b represents the radius of curvature at the coordinate origin. β is a dimensionless number introduced to simplify the expression [8]:

$$\beta = {-g\Delta\rho b^2}/{\gamma} \tag{2}$$

where g is the acceleration of gravity, $\Delta\rho$ is the density difference between the two sides of the interface (in this experiment, the density difference is between water and air), γ is the surface tension coefficient and can be calculated by measuring β, b, g, $\Delta\rho$.

In practice β and b are difficult to measure, so this calculation is discarded. After simplifying and approximation, γ is calculated using the following equation:

$$\gamma = {gD_e^2\Delta\rho}/{H} \tag{3}$$

In this context, D_e represents the maximum diameter of the liquid droplet. In the experiment, the edge of the water droplet image obtained through Matlab from the smartphone is extracted, and the number of pixels at the location of the maximum edge diameter is calculated. Combining the image magnification (which can be calculated from the actual width of the reference object and its corresponding pixel count on the image), D_e can be obtained. H represents the shape factor of the liquid droplet, and it is the corrected shape factor S. There is an empirical relationship between the two, which has been summarized in an empirical table [9]. The expression for S is as follows:

$$S = D_s / D_e \tag{4}$$

In this case, D_s represents the diameter on the edge of the liquid droplet at a distance D_e from the bottom. In the experiment, using the extracted edge, the corresponding lengths of pixels for D_s and D_e can be obtained to calculate the shape factor S.

2 Experimental hardware and software design

2.1 Experimental setup

A glass pipette or a regular straw with an approximate mouth diameter of 3 mm, open at both ends, is fixed upside down next to a reference object on a stand similar to an iron frame. The two are arranged parallel and can be secured using perforated hard cardboard combined with clay. If there is no iron frame, the hard cardboard can be fixed at both ends on objects such as boxes or chairs of the same height to ensure that the dropper can hang vertically. In this study, a steel rod with a marked diameter (6.04 mm) was chosen as the reference object, and at home, a wire with a known standard diameter can be used as a substitute. The bottom of the steel rod is positioned lower than the bottom of the glass pipette, and the distances to the camera are roughly equal, ensuring that the steel rod can be parallel to the water droplet for comparison during photography. The background for the experiment is black or white, and the light source is natural indoor light or a desk lamp. The physical setup of the home experiment is shown in Figure 2. Compared to using the mouth diameter of the capillary as a reference object, selecting an item with a known actual length can avoid measuring the diameter, thereby simplifying the steps and reducing random errors.

Fig. 2 Home experimental setup diagram.

2.2 Matlab algorithm and GUI interface

Drip water into the upper end of the glass pipette, trying to avoid vibrations. Let the water flow slowly to the lower end of the pipette and form a relatively stable droplet. After that, use a smartphone to take a photo to obtain an image as shown in Figure 3.

Fig. 3 Pictures collected by smartphones.

Due to factors such as liquid reflection, low contrast with the environment, and a high level of image noise both inside and outside the water droplet, the quality of the obtained images is not optimal. It is not possible to directly use the edge detection function in Matlab to accurately extract the edges. Therefore, it is necessary to analyze the characteristics of common water droplet edge types and design a program tailored to ensure obtaining clear edges.

(a) (b) (c) (d)

Fig. 4 (a) Overall edge is clear; (b) significant difference in clarity between the left and right halves of the edge; (c) scattered blurry segments along the edge; (d) overall edge not clear due to low light.

Considering the variations in brightness, lighting intensity, and other factors, four different clear water droplet edge situations are observed, as shown in Figure 4: overall clear edge, significant difference in clarity between the left and right halves of the edge, scattered blurry segments along the edge, and overall edge not clear due to low light. In the following text, these four situations are referred to as "Scenario 1," "Scenario 2," "Scenario 3," and "Scenario 4." Specific processing algorithms are designed for each situation to achieve high-quality edge extraction. To determine the number of pixels representing the width of the steel rod in the image, it is necessary to accurately extract the edges on both sides of the rod. When the edges of the steel rod are clear, they can be directly extracted using the "edge" function. However, if the edges of the steel rod are not easily distinguishable due to reflection or other reasons, the two edges need to be manually selected.

2.2.1 Edge extraction of water drops under different lighting conditions

1) Scenario 1

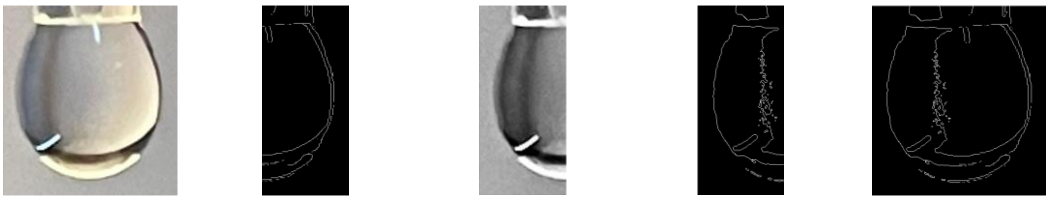
For the captured image shown in Figure 5(a) representing ‘Scenario 1’, a binary image can be obtained as shown in Figure 5(b). The threshold value τ for binarization is determined using the maximum inter-class variance method, implemented in Matlab using the ‘graythresh’ function. Based on the Canny edge detection algorithm, the binary image is processed to extract the water droplet's edge, resulting in the edge image shown in Figure 5(c).

(a) (b) (c)

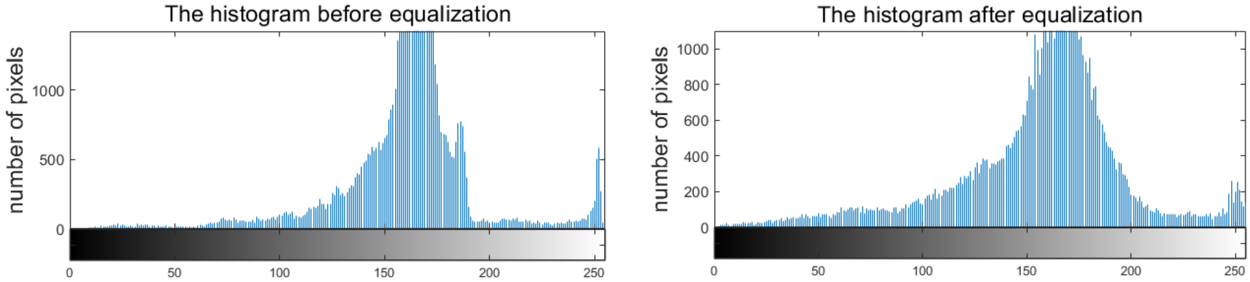
Fig. 5 (a) Original image of water droplets; (b) binarized map; (c) water droplet edge image.

2) Scenario 2

For the captured image shown in Figure 6(a), representing ‘Scenario 2’, a method of extracting and merging the edges for each half of the water droplet is employed. The grayscale image of the original image is divided into two halves (left and right) and denoted as Image A (the half with relatively clear edges) and Image B (the other half). For Image A, the edge is directly extracted using Sobel or Canny algorithms, and the result is shown in Figure 6(b). For Image B, contrast-limited adaptive histogram equalization is applied to the grayscale image to obtain Image C (Figure 6(c)). After binarization and edge extraction, the edge image as shown in Figure 6(d) is obtained. The extracted edge images are then recombined to produce a clear edge image, as shown in Figure 6(e). The histograms before and after the equalization of Image B are shown in Figure 6(f) and (g) respectively. It can be observed that this process spreads the grayscale distribution of Image B across the entire color space, making it almost uniformly distributed, thus enhancing the image contrast.



(a) (b) (c) (d) (e)



(f) (g)

Fig. 6 (a) Original image of water drop; (b) the right half of the edge image; (c) the left half after histogram equalization; (d) the left half of the edge image; (e) the whole waterdrop edge image; (f) histogram distribution before equalization; (g) histogram distribution after equalization.

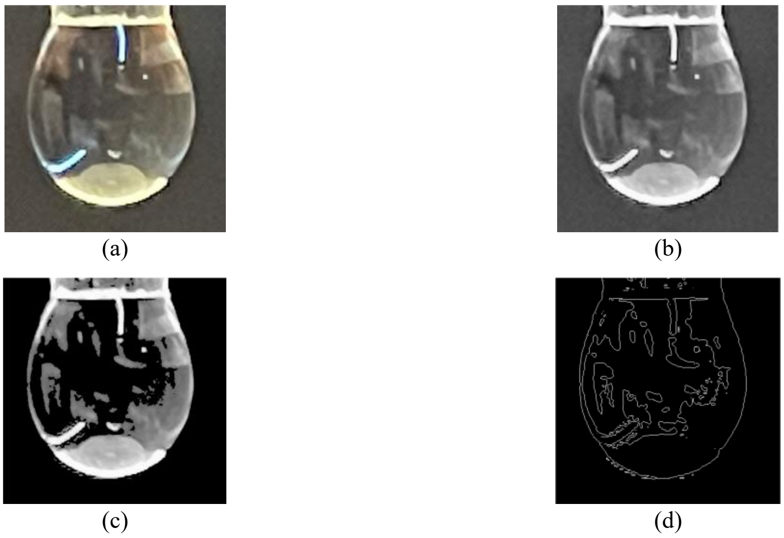
3) Scenario 3

Figure 7(a) shows the water droplet image obtained under ‘Scenario 3’. After converting the original image to a grayscale image, denoted as I (Figure 7(b)), an interactive method is used to select a point (x, y) in the inner area of the water droplet, adjacent to the blurry segment. The grayscale value of this point is taken as the reference grayscale, denoted as $I(x, y)$, and the appropriate grayscale range is $I(x, y) \pm \tau$ (where τ is taken as 0.001 in this case). The entire image is then examined pixel by pixel. For each pixel at position (m, n), if its grayscale value falls within this range, its grayscale value is set to 0, otherwise, it remains unchanged. The grayscale value is set as

$$I(m,n) = \begin{cases} 0, & a \le \tau \\ I(m,n), & a > \tau \end{cases} \tag{5}$$

where a is $|I(m, n) - I(x, y)|$.

The threshold τ=0.001 is determined based on a statistical analysis of the experimental environment and is not absolute. The enhanced image of Figure 7(b) is shown in Figure 7(c), where it can be observed that the leftmost part of the contour is significantly clearer compared to the original image. At this point, the entire water droplet's edge is clear, as shown in Figure 7(d).

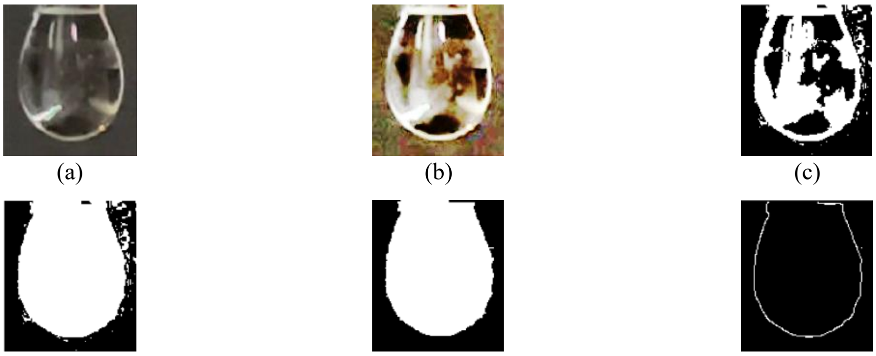


(a) (b) (c) (d)

Fig. 7 (a) Original image of the waterdrop; (b) grayscale image of the waterdrop; (c) grayscale image of the waterdrop after enhancement; (d) edge image of the waterdrop.

4) Scenario 4

Under 'Scenario 4', the obtained image is shown in Figure 8(a). The following processing steps are taken: histogram equalization is performed on the water droplet image to enhance the edges, resulting in Figure 8(b). The image in Figure 8(b) is subjected to grayscale processing and binarization to obtain Figure 8(c). The 'imfill' function in Matlab is applied to fill holes in Figure 8(c). Since the characteristic of this scenario is that the edge and its adjacent inner part of the water droplet are brighter than the outer part (the entire edge of the water droplet is continuous, and it can completely enclose the internal noise), after filling the holes, the resulting image I should have a single white region in the inner part of the water droplet, as shown in Figure 8(d), where $I(m_1, n_1) = 1$ for points (m_1, n_1) inside the water droplet. Noise reduction is performed on the background outside the water droplet: Starting from the vertical midline position of the black and white image (approximately the midline of the water droplet), each pixel point is checked towards the left and right sides. When encountering the first black point, it is considered as the edge, and the points $(m_2,\ n_2)$ detected beyond the edge are considered as the background outside the water droplet, and all the background points are set to black $I(m_2, n_2) = 0$. This results in a clear black and white image with a distinct boundary for both the inner and outer parts of the water droplet, as shown in Figure 8(e). Edge extraction is performed on the resulting image to obtain a clear edge, as shown in Figure 8(f).



(a) (b) (c)

(d) (e) (f)

Fig. 8 (a) Original image of water droplets; (b) histogram equalization image; (c) binarization image; (d) hole-filling" image; (e) background denoising image; (f) drop edge image.

2.2.2 Extraction of the left and right contours of the steel rod

To extract the steel rod portion from the original image captured by the smartphone, follow these steps: if the steel rod edge is clear, convert the color image to grayscale and apply binarization to obtain a binary image. Then use the 'edge' function to detect the outermost two white lines in the binary image, which represent the edges of the steel rod. If the steel rod edge is not clear, ask the user to select a point on each side of the steel rod to represent the edges. Then use the selected points as the x-coordinates to draw two vertical lines on the image, representing the two edges of the steel rod.

2.2.3 Derivation of surface tension coefficient

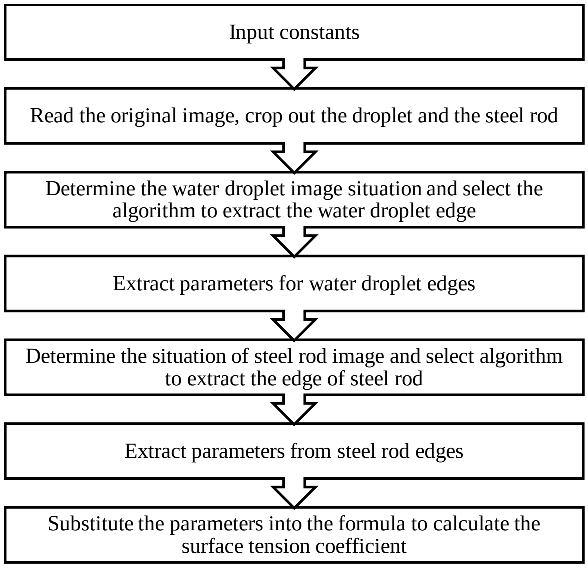


Fig. 9 Flow chart of surface tension coefficient calculation.

The Matlab program used in this study to calculate the surface tension coefficient follows the flowchart depicted in Figure 9. Initially, the program requires setting the system constants: air density ρ_{air}, water density ρ_{water}, gravity acceleration g, and the diameter of the steel rod D. After reading in the original image, an interactive interface is employed to allow users to manually select and extract the regions corresponding to the water droplet and the steel rod. These regions are processed separately. For the water droplet image, based on the discussion of its edge characteristics mentioned earlier, the program determines its edge type and selects the most suitable algorithm for edge extraction. The resulting edge image shows white lines representing the water droplet edges against a black background. Utilizing this characteristic, the program measures the distance between the two outermost white pixels in each row as the horizontal diameter, using pixels as the unit. The largest value d_e corresponds to the actual maximum diameter D_e of the water droplet in terms of pixel count in the image. Identify the horizontal diameter d_s at a distance of d_e pixels from the bottom of the water droplet, representing the actual distance denoted as D_s. Similarly, the pixel diameter d of the steel rod can be calculated. The relationships are as follows:

$$D_e = \eta * d_e \tag{6}$$

where η is the ratio of the actual length of the steel rod to its pixel grid number D/d. The shape factor S can be derived from the following equation:

$$S = D_s / D_e = d_s / d_e \tag{7}$$

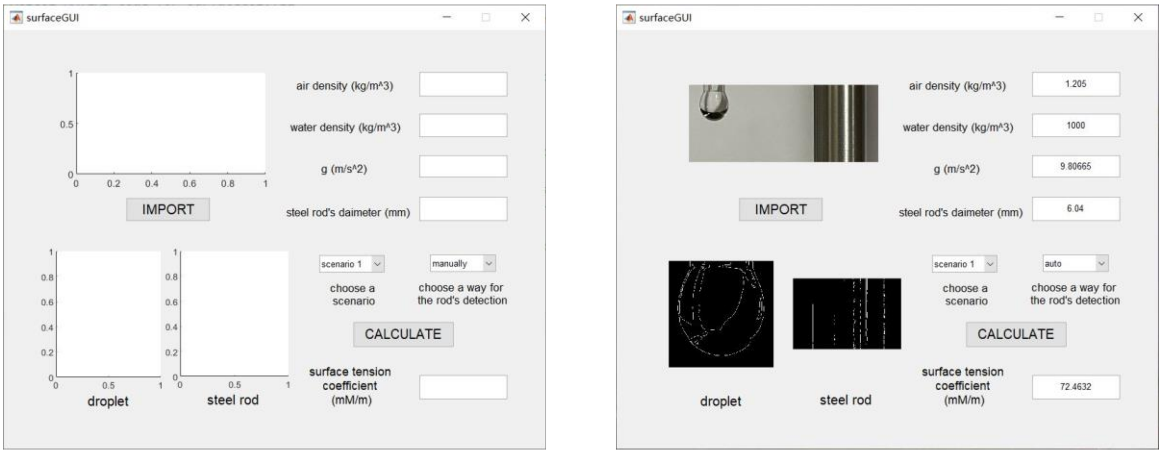
Then the surface tension coefficient of water is

$$\gamma_{water} = g\Delta\rho D_e^2 / H \tag{8}$$

where $\Delta\rho = \rho_{water} - \rho_{air}$, H is the shape factor of the liquid droplet, which is corrected by the factor S. The one-to-one correspondence between them has been summarized in an empirical table, which has been preloaded into Matlab (with S values ranging from 0.660 to 1.003, with an interval of 0.001) for easy access during program execution to find the corresponding H value for a given S. Since the values of S in the table are discrete, interpolation needs to be employed in the program to calculate the corresponding H value. By combining equations (6) to (8), the surface tension coefficient γ_{water} can be calculated.

2.2.4 Design of the experimental GUI interface

Designed with Matlab, the GUI interface shown in Figure 10(a) enables interactive data processing. Four constants are input as prompted. Based on the determination of the droplet and steel rod conditions, suitable image processing methods are chosen from two drop-down menus. After importing the original image, clicking the 'Calculate' button initiates automatic program execution. Following the prompts on the interactive interface, the results of the measurement can be viewed in the 'surface tension coefficient' section of the interface, as shown in Figure 10(b).



(a) (b)

Fig. 10 (a) Experiment GUI interface; (b) results display interface.

3 Experimental results and discussion

To assess the accuracy of the water droplet edge detection algorithms designed for the four different scenarios, each scenario was tested using five images. Each image underwent five repetitions of the operation to minimize random errors, and the average of these five results was taken as the measured surface tension value for that specific image. This approach aimed to calculate the relative standard deviation and relative error for each algorithm in each scenario, reflecting their stability and precision, respectively. A total of seven data points were recorded for each scenario, resulting in a total of 28 data points, as illustrated in Table 1. The environmental temperature for this experiment was measured at 20°C using a mercury thermometer. Consequently, the following parameters were established: air density of 1.205 kg/m³ [10], water density of 1000 kg/m³, gravity acceleration of 9.80665 m/s², and a steel rod diameter of the factory-calibrated value of 6.04 mm. The theoretical surface tension coefficient of water was assumed to be 73 mN/m [10].

3.1 Calculation results and error analysis of four edge detection methods

Surface tension coefficient measurement results and error analysis for four scenarios are recorded as shown in

Table 1 [11]. Taking scenario one as an example, the steps of error analysis are demonstrated. The average value of the measurement results is calculated.

$$\bar{x} = \sum_{i=1}^{N} x_i \Big/ N \approx 72.9 \text{ mN/m} \tag{9}$$

Table 1 Experimental results and error analysis in four cases

Scenario	Image serial number	1	2	3	4	5	Relative Error (%)	Relative standard deviation (%)
1	γ_{water}(mN/m)	72.4	75.1	72.2	71.8	72.8	0.14	1.78
2	γ_{water}(mN/m)	82.5	68.0	76.3	69.5	69.8	0.27	8.32
3	γ_{water}(mN/m)	72.7	74.9	74.2	68.2	71.7	0.96	3.64
4	γ_{water}(mN/m)	73.5	71.1	70.4	74.1	76.3	0.11	3.26

where N is the number of samples, which is 5. x_i (i =1, 2, 3, 4, 5) is the measurement result of 5 images in this case.

The relative error is

$$d_r = |\bar{x} - x_0| \Big/ x_0 \times 100\% \approx 0.14\% \tag{10}$$

where x_0 is the theoretical value of the water surface tension coefficient.

The standard deviation is

$$s = \sqrt{\sum_{i=1}^{N} (x_i - \bar{x})^2 \Big/ N - 1} \approx 1.30 \text{ mN/m} \tag{11}$$

The relative standard deviation is

$$S_r = s \big/ \bar{x} \times 100\% \approx 1.78\% \tag{12}$$

Similarly, the relative errors and relative standard deviations for cases two, three, and four can be obtained. Based on the above analysis, the following can be concluded: 1) Among the water surface tension measurement experiments based on the pendant drop method, Wan's results [6] have a relative error of 0.00014%; Zhao's [5] is 2.2%; Goy's [7] is 6.8%. Among experiments using different principles, the relative errors for methods based on the capillary rise method [2] is 0.14%; based on the detachment method combined with a sensor [4] is 1.1%; and based on the capillary probe method [3] is 0.12%. The accuracy of the method proposed in this paper is comparable to general laboratory methods and higher than Goy's [7] manual measurement accuracy. Compared to methods under other principles, the errors are within a reasonable range. 2) The relative standard deviations of the algorithms are within an acceptable range, indicating good stability and repeatability under the same conditions.

4 Conclusion

This study investigated a home-based surface tension measurement experiment using smartphones and Matlab. By utilizing devices such as smartphones, glass pipettes, and steel rods, along with Matlab image processing algorithms designed for four different unclear edge water droplet scenarios, a method for measuring water surface tension at home was developed. This method is simpler and more convenient compared to previous setups. When compared to experiments under other principles and methods, the proposed approach demonstrates higher measurement accuracy and better stability. This experiment contributes to enhancing students' understanding of experimental principles and their skills in Matlab programming. Additionally, it provides a feasible and innovative solution for online experimental teaching in the post-pandemic era.

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