Using Continuous Greedy Algorithm to Solve Multi-Robot Multi-Target Coverage Problem

Yifan Su

Carnegie Mellon University, Computer Science Department

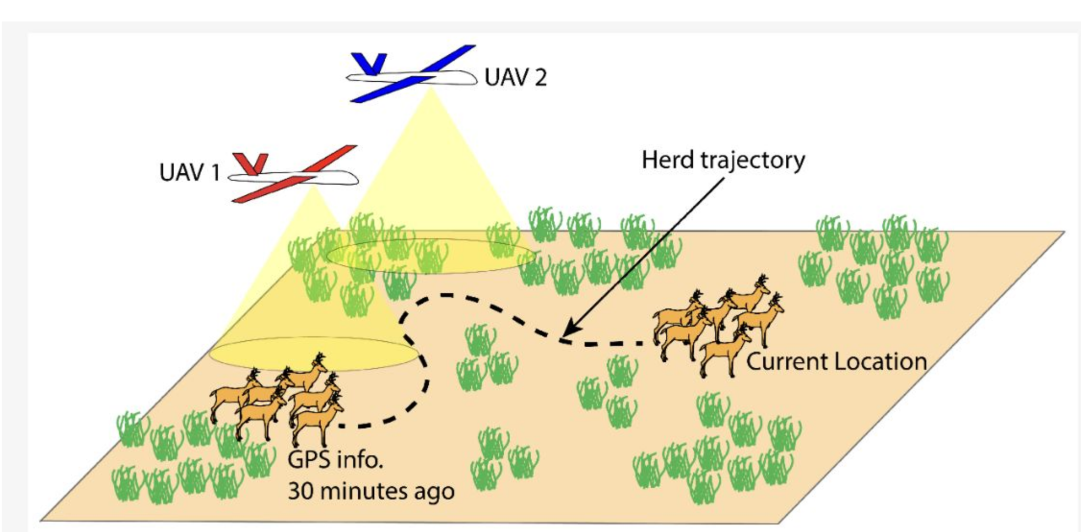
Advised by: **Dr. Micah Corah and Prof. Sebastian Scherer**

Abstract/overview

The Multi-Robot Multi-Target Tracking Problem involves multiple robots tracking multiple targets in an environment. The primary objective is to allocate the robots to the targets effectively and optimize their trajectories to ensure accurate monitoring of the targets. In this study, our focus is on maximizing the robots' coverage of the targets. This problem is usually an NP-hard problem. We try to get the suboptimal result of (1-1/e)OPT using the continuous greedy algorithm.

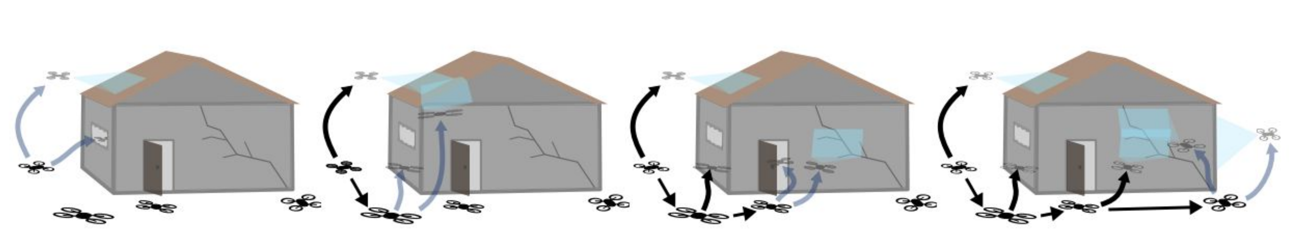
Introduction/motivation

The Multi-Robot Multi-Target Tracking Problem involves multiple robots tracking multiple targets in an environment. The primary objective is to allocate the robots to the targets effectively and optimize their trajectories to ensure accurate monitoring of the targets.



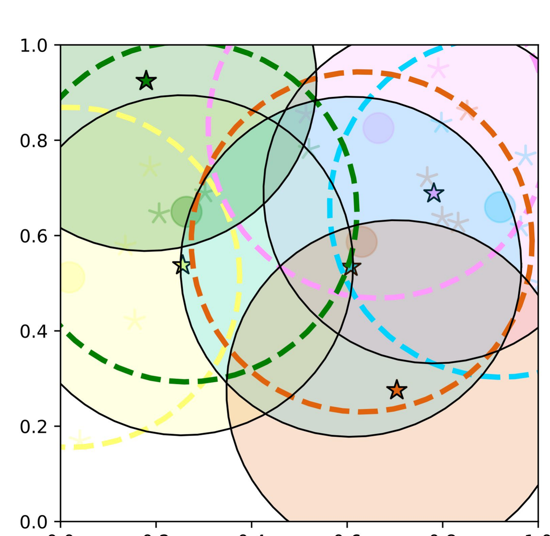
Wildlife Monitoring Using a Multi-UAV System [1]

Aerial Photography and Videography



Infrastructure Inspection

In this study, our focus is on maximizing the robots' **coverage of the targets.** This study adapted the submodular welfare problem for multi-robot coverage/trajectory planning and implemented Julia code for the continuous greedy algorithm.



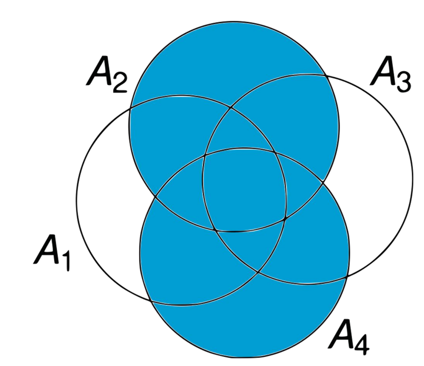
Proposed Methods/Description

Submodularity

For any subsets A and B of S, the marginal gain of adding an element to A (i.e., $f(A \cup \{x\}) - f(A)$) is greater than or equal to the marginal gain of adding the same element to B, given that A is a subset of B (i.e. $f(B \cup x) - f(B)$)

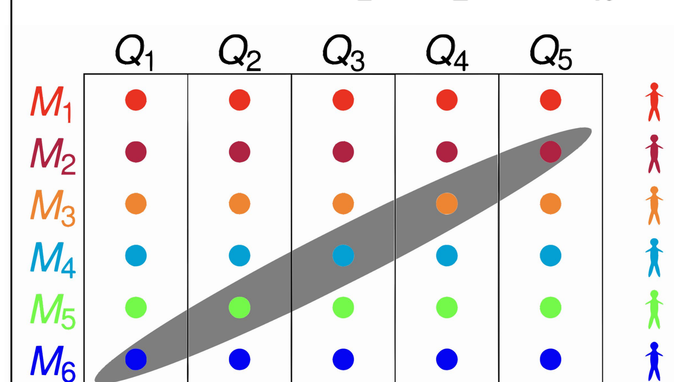
Coverage function:
Given $A_1, \ldots, A_n \subset U$,

$$f(S) = \left|\bigcup_{j\in S} A_j\right|.$$



Submodular Welfare Problem

Given n players with submodular valuation function $w_i: 2^M \to \mathbb{R}_+$.
Partition $M = S_1 \cup S_2 \cdots \cup S_n$ so as to maximize $\sum_{i=1}^{N} w_i(S_i)$.

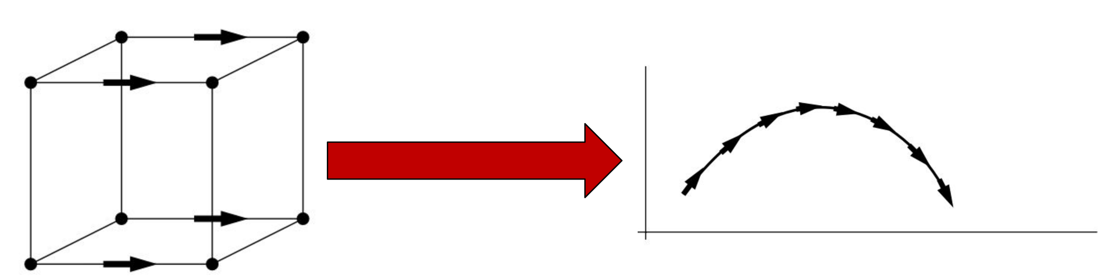


To Our Problem … :
Q_i : actions of ith robot;
M_i : ith action of all robots
After reduction, Submodular Welfare Maximization is equivalent to
$\max\{f(S) : S \in I\}$.
Simple greedy algorithm can give **1/2-approximation.**

But We can make it better …

Multilinear Extension

Discrete Problem	→	(Multilinear) Continuous Problem
$\max\{f(S) : S \in F\}$		$\max\{F(y) : y \in P(F)\}$



$$F(y) = \mathbb{E}_{S\sim y}[f(S)] = \sum_{R\subseteq N} f(R) \prod_{i\in R} y_i \prod_{i\notin R} (1-y_i)$$

Find the maximum value of F(y) would be **NP-hard**, then continuous greedy algorithm can give us **(1-1/e)-approximation** algorithm

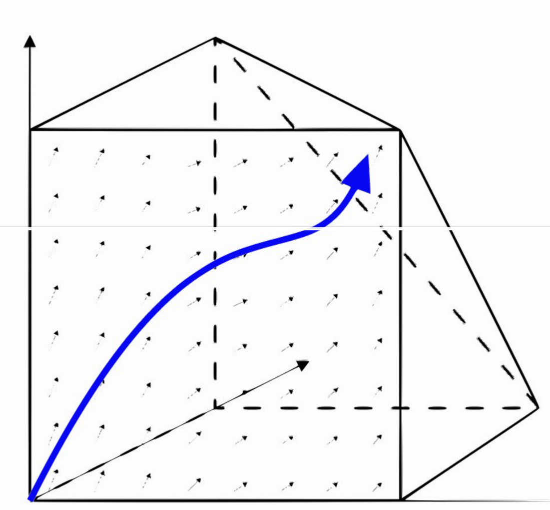
Continuous Greedy Algorithm

For each $y \in P$, define $v(y)$ by $v(y) = argmax_{\{v\in P\}}(v \cdot \nabla F\ |y)$

Define a curve $y(t)$: $y(0) = 0, \frac{dx}{dt} = v(x)$

Run this process for $t \in [0,1]$ and return $y(1)$.

Explanation: The function $y(t)$ can be seen as a particle moving from $t = 0$ to $t = 1$ in the feasible region to maximize local gain.
Hence, $y(t)$ can be seen as the blue curve in the right graph.



Algorithm

Algorithm 1 Continuous Process

$y(0) \leftarrow 0$
$v(y) \leftarrow argmax_{v\in P}(v \cdot \nabla F(y))$
$\frac{dy}{dt} \leftarrow v(y)$
return $y(1)$

Now, we need discretize it to obtain a polynomial time algorithm …

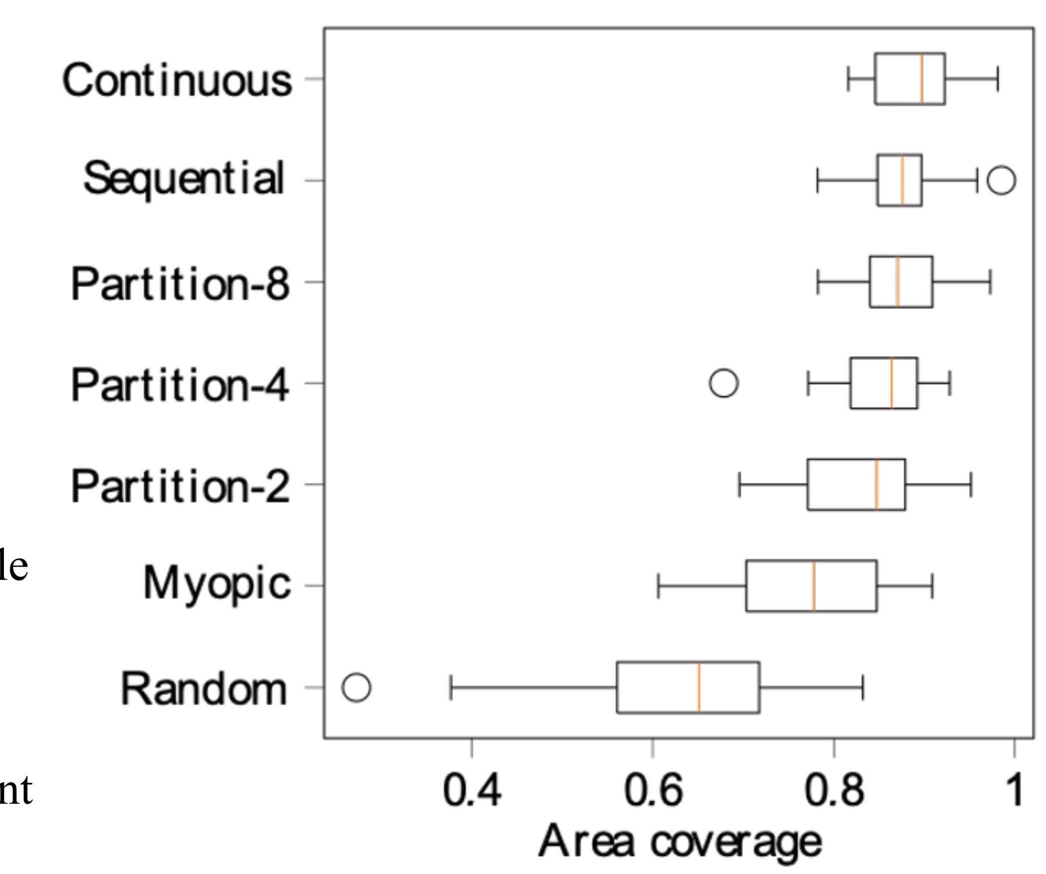
Algorithm 2 Multi-Robot Multi-Target Coverage Problem

$\delta \leftarrow \frac{1}{(mn)^2}$
$t \leftarrow 0$; $y_{ij}(0) = 0, \forall i, j$
while $t < 1$ **do**
  $w_{ij} \leftarrow \mathbb{E}[g(R(t)+j) - g(R(t))]$ by taking $(mn)^5$ samples
  for each j **do**
    $i_j(t) \leftarrow argmax_i w_{ij}(t)$
    $y_{ij} \leftarrow y_{ij} + \delta$
  end for
  $t \leftarrow t + \delta$
end while
Assign each robot j to its ith action independently with probability y_{ij}

Continuous greedy algorithm for submodular welfare problem allow us to get result distribution y which **F(y) ≥ (1 − 1/e)OPT**. [2]

Result

We set the number of drones to five and each drone has five possible motions (four possible ways of coverage). Continuous Greedy (median: 89.7%) gives slightly better results than Sequential Greedy (median: 87.5%).



Limitation:

- It is a simple problem, since just covering a single plane with simple shapes—versus trajectories and views over a long horizon (more room to make poor decisions)
- Problems too small (less significant variation in performance)

References

[1] R. H. Kabir and K. Lee, "Wildlife monitoring using a Multi-UAV system with optimal transport theory," *MDPI*, 29-Apr-2021. [Online]. Available: https://www.mdpi.com/2076-3417/11/9/4070. [Accessed: 28-Apr-2023].

[2] J. Vondrák, "Optimization of submodular functions tutorial - lecture I." [Online]. Available: https://theory.stanford.edu/ jvondrak/data/submod-tutorial-1.pdf. [Accessed: 01-Apr-2023].