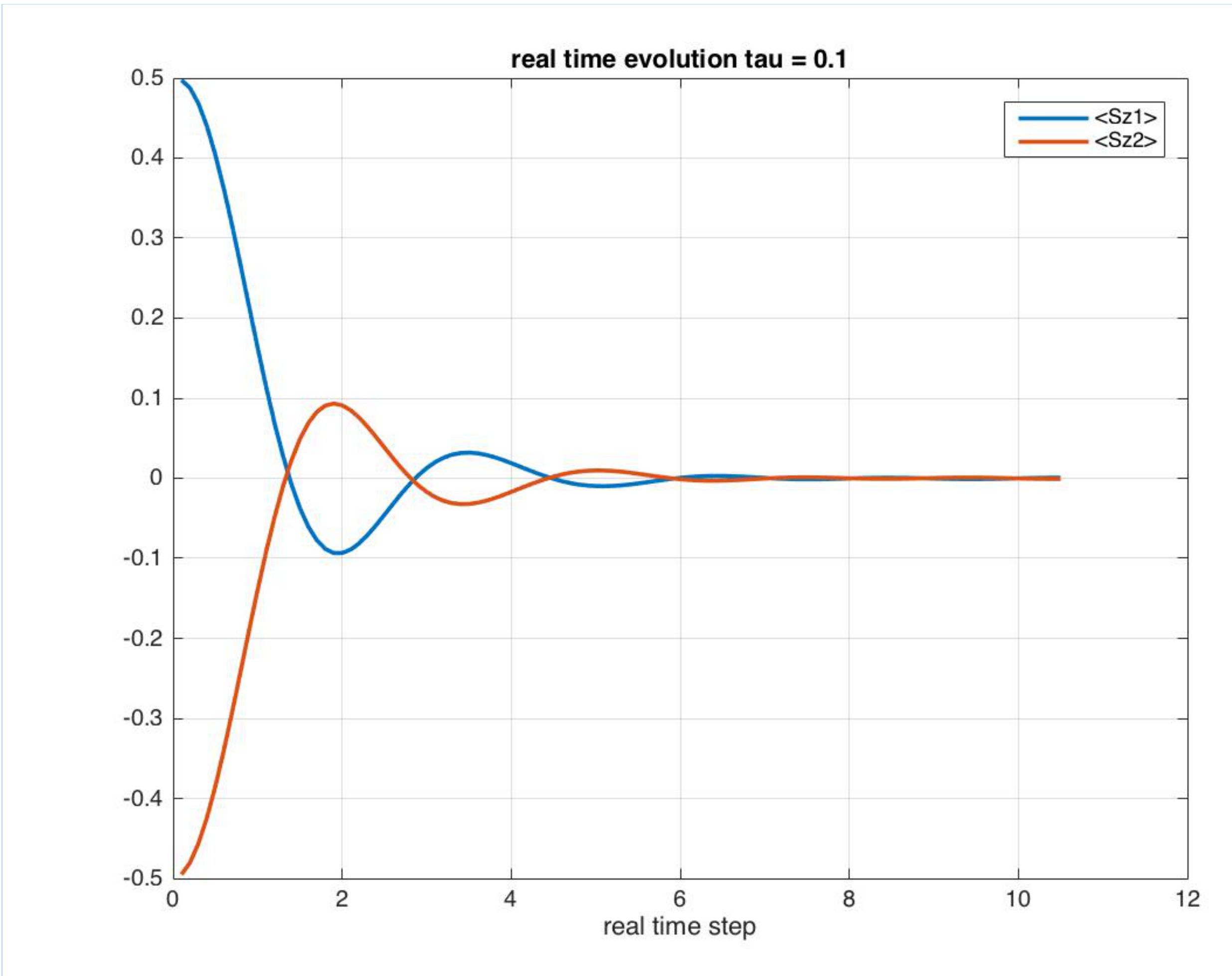
Name: Yifan Tian ID: 78921267   HW8

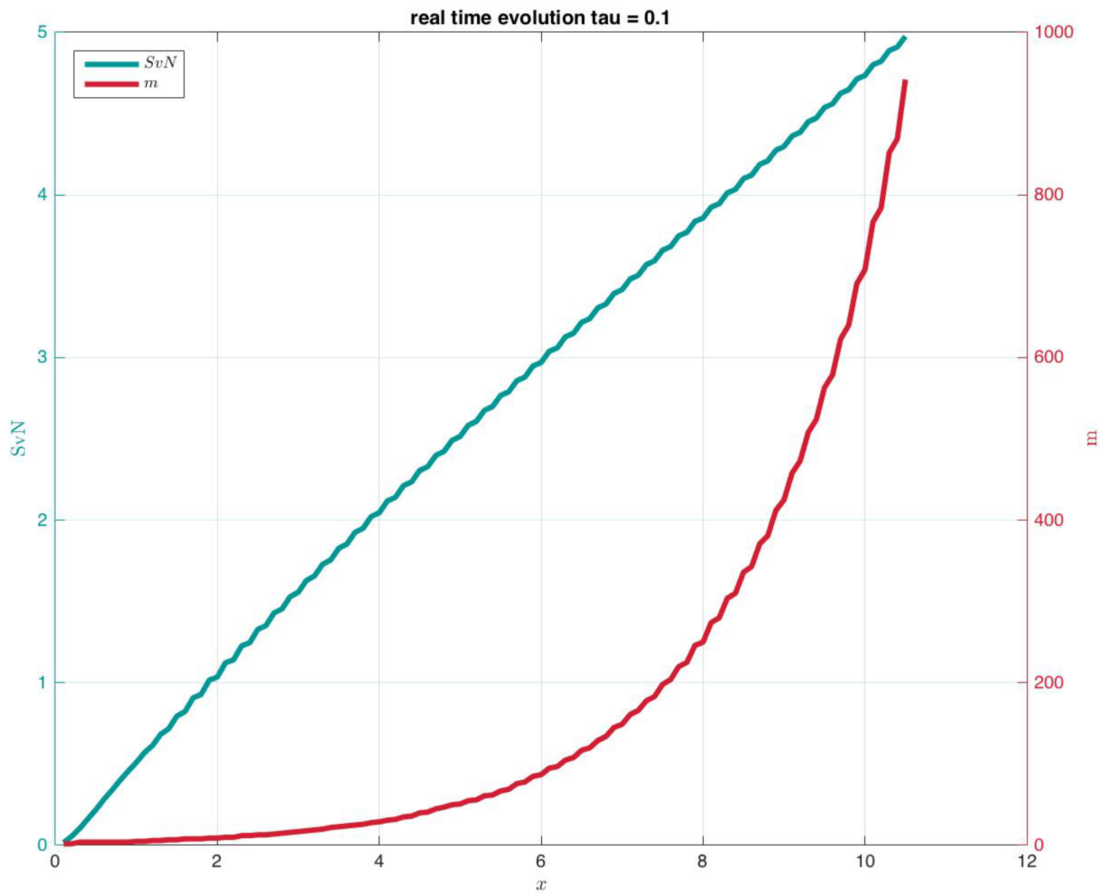
1.

Iniitial state is Ising ground state: $|\psi\rangle = |\uparrow\downarrow\rangle$,
then evolved in real time under Heisenberg Hamiltonian $H = 0.5(S_+S_- + S_-S_+) + S_zS_z$

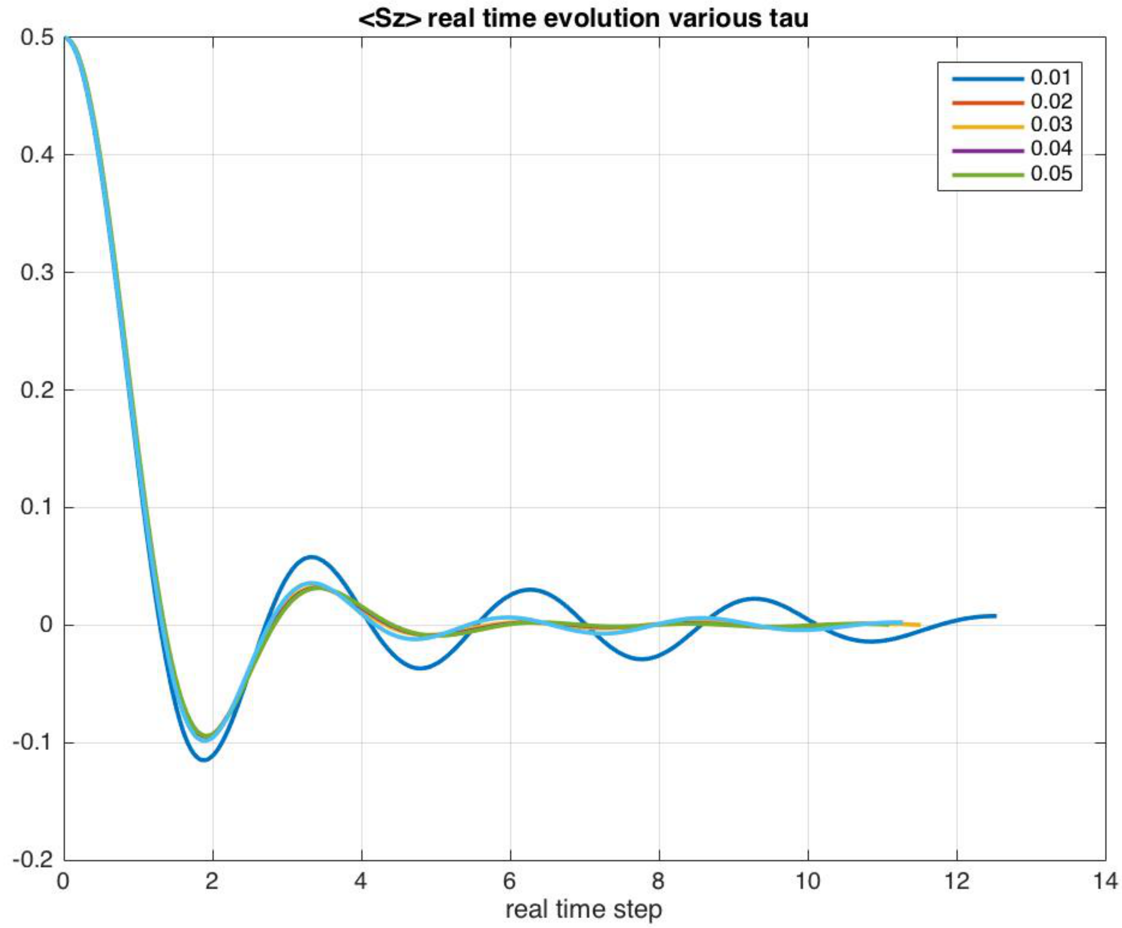
tau = 0.1, cutoff = 1E-6



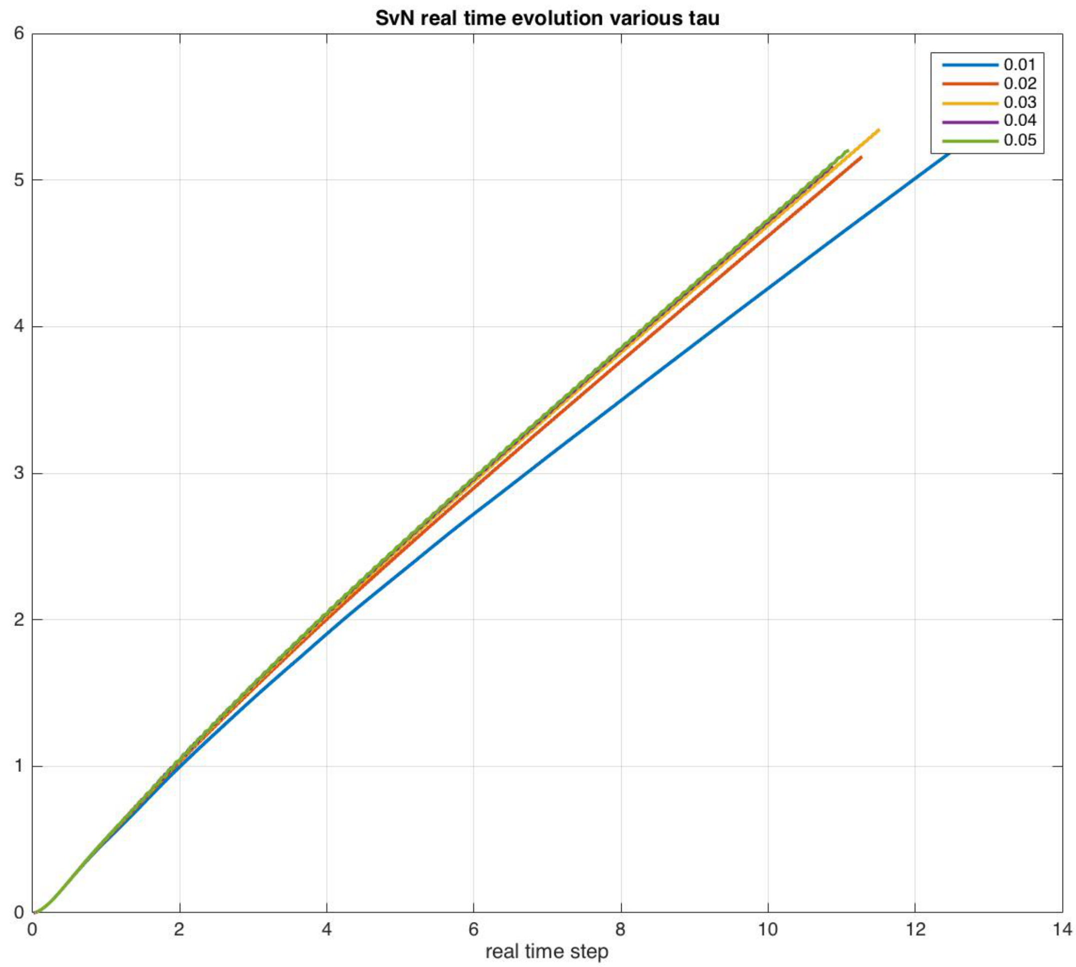
The change of SvN over time.



The result of various τ. From here we see that except for τ = 0.01, Other τ are pretty close, means they reach convergence.

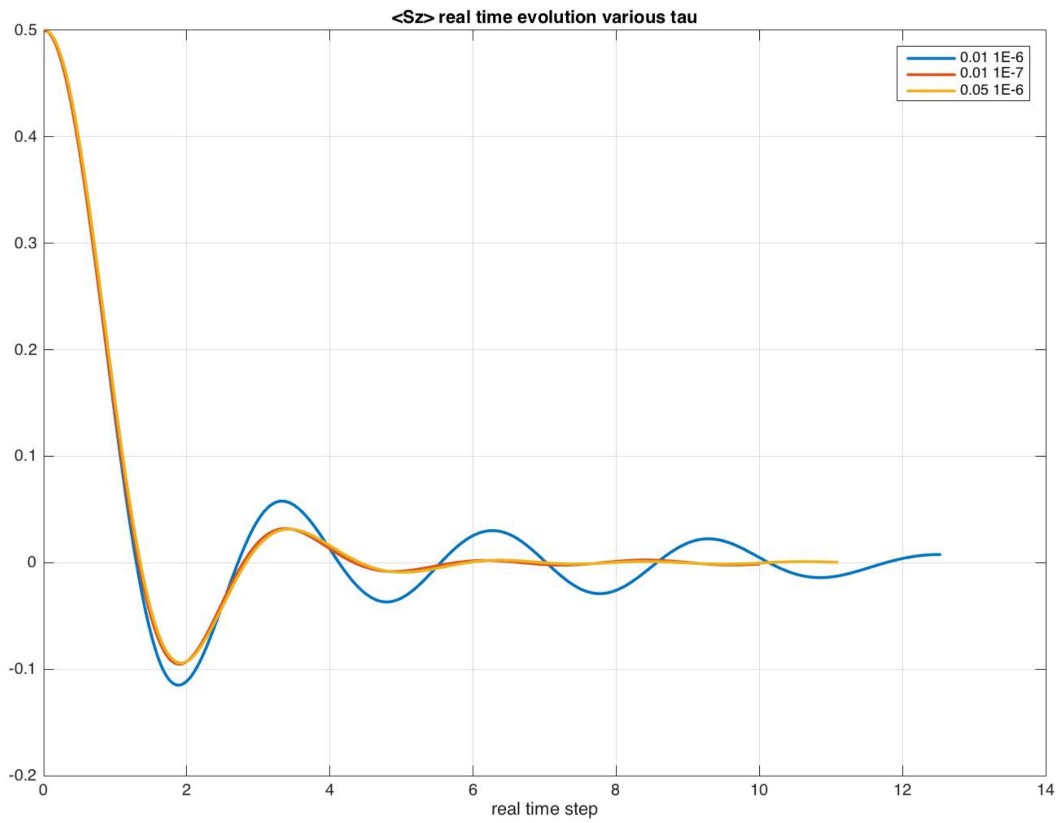


The corresponding SvN, also τ = 0.01 diverge faw away.



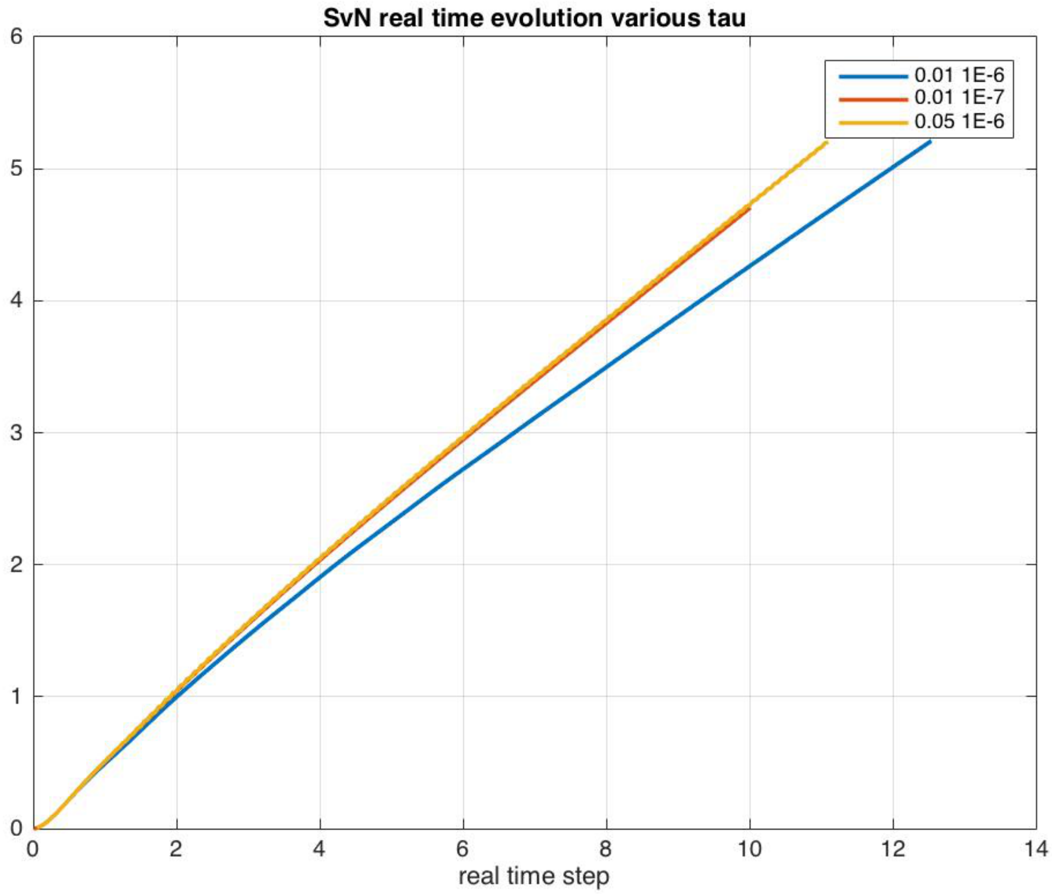
The reason why τ=0.01 is not good is that even though smaller τ reduce trotter error, but more steps cause bigger truncation error.

The result of same τ = 0.01 with smaller trotter error:



<Sz> real time evolution various tau
0.01 1E-6
0.01 1E-7
0.05 1E-6
real time step
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
0
2
4
6
8
10
12
14

And SvN:



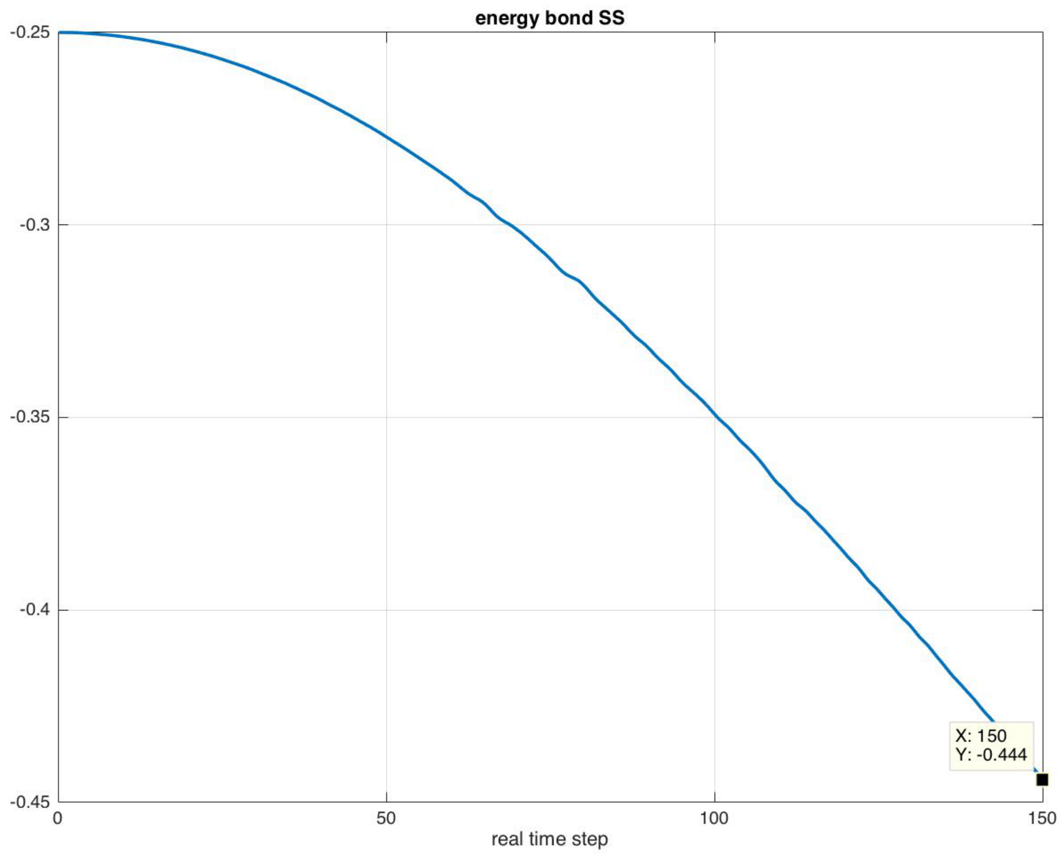
2.

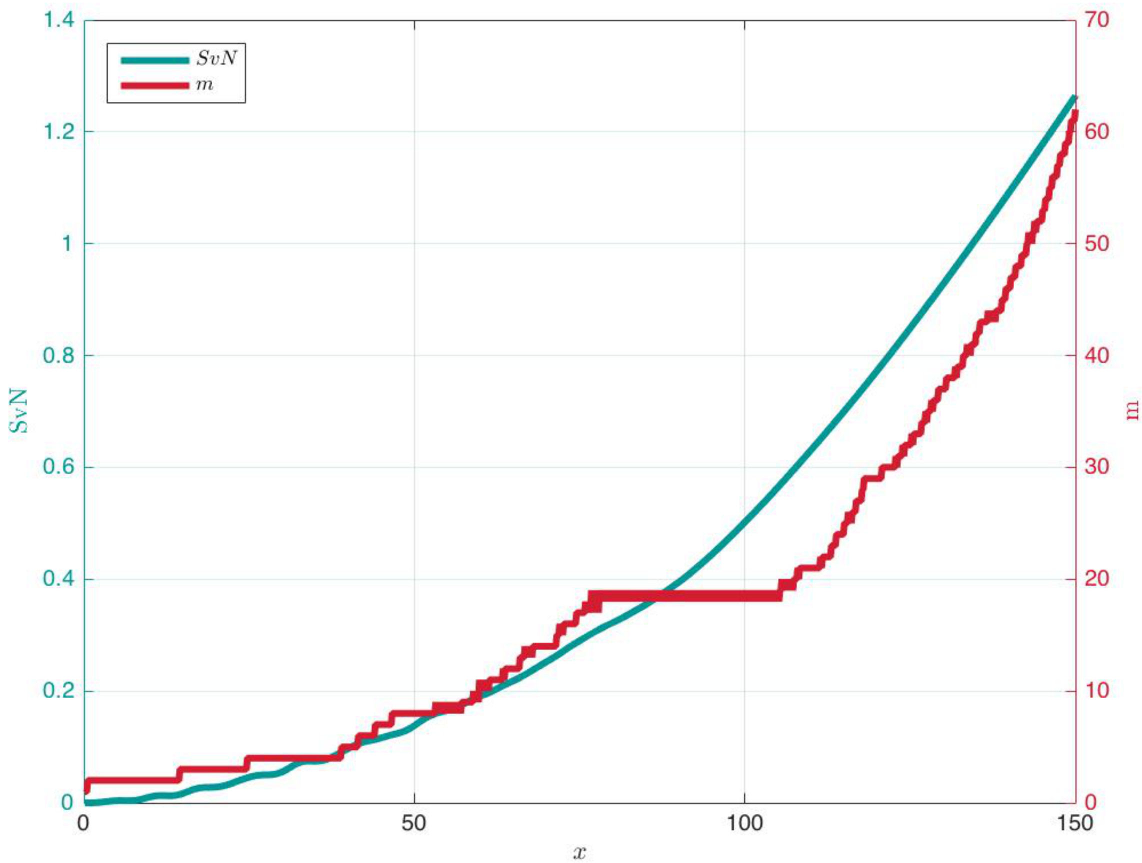
Then vary H slowly with time from Ising Hamiltonian to Heisenberg Hamiltonian to perform adiabatic evolution.

Exact infinite Heisenberg SS bond:
(1/4-log(2)) = -0.44315

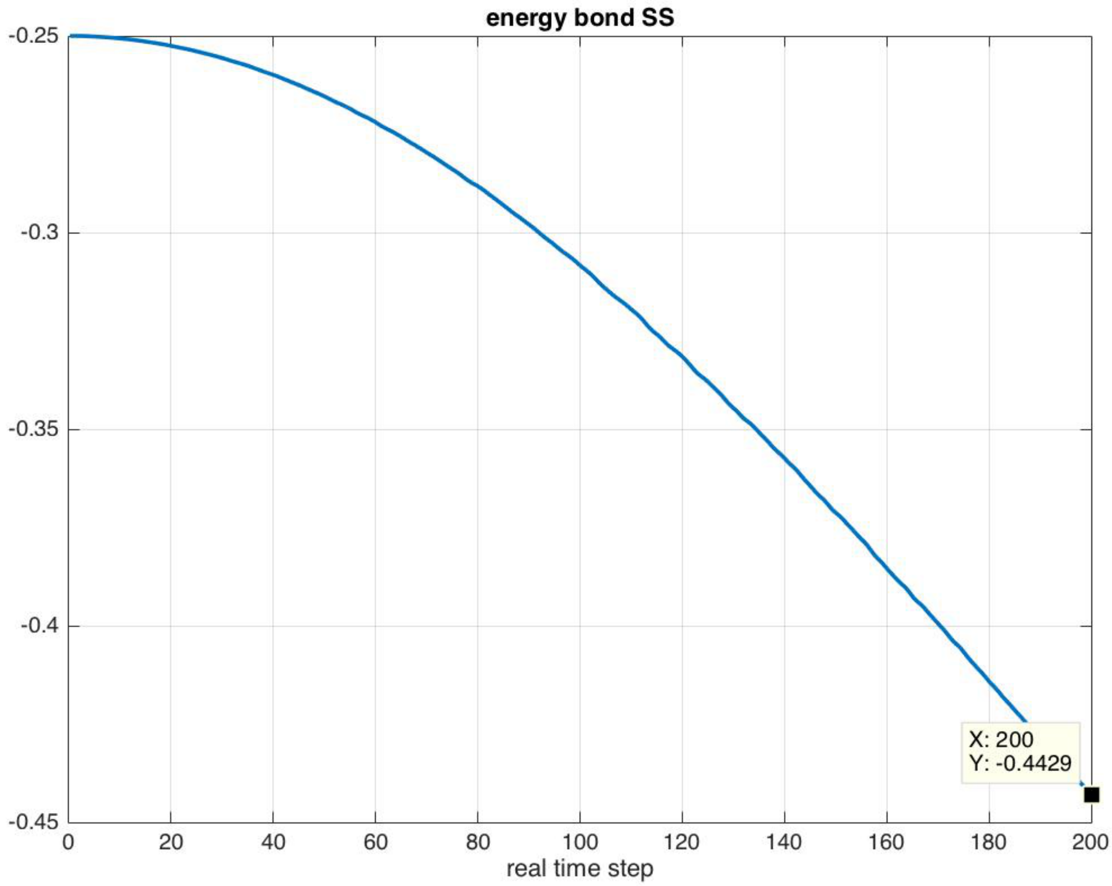
Good scheme:
(τ = 0.15~0.2; smaller Nsweeps because truncation error will accumulate; ensure that m is not very small; and change H every step to quench smoothly.

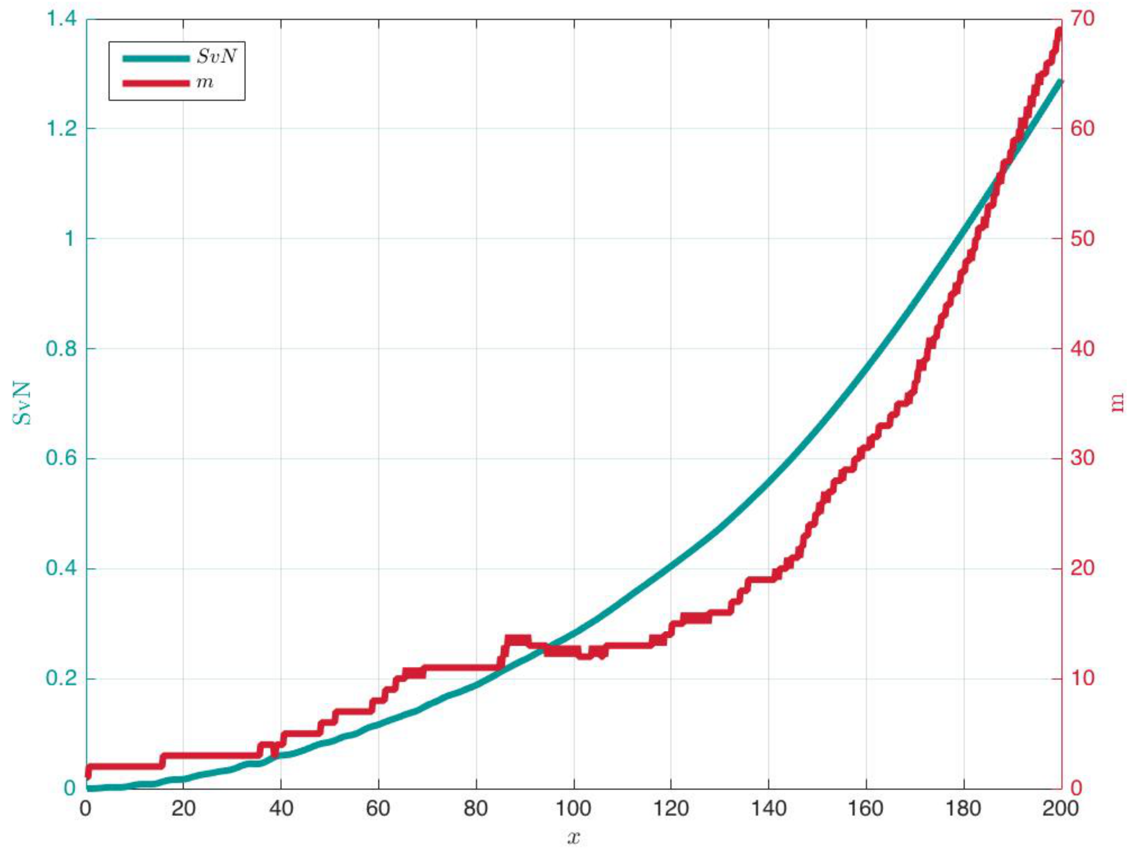
tau = 0.15 cutoff = 1E-7 nsweep = 1000 ,E = -0.444:


energy bond SS
-0.25
-0.3
-0.35
-0.4
-0.45
0
50
100
150
real time step
X: 150
Y: -0.444



tau = 0.2 cutoff = 1E-7, nsweep = 1000:


energy bond SS
-0.25
-0.3
-0.35
-0.4
-0.45
0
20
40
60
80
100
120
140
160
180
200
real time step
X: 200
Y: -0.4429



tau = 0.15 cutoff = 1E-8   nsweep = 2000,   **E = -0.443195(0.0107%)**,   m = 130
tau = 0.2   cutoff = 1E-8   nsweep = 1000,   E = -0.4430407207(0.024),   m = 359
tau = 0.1   cutoff = 1E-8   nsweep = 2000,   E = , m = -0.443802(0.148) , m = 91

So we see that tau = 0.15 here works best.