CSCB63 – Design and Analysis of Data Structures

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[1]with huge thanks to Anna Bretscher and Albert Lai

priority queue

Collection of priority-job pairs; priorities must be comparable.

- `insert(`p`, `j`)`: insert job j with priority p
- `max()`: return job with max priority

O(height)

- `extract-max()`: remove and return job with max priority

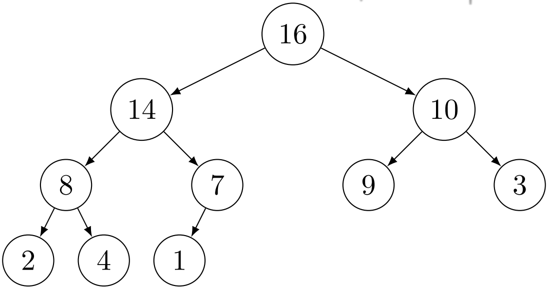
O(1)

heap

A heap is one way to store a priority queue. A heap is:

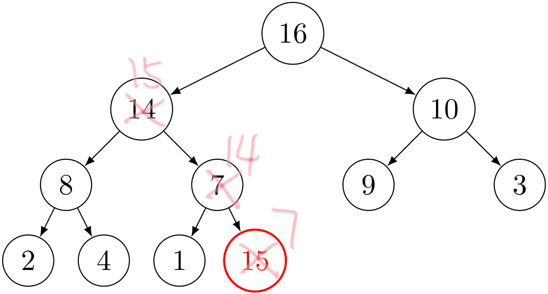
- a binary tree
- "nearly complete": every level i has 2^i nodes, except the bottom level; the bottom nodes flush to the left
- at each node n: $\mathit{priority}(n) \geq \mathit{priority}(n.\mathit{left})$ and $\mathit{priority}(n) \geq \mathit{priority}(n.\mathit{right})$

priority = value



heap insert: example

Insert job with priority 15.



√ The tree is still "nearly-complete". But:

solution: Swap 15 with 7 and 15 with 14

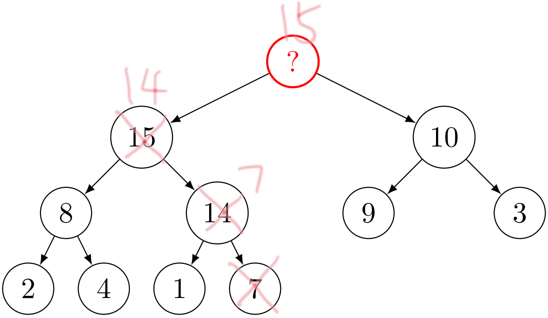
heap insert: algorithm

`insert(p, j)`:

1. `v := new node(p, j)`
2. insert `v` at bottom level, leftmost free place
   (keep the tree "nearly-complete")
3. while `v` has parent `p` with `p.priority < v.priority`:
   - swap `v.priority` and `p.priority`
   - swap `v.job` and `p.job`
   - `v := parent(v)`

Worst case time: O(height)

heap extract-max: example



new root?

move 7 to root

swap 7 and 15

swap 7 and 14

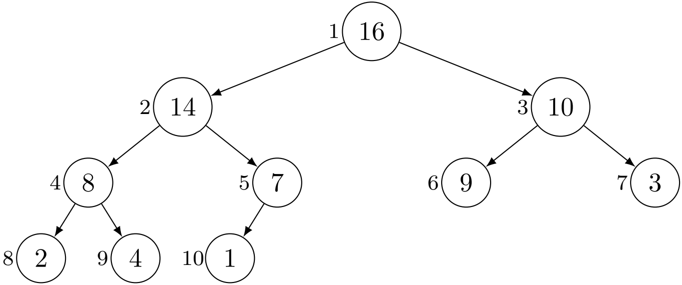
heap extract-max: algorithm

`extract-max()`:

1. `max_p, max_j = root.priority, root.job`
2. move `(priority, job)` from last (bottom, rightmost) node into root
3. remove last node
4. `v := root`
5. while `v` has child `c` with `c.priority > v.priority`:
   - `c :=` child of `v` with largest priority
   - swap `v.priority` and `c.priority`
   - swap `v.job` and `c.job`
   - `v := c`
6. return `max_p, max_j`

Worst case time: $O(\text{height})$

heap in array/vector



	16	14	10	8	7	9	3	2	4	1	
0	1	2	3	4	5	6	7	8	9	10	11

heap in array/vector

	16	14	10	8	7	9	3	2	4	1	
0	1	2	3	4	5	6	7	8	9	10	11

Easy:

- where to insert/remove?
- saves space:

Where are children/parents?

- left child of node at index i: $2i$
- right child of node at index i: $2i+1$
- parent of index node at i: $\lfloor \frac{i}{2} \rfloor$

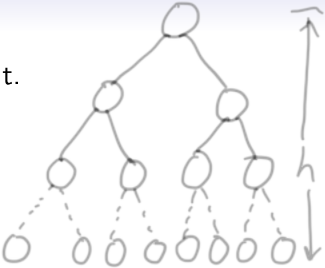
Downside?

heap: height

Let n be the number of nodes, h be the height.

- largest n: bottom level is full
    -
- smallest n: only 1 node at bottom level
    - $h-1$ levels are full
    -

Proof: $h \in \Theta(\log_2 n)$



maximum number of nodes with height h: $2^h - 1$

minimum number of nodes with height h:

$$2^{h-1} - 1 + 1 = 2^{h-1}$$

At height $2^{h-1} \leq n \leq 2^h - 1 \Rightarrow h = \lfloor \log_2 n \rfloor + 1$

$$(2^{h-1} \leq n < 2^h)$$