STAT2203/7203: Week 9 Practical Questions

1. Let X_1, X_2, \ldots, X_n be a collection of n independent random variables with $\mathbb{E}X_i = \mu$ and $\text{Var}(X_i) = \sigma^2$. Define the random variables

$$\bar{X} = \frac{1}{n} \sum_{i=1}^{n} X_i$$
, and $S^2 = \frac{1}{n-1} \sum_{i=1}^{n} (X_i - \bar{X})^2$.

- (a) What is the expected value and variance of \bar{X} ?
- (b) Compute the covariance of X_1 and \bar{X} .
- (c) What is the expected value of S^2 ?
- 2. Serious gaming technology is increasingly being used as a method of training. A 2010 study compared the efficacy of a serious game 'Triage Trainer' to traditional card-sort exercises in preparing learners for a major incident triage. In this study 91 learners were randomly assigned to one of two training groups: 44 participants practiced triage sieve protocol using a card-sort exercise, whilst the remaining 47 participants used 'Triage Trainer'. After the training sessions, each participant was evaluated by triaging eight casualties in a simulated live exercise. Their performance was assessed in terms of accuracy and speed.

The file triage.csv contains the following variables:

Group: Either CS for the card-sort group or TT for the 'Triage Trainer' group.

Accuracy: True if participant correctly triaged all eight casualties in the simulated exercise. Otherwise False.

Time: The time (in seconds) for the participant to triage all eight casualties.

Assuming we have set the working directory to be the folder containing Triage.csv, then we can obtain summary statistics.

```
> table(Triage$Group, Triage$Accuracy)
     False True
  CS
        20
              24
        13
  TT
> aggregate(Time ~ Group, data = Triage, mean)
  Group Time
     CS
1
         435
         456
> aggregate(Time ~ Group, data = Triage, sd)
  Group Time
     CS
1
          74
2
     TT
          62
```

Accuracy: In the card-sort group, 24 of the 44 participants got a score of 8/8. In the triage trainer group, 34 of the 47 participants got a score of 8/8.

Speed: The average time taken to triage all eight casualties in the card-sort group was 435s with a sample standard deviation of 74s, whereas the average time taken to triage all eight casualties in the 'Triage Trainer' group was 456s with a sample standard deviation of 62s.

(a) Describe the distribution of time take to triage all eight casualties for both groups. Is it reasonable to construct confidence intervals for the means based on the t-distribution with this data?

The following code may be useful to construct plots for the time taken split by group.

```
qqmath(~ Time | Group, data = Triage)
bwplot(Group ~ Time, data = Triage)
histogram(~ Time|Group, data = Triage)
```

- (b) Construct a 95% confidence interval for the mean time taken to triage eight casualties by a person trained using the 'Triage Trainer' game.
- (c) Construct a 99% confidence interval for the true proportion of learners from the card-sort method who correctly assign all eight casualties.
- (d) How many participants using the card-sort method would be need to reduce the margin of error of the 99% confidence interval to 0.1?