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1.

(a) Sum of overheads $= 5 \times 5 = 25$

profit: $2 \times 15 - 25 > 0$

So we need to calculate number of hired machine ≥ 2

$P(X=2) = \frac{4^2}{2!} e^{-4} = 0.1465$ $\Rightarrow P(X \geq 2) = 0.6933$

$P(X=3) = \frac{4^3}{3!} e^{-4} = 0.1953$

$P(x=4) = \frac{4^4}{4!} e^{-4} = 0.1953$

$P(x=5) = \frac{4^5}{5!} e^{-4} = 0.1562$ #

(b) $P(x=2) = \frac{4^2}{2!} e^{-4} = 0.1465$

$P(x=3) = \frac{4^3}{3!} e^{-4} = 0.1953$

$P(x=4) = \frac{4^4}{4!} e^{-4} = 0.1953$

$P(x=5) = \frac{4^5}{5!} e^{-4} = 0.1562$

profit: $X=0 \Rightarrow -25$, $X=1 \Rightarrow -10$, $X=2 \Rightarrow 5$

$X=3 \Rightarrow 20$, $X=4 \Rightarrow 35$, $X=5 \Rightarrow 50$

excepte profit $= (-25) \cdot 0.0183 + (-10) \cdot 0.0733 + 5 \times 0.1465$
$+ 20 \times 0.1953 + 35 \times 0.1953 + 50 \times 0.1562 = 18.0935$

#

2.

(a) $\begin{cases} 1 - e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$

$0.5 = 1 - e^{-\lambda x}$

$e^{-\lambda x} = 0.5$

$\ln(0.5) = -\lambda x$

$x = -\frac{\ln(0.5)}{\lambda}$

$\lambda = \frac{1}{35200}$

$x = 24398.7$

#

(b) $P(X<10000) = \int_0^{10000} \lambda e^{-\lambda x}\,dx$

$= \int_0^{10000} \frac{1}{35200} e^{-\frac{x}{35200}}\,dx$

$= 0.2473$ #

(c) $P(X \geq x) = 1 - P(X<x)$

$= 1 - \int_0^x \lambda e^{-\lambda x}\,dx$

$0.01 = 1 - \int_0^x \frac{1}{35200} e^{-\frac{1}{35200}x}\,dx$

$= 1 - (-e^{-\frac{1}{35200}x} + e^0)$

$= e^{-\frac{1}{35200}x}$

$x = 162102$ #

3. $X \sim N(1.4)$

(a)

$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

$= \frac{1}{\sqrt{8\pi}} e^{-\frac{(x-1)^2}{8}}$

$f_Y(y) = f_X(x)\frac{dx}{dy}$

$\because Y = e^x$

$\therefore x = \ln y$

$f_Y(y) = f_X(\ln y)\frac{\ln y}{dy}$

$= \frac{1}{y\sqrt{8\pi}} e^{-\frac{(\ln y-1)^2}{8}}$ #

(b) $E(Y) = M(t) = E(e^{tX})$, $t=1$

$= \int_{-\infty}^{+\infty} e^{tx} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}\,dx$

$= \frac{1}{\sqrt{2\pi}\sigma}\int_{-\infty}^{+\infty} e^{-\frac{x^2-2\mu x+\mu^2-2\sigma^2 tx}{2\sigma^2}}\,dx$

$= \frac{e^{-\frac{\mu^2-(\mu+\sigma^2 t)^2}{2\sigma^2}}}{2\sqrt{\pi}\sigma}\int_{-\infty}^{+\infty} e^{-\frac{(x-(\mu+\sigma^2 t)^2)}{2\sigma^2}}\,dx$

$= e^{\mu t+\frac{\sigma^2}{2}t^2}$ $\because t=1$ $\therefore E(Y) = e^{\mu+\frac{\sigma^2}{2}}$

$= e^3$

$E(Y^2)$

$= \int_0^{+\infty} y^2 f(y)\,dy$

$= \int_0^{+\infty} \frac{y}{\sqrt{2\pi}\sigma} e^{-\frac{(\ln y-\mu)^2}{2\sigma^2}}\,dy$

$= e^{2\mu+2\sigma^2}$

$Var(Y) = E(Y^2) - E(Y)^2$

$= e^{10} - e^6$ #

4.

(a) $X \sim Exp(1)$ $\lambda = 1$

$$\Rightarrow f_X(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases} \Rightarrow \begin{cases} e^{-x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

$\because y \geq -x$, $x \geq 0$

$\therefore y \geq -x$

$$f_Y(y) = \int_{-\infty}^{+\infty} f(x,y)\,dx$$

$$= \int_{-\infty}^{+\infty} f_{Y|X}(y|x) \cdot f_X(x)\,dx$$

$$= \int_{-\infty}^{+\infty} e^{-(y+x)} \cdot e^{-x}\,dx$$

$$= \int_0^{\infty} e^{-y-2x}\,dx$$

$$= e^{-y}\int_0^{\infty} e^{-2x}\,dx$$

$$= \frac{1}{2}e^{-y} \quad , y \geq -x, x \geq 0$$

$$f_Y(y) = \begin{cases} \frac{1}{2}e^{-y}, & y \geq -x, x \geq 0 \\ 0, & \text{else} \end{cases}$$

#

(b) $Cov(X,Y) = E(XY) - E(X)E(Y)$

$\because X \sim exp(1)$

$\therefore E(X) = 1$

$$E(XY) = E(X\,E(Y|X))$$

$$E(Y|X) = \int_{-x}^{+\infty} y \cdot f_{Y|X}(y|x)\,dy$$

$$= \int_{-x}^{+\infty} y \cdot e^{-(y+x)}\,dy$$

$$= e^{-x}\int_{-x}^{+\infty} y \cdot e^{-y}\,dy$$

$$= e^{-x}(-x \cdot e^{x} + e^{x})$$

$$= 1 - x$$

$$E(Y) = E(E(Y|X))$$

$$= E(1-x)$$

$$= 0$$

$$E(XY) = E(X - X^2)$$

$$E(X^2) = \int_0^{+\infty} x^2 \cdot f_X(x)\,dx$$

$$= \int_0^{+\infty} x^2 \cdot e^{-x}\,dx$$

$$= 2$$

$$Cov(X,Y) = E(1-2) - E(1)\cdot E(0) = -1$$

#

5.

(a)

$$\int_0^1 \int_0^1 f_{X,Y}(x,y)\,dx\,dy = 1$$

$$= \int_0^1 \int_0^1 cxy\,dx\,dy$$

$$= \int_0^1 \frac{1}{2}cx^2y \Big|_0^1 dy \qquad \Rightarrow C = 4$$

$$= \int_0^1 \frac{1}{2}cy\,dy$$

$$= \frac{1}{4}cy^2 \Big|_0^1 = \frac{1}{4}C = 1 \qquad \#$$

(b) $f_{X,Y}(x,y) = \begin{cases} 4xy, & 0 \le x \le 1,\ 0 \le y \le 1 \\ 0 & \text{else} \end{cases}$

if $f_{X,Y}(x,y) = f_X(x) \cdot f_Y(y) \Rightarrow$ independent

$$f_Y(y) = \int_{-\infty}^{+\infty} f_{X,Y}(x,y)\,dx \qquad \Rightarrow \begin{cases} 2y & 0 \le y \le 1 \\ 0 & \text{else.} \end{cases}$$

$$= \int_0^1 4xy\,dx$$

$$= 2y$$

$$f_X(x) = \int_{-\infty}^{+\infty} f_{X,Y}(x,y)\,dy \qquad \Rightarrow \begin{cases} 2x & 0 \le x \le 1 \\ 0 & \text{else.} \end{cases}$$

$$= 2x$$

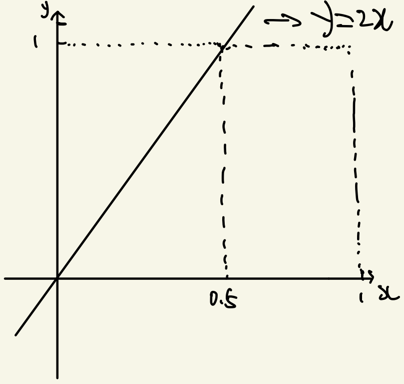
So we can get $f_{X,Y}(x,y) = 4xy = f_X(x) \cdot f_Y(y) = 2x \cdot 2y = 4xy$

X and Y independent #

(c)

$$2X > Y \qquad \Rightarrow \qquad 0 \le x \le 1$$

$$2X - Y = 0 \qquad\qquad 0 \le y \le 2x$$



$$\int_0^{0.5}\int_0^{2x} 4xy\,dy\,dx + \int_0^1\int_{0.5}^1 4xy\,dx\,dy$$

$= 0.875$

#

6.

```
>
> data <- read.csv("/Users/meviusz/UQ/sem2-23/STAT7203/ass2/TextSpeed.csv")
>
> x <- data$SitWPM
> n <- length(x)
> sample_mean <- mean(x, na.rm = TRUE)
> sample_std_dev <- sd(x, na.rm = TRUE)
>
>
> alpha <- 0.1
> z <- qnorm(1 - alpha / 2)
>
> margin_of_error <- z * (sample_std_dev / sqrt(n))
> lower_limit <- sample_mean - margin_of_error
> upper_limit <- sample_mean + margin_of_error
>
> cat("Lower limit of the 90% confidence interval: ", lower_limit, "\n")
Lower limit of the 90% confidence interval:  40.0355
> cat("Upper limit of the 90% confidence interval: ", upper_limit, "\n")
Upper limit of the 90% confidence interval:  42.13116
```

#