

**Question 1 – Prove the following using only the three core Armstrong's Axioms**

Union: If  $X \rightarrow Y$  and  $X \rightarrow Z$ , then  $X \rightarrow \{Y, Z\}$

$$\begin{aligned} & \{X\} \rightarrow \{Y, Z\} \\ & \{Y, Z\} \rightarrow \{Z, Y\} \\ & X \rightarrow \{Y, Z\} \end{aligned}$$

Decomposition: If  $X \rightarrow \{Y, Z\}$ , then  $X \rightarrow Y$  and  $X \rightarrow Z$

$$\begin{aligned} & \{Y, Z\} \rightarrow Y, \quad \{Y, Z\} \rightarrow Z \\ & X \rightarrow Y, \quad X \rightarrow Z. \end{aligned}$$

Pseudo-transitivity: If  $X \rightarrow Y$  and  $\{Y, Z\} \rightarrow A$ , then  $\{X, Z\} \rightarrow A$

$$\{X, Z\} \rightarrow \{Y, Z\} \rightarrow A.$$

**Question 2 – Given the following data, identify which of the options are \*potential\* Functional Dependencies**

A	B	C	D	E
1	X	1	M	1
2	Y	1	M	1
3	Y	4	N	3
4	W	2	L	5
5	W	2	M	1
6	T	5	O	2

- |                               |                                |
|-------------------------------|--------------------------------|
| a) $A \rightarrow B$ ✓        | b) $B \rightarrow A$ ✗         |
| c) $A \rightarrow C$ ✓        | d) $B \rightarrow C$ ✗         |
| e) $C \rightarrow D$ ✓        | f) $C \rightarrow E$ ✗         |
| g) $D \rightarrow E$ ✓        | h) $\{A, B\} \rightarrow C$    |
| i) $\{B, C\} \rightarrow E$ ✗ | j) $\{B, C, D\} \rightarrow E$ |

**Question 3 – Fill in the following table using the stated Functional Dependencies**

FD 1:  $A \rightarrow B$

FD 2:  $B \rightarrow C$

FD 3:  $\{C, D\} \rightarrow E$

A	B	C	D	E
1	2	1	6	2
1	2	1	4	3
2	4	2	7	4
3	2	1	4	3

**Question 4 - Find all candidate keys for the given relations and functional dependencies.**

a) R [A, B, C, D, E, F, G, H, I]

$\{A, B\} \rightarrow \{C, D\}$

$\{A\} \rightarrow \{E\}$

$\{B\} \rightarrow \{F, H\}$

$\{C\} \rightarrow \{G\}$

$\{D\} \rightarrow \{B\}$

$\{G\} \rightarrow \{C\}$

$\{H\} \rightarrow \{I\}$

Candidate Key(s):  $\{AB\}, \{D, A\}$ .

b) R [A, B, C, D, E] ABCD E

$\{A\} \rightarrow \{B, C\}$

$\{C, D\} \rightarrow \{E\}$

$\{A, C\} \rightarrow \{E\}$

$\{B\} \rightarrow \{D\}$

$\{E\} \rightarrow \{A, B\}$

Candidate Key(s):  $\{A\}, \{E\}, \{C, D\}, \{B, C\}$

c) R [A, B, C, D]

$\{A, B\} \rightarrow \{C, D\}$

$\{C\} \rightarrow \{A, B, D\}$

$\{D\} \rightarrow \{C\}$

Candidate Key(s):  $\{A, B\}, C, D$

d)  $R [A, B, C, D, E, F, G, H, I, J]$

$\{A, B\} \rightarrow \{C\}$

$\{A\} \rightarrow \{D, E\}$

$\{B\} \rightarrow \{F\}$

$\{F\} \rightarrow \{G, H\}$

$\{D\} \rightarrow \{I, J\}$

Candidate Key(s):  $\{A, B\}$

e)  $R [A, B, C, D, E, F, G, H, I, J]$

$\{A, B\} \rightarrow \{C\}$

$\{B, D\} \rightarrow \{E, F\}$

$\{A, D\} \rightarrow \{G, H\}$

$\{A\} \rightarrow \{I\}$

$\{H\} \rightarrow \{J\}$

$\{J\} \rightarrow \{A, B, D\}$

Candidate Key(s):  $J, H, \{A, D\}$

**Question 5 – Normalise the following relations into BCNF**

a)

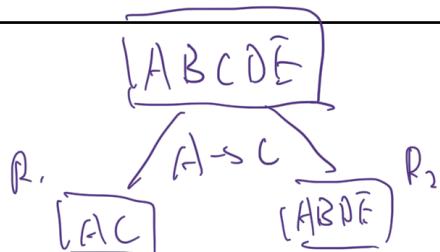
R [A, B, C, D, E]

FD 1: A → C

FD 2: {A, B} → {D, E}

$$A^+ \rightarrow \{AC\}$$

$$\{AB\}^+ \rightarrow \{ABCDE\}$$



$$R_1 [A, C] \quad \text{FD1: } A \rightarrow C$$

$$R_2 [A, B, D, E] \quad \text{FD2: } \{A, B\} \rightarrow \{D, E\}$$

b)

R [A, B, C, D, E]

FD 1: A → B

FD 2: C → {D, E}

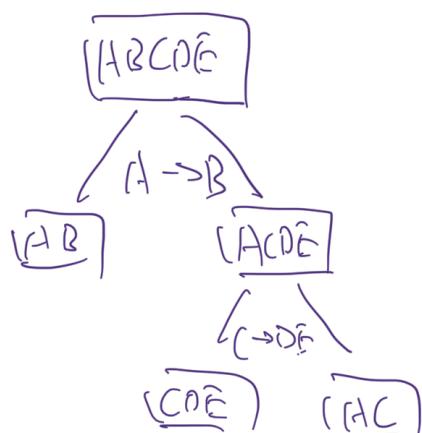
$$A^+ = AB$$

$$C^+ = CDE$$

$$R_1 [AB]$$

$$R_3 [\underline{CDE}]$$

$$R_4 [\underline{AC}]$$



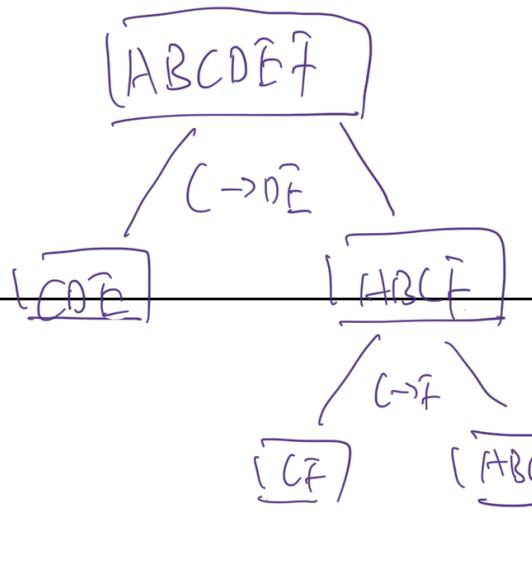
c)

R [A, B, C, D, E, F]

FD 1: A → {B, C}

FD 2: C → {D, E}

FD 3: E → F



$$A^+ = ABCD\bar{E}\bar{F}$$

$$C^+ = CO\bar{E}\bar{F}$$

$$\bar{E}^+ = \bar{E}\bar{F}$$

d)

R [A, B, C, D, E]

FD 1: A → B

FD 2: C → {D, E}

FD 3: {A, D, E} → C

FD 4: {B, C} → A

$$A^+ = AB$$

$$C^+ = CD\bar{E}$$

$$ADE^+ = AD\bar{E}CB$$

$$BC^+ = ABCD\bar{E}$$

