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1. $\sinh(x) = \dfrac{e^{2x}-1}{2e^x}$ $\sinh'(x) = \dfrac{e^{2x}+1}{2e^x}$ $\sinh''(x) = \dfrac{e^{2x}-1}{2e^x}$ $\sinh'''(x) = \dfrac{e^{2x}+1}{2e^x}$

$\sinh^{(4)}(x) = \dfrac{e^{2x}-1}{2e^x}$ ...... $\Rightarrow f(0)=0.\ f'(0)=1\ f''(0)=0.\ f'''(0)=1.\ f^{(4)}(0)=0$ ....

we can get $\sinh^n(0) = \begin{cases} 0, & n=2m \\ 1, & n=2m+1, \end{cases}$ $m \geq 0, m \in \mathbb{Z}$

Such that $\sinh(x) = \sum_{n=0}^{\infty} \dfrac{f^{(n)}(a)}{n!}(x-a)^n = \dfrac{x}{1!} + \dfrac{x^3}{3!} + \dfrac{x^5}{5!} + \dfrac{x^7}{7!} + \cdots + \dfrac{x^{2n+1}}{(2n+1)!}$

$= \sum_{n=0}^{\infty} \dfrac{x^{2n+1}}{(2n+1)!}$

$$L = \lim_{n\to\infty} \left| \frac{\frac{x^{2n+3}}{(2n+3)!}}{\frac{x^{2n+1}}{(2n+1)!}} \right| = \lim_{n\to\infty} \left| \frac{x^2}{(2n+2)(2n+3)} \right| = 0$$

By the Ratio Test, the series converges for $0<1$.

$\cosh(x) = \dfrac{e^{2x}+1}{2e^x}$ $\cosh'(x) = \dfrac{e^{2x}-1}{2e^x}$ $\cosh''(x) = \dfrac{e^{2x}+1}{2e^x}$ ......

$\Rightarrow \cosh(0)=1$, $\cosh'(0)=0$ $\cosh''(0)=1$ $\cosh'''(0)=0$ ....

we can get $\cosh^n(0) = \begin{cases} 0, & n=2m+1 \\ 1, & n=2m, \end{cases}$ $m \geq 0, m \in \mathbb{Z}$

Such that $\cosh(x) = \sum_{n=0}^{\infty} \dfrac{f^{(n)}(a)}{n!}(x-a)^n = \dfrac{x^0}{0!} + \dfrac{x^2}{2!} + \dfrac{x^4}{4!} + \cdots + \dfrac{x^{2n}}{(2n)!}$

$= \sum_{n=0}^{\infty} \dfrac{x^{2n}}{(2n)!}$

$$L = \lim_{n\to\infty} \left| \frac{\frac{x^{2n+2}}{(2n+2)!}}{\frac{x^{2n}}{(2n)!}} \right| = \lim_{n\to\infty} \left| \frac{x^2}{(2n+1)(2n+2)} \right| = 0$$

By the Ratio Test, the series converges for $0<1$

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2. $\lim_{n\to\infty}\left(\frac{n^2}{n^3}+\frac{n^2}{(n+1)^3}+\frac{n^2}{(n+2)^3}+\cdots+\frac{n^2}{(2n-1)^3}+\frac{n^2}{8n^3}\right)$

$\Rightarrow \lim_{n\to\infty}\sum_{i=0}^{n}\frac{n^2}{(n+i)^3} \Rightarrow \lim_{n\to\infty}\sum_{i=0}^{n}\frac{1}{n}\cdot\left(\frac{n}{n+i}\right)^3 \Rightarrow \lim_{n\to\infty}\sum_{i=0}^{n}\frac{1}{n}\cdot\left(\frac{1}{1+\frac{i}{n}}\right)^3$

Such that $f(n)=\left(\frac{1}{1+\frac{i}{n}}\right)^3 \Rightarrow f(x)=\left(\frac{1}{1+x}\right)^3$

$\int_0^1\left(\frac{1}{1+x}\right)^3dx = -\frac{1}{2(1+x)^2}\Big|_0^1 = \frac{3}{8}$

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3. $f(x)=|x(x-a)(x-2a)(x-3a)|$ $\begin{cases} f(x)>0, & x\in[-a,0] \\ f(x)\leq 0, & x\in[0,a] \end{cases}$

$f(x)=x^4-6ax^3+11a^2x^2-6a^3x$

$\int_{-a}^{0}x^4-6ax^3+11a^2x^2-6a^3x - \int_0^a x^4-6ax^3+11a^2x^2-6a^3x$

$\Rightarrow \frac{x^5}{5}-\frac{3ax^4}{2}+\frac{11a^2x^3}{3}-3a^3x^2\Big|_{-a}^{0}-\left(\frac{x^5}{5}-\frac{3ax^4}{2}+\frac{11a^2x^3}{3}-3a^3x^2\right)\Big|_0^a$

$=9a^5$

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4. $\int_0^2\lfloor x^2\rfloor dx = 1(\sqrt{2}-\sqrt{1})+2(\sqrt{3}-\sqrt{2})+3(\sqrt{4}-\sqrt{3}) = 6-(\sqrt{1}+\sqrt{2}+\sqrt{3})$

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5. assume $f(x)=\sqrt{x^4+1} \Rightarrow F(1+h)-F(1)=\int_1^{1+h}f(x)dx$

$\lim_{h\to 0}\frac{F(1+h)-F(1)}{h} = F'(1) = f(1) = \sqrt{2}$

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6. $f(x)=x^3 \quad g(x)=x$

$A=\int_{-1}^{1}[f(x)-g(x)]dx = \int_{-1}^{1}(x^3-x)dx$

$=(\frac{1}{4}x^4-\frac{1}{2}x^2)\Big|_{-1}^{1} = -\frac{1}{2}$ Such that the area is $\frac{1}{2}$

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7. $\frac{d f(\lambda x,\lambda y)}{d\lambda} = \frac{d(\lambda^n f(x,y))}{d\lambda}$ by homogeneous, we can gee

$\frac{\partial f}{\partial x}\cdot\frac{dx}{d\lambda}+\frac{\partial f}{\partial y}\cdot\frac{dy}{d\lambda} = \frac{\partial f}{\partial x}\cdot x+\frac{\partial f}{\partial y}y = n\cdot\lambda^{n-1}\cdot f(x,y)$

when $\lambda=1$, we get $x\frac{\partial f}{\partial x}+y\frac{\partial f}{\partial y}=n\cdot f(x,y)$ #

8. $\frac{\partial f}{\partial x}=2xy\cdot(x^4+y^2+1)^{-1}-4x^3y(x^4+y^2+1)^{-2} = \frac{2}{9}$

$\frac{\partial f}{\partial y}=2xy(x^4+y^2+1)^{-1}-2x^2y^2(x^4+y^2+1)^{-2} = \frac{1}{9}$

$f(1,1)=\frac{1}{3} \quad f(x,y)\approx\frac{1}{3}-\frac{2}{9}(x-1)-\frac{1}{9}(y-1)$

$=\frac{2}{9}x+\frac{1}{9}y$

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9. $p'(x)=3x^2 \quad q'(x)=-2\sin 2x \quad r'(x)=4 \quad \frac{p^2-r}{q^4}$

$\frac{df}{dx}=\frac{\partial f}{\partial p}\cdot\frac{dp}{dx}+\frac{\partial f}{\partial q}\cdot\frac{dq}{dx}+\frac{\partial f}{\partial r}\cdot\frac{dr}{dx}$

$$= \frac{2p}{q^4} \cdot 3t^2 - \frac{p^2-r}{4q^5} \cdot (-2\sin(2t)) - \frac{1}{q^4} \cdot 4.$$

$$= \frac{2t^3+14}{\cos^4(2t)} \cdot 3t^2 + \frac{(t^3+7)^2-4t}{4\cos^5(2t)}(2\sin(2t)) - \frac{4}{\cos^4(2t)}$$

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