Lecture 2.x

Review of Week 2

Question - Probability measures

Let A and B be two disjoint events such that $\mathbb{P}(A) > 0$ and $\mathbb{P}(B) > 0$. Which of the following statements is FALSE?

(a) $\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B)$, $\checkmark$ [sum rule]

(b) $\mathbb{P}(A^c \cup B^c) = 1$, $\checkmark$ [$A^c \cup B^c = \Omega$ as $A \cap B = \emptyset$.]

(c) $\mathbb{P}(A \cap B) = \mathbb{P}(A)\,\mathbb{P}(B)$, $\times$ [$A \cap B = \emptyset$ so $\mathbb{P}(A \cap B) = 0$]

(d) $\mathbb{P}(A|B) = 0$, $\checkmark$ [$\mathbb{P}(A \cap B) = 0$ so $\mathbb{P}(A \cap B)/\mathbb{P}(B) = 0$]

Question - Counting

Vinny from Vegas has received a "lucky" coin from "Tricky" Trish. Tricky has assured Vinny that the coin is biased towards heads. Vinny is skeptical, so decides to flip the coin eight times with the aim to investigate Tricky's claim.

How many possible outcomes are there for this random experiment?

(a) $2 \times 8 = 16$

(b) $\binom{8}{2} = 28$

(c) $2^8 = 256$ ✓

(d) $8! = 40320$

Question - Counting

Assuming the coin is fair, what is the probability of Vinny getting 8 heads from the 8 flips of the coin?

Answer: There are 256 possible outcomes from the 8 flips. There is only one way to get 8 heads. The probability of 8 heads is $1/256$.

Assuming the coin is fair, what is the probability of Vinny getting 6 heads from the 8 flips of the coin?

Answer: The number of ways to get 6 heads from 8 flips is $\binom{8}{6}$. The probability of 6 heads from 8 flips is $\binom{8}{6}/256 \approx 0.109$

Question - Counting

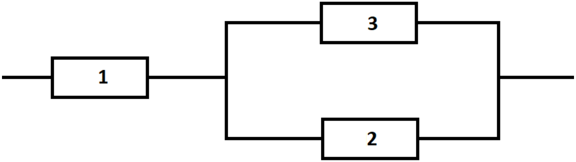
Vinny conducts his experiment and finds 7 heads out of 8 flips. Assuming the coin is fair, what is the probability of getting 7 or more heads. What should Vinny conclude?

Answer: The probability of getting 7 or more heads is

$$\begin{aligned}\mathbb{P}(\text{7 or more heads}) &= \mathbb{P}(\text{exactly 7 heads}) + \mathbb{P}(\text{exactly 8 heads}) \\ &= \frac{\binom{8}{7}}{256} + \frac{\binom{8}{8}}{256} = 0.035\end{aligned}$$

There is moderate evidence to suggest that the coin is biased towards heads.

Question - System reliability



Components in the system fail independently with probability 0.6. Let A_i be the event that component i is working. What is the event that the system is working?

(a) $A_1 \cap A_2 \cap A_3$

(b) $A_1 \cap (A_2 \cup A_3) \ = (A_1 \cap A_2) \cup (A_1 \cap A_3)$

(c) $A_1 \cup (A_2 \cap A_3)$

(d) $A_1 \cup A_2 \cup A_3$

Question - System reliability

What is the probability that component i is working? Component i fails with probability 0.6. Therefore,

$$\mathbb{P}(A_i) = 1 - 0.6 = 0.4$$

What is the probability that components 2 and 3 are working?

$$\mathbb{P}(A_2 \cap A_3) = \mathbb{P}(A_2)\,\mathbb{P}(A_3) = 0.4 \times 0.4 = 0.16, \quad \text{[by independence]}$$

Question - System reliability

What is the probability that components 2 or 3 are working?

(a) 0.16

(b) 0.4

(c) 0.64 ✓ $\mathbb{P}(A_2 \cup A_3) = \mathbb{P}(A_2) + \mathbb{P}(A_3) - \mathbb{P}(A_2 \cap A_3)$

(d) 0.8

What is the probability that the system is working?

$$\begin{aligned}
\mathbb{P}(A_1 \cap (A_2 \cup A_3)) &= \mathbb{P}((A_1 \cap A_2) \cup (A_1 \cap A_3)) \\
&= \mathbb{P}(A_1 \cap A_2) + \mathbb{P}(A_1 \cap A_3) - \mathbb{P}(A_1 \cap A_2 \cap A_3) \\
&= 0.4 \times 0.4 + 0.4 \times 0.4 - 0.4 \times 0.4 \times 0.4 = 0.256
\end{aligned}$$

Question - Hypothesis tests

Testing a hypothesis at the 5% level means that there is a 0.05 probability of rejecting the null hypothesis when in fact it is true. Suppose 12 research teams around the world independently carry out trials of the same chemical compound to see whether it is effective against HIV.

If the compound is actually not effective against HIV, what is the probability that at least one research team will find significant evidence of an effect at the 5% level?

Question - Hypothesis tests

Answer: Let A_i be the event that the i-th research team finds significant evidence at the 5% level. Assuming the compound is not effective, i.e., null hypothesis is true, $\mathbb{P}(A_i) = 0.05$.

The event that at least one team finds evidence is the *complement* of the event that no team finds evidence:

$$(A_1 \cup A_2 \cup \ldots \cup A_{12})^c = A_1^c \cap A_2^c \cap \ldots \cap A_{12}^c.$$

So

$$\mathbb{P}(A_1 \cup A_2 \cup \ldots \cup A_{12}) = 1 - \mathbb{P}(A_1^c \cap A_2^c \cap \ldots \cap A_{12}^c)$$

Question - Hypothesis tests

We know $\mathbb{P}(A_i^c) = 1 - \mathbb{P}(A_i) = 0.95$.

As the events are independent

$$\mathbb{P}(A_1^c \cap A_2^c \cap \ldots \cap A_{12}^c) = \mathbb{P}(A_1^c) \ldots \mathbb{P}(A_{12}^c) = 0.95^{12} \approx 0.54$$

The probability that at least one research team will find significant evidence of an effect at the 5% level is

$$\mathbb{P}(A_1 \cup A_2 \cup \ldots \cup A_{12}) = 1 - \mathbb{P}(A_1^c \cap A_2^c \cap \ldots \cap A_{12}^c) = 1 - 0.54 = 0.46$$