1.

(a)

$f(z) = f(-1, 1) = [-1 \quad -2]^T$ #

(b)

by (a) we can get $f(z) = [-1 \quad -2]^T$

$$g(z) = \begin{bmatrix} -1 \\ -2 \end{bmatrix} [2 \quad 1] = \begin{bmatrix} -2 & -1 \\ -4 & -2 \end{bmatrix}$$ #

(c)

$$g(z)\,z = \begin{bmatrix} -2 & -1 \\ -4 & -2 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\|g(z)z\| = \sqrt{1^2+2^2} = \sqrt{5}$$ #

(d)

$[-2 \quad -4]^T$

$[-1 \quad -2]^T$

inner product $\Rightarrow (-2\times -1) + (-4\times -2) = 10$

#

(e) $g(x) = \begin{bmatrix} u^3 v \\ uv^2 - e^{u+v} \end{bmatrix} [2 \quad 1] = \begin{bmatrix} 2u^3 v & u^3 v \\ 2uv^2 - 2e^{u+v} & uv^2 - e^{u+v} \end{bmatrix}$

$\det(g(x)) = 0 \Rightarrow$ so the answer does not depend on x #.

(f) $Df(u, v) = \begin{bmatrix} 3u^2 v, & u^3 \\ v^2 - e^{u+v}, & 2uv - e^{u+v} \end{bmatrix}$

#

(g)

$$\hat{f}(x) = \begin{bmatrix} -1 \\ -2 \end{bmatrix} + \begin{bmatrix} 3 & -1 \\ 0 & -3 \end{bmatrix} (x - z)$$

$$\hat{f}(x^0) = 0 \Rightarrow \begin{bmatrix} -1 \\ -2 \end{bmatrix} + \begin{bmatrix} 3 & -1 \\ 0 & -3 \end{bmatrix} [x_1+1 \quad x_2-1]$$

$$\Rightarrow \begin{bmatrix} -1 \\ -2 \end{bmatrix} + \begin{bmatrix} 3x_1+4-x_2 \\ -3x_2+3 \end{bmatrix}$$

$$\begin{cases} 3x_1+4-x_2 = 1 \\ -3x_2+3 = 2 \end{cases} \Rightarrow \begin{cases} x_1 = -\frac{11}{9} \\ x_2 = \frac{1}{3} \end{cases}$$

So we can find $x^0 = [-\frac{11}{9} \quad \frac{1}{3}]$ #

2.

proof: $\because$ A and B is lower triangular matrix.

$e_1 = [1, 0 \cdots 0]^T$

$e_1^T A = [1\ 0 \cdots 0] \cdot [A_{11}\ 0 \cdots 0]$

$Be_1 = [B_{11}\ 0 \cdots 0]^T \cdot [1\ 0\ 0 \cdots 0]$

A and B multiple to e_1 it just use first column

So we can get $e_1^T ABe_1 = [1\ 0 \cdots 0][A_{11}\ 0 \cdots 0][B_{11} \cdots 0]^T [1\ 0 \cdots 0]$

$\Rightarrow A_{11} \cdot B_{11}$ #

3.

$$\|u+v\|^2 = \|u\|^2 + 2u^Tv + \|v\|^2$$

$\because u^Tv = 0$

$\therefore \|u+v\|^2 = \|u\|^2 + \|v\|^2$

So $\|u+v\|^2 - \|u\|^2 - \|v\|^2 = 0$ #.