

Exam information	
Course code and title	MATH7501 Mathematics for Data Science 1
Semester	Semester 1, 2021
Exam type	Online, non-invigilated, final examination
Exam technology	Sending file and voice recording to the course e-mail: mathdatasciencebridgingsubmissions@uq.edu.au Using Zoom for communication: https://uqz.zoom.us/j/88266918039
Exam date and time	The exam time is 12:00pm (noon), BNE-Time, Saturday June 5. Note that your personalized timetable will still show the wrong time, (8AM BNE time).
Exam window	You must commence your exam at 12:00pm (noon), BNE Time, Saturday June 5. All submissions must be sent/received to the course e-mail address, no later than 2:00pm.
Permitted materials	This is a closed book exam. Only a double sided A4 self-made summary sheet is permitted. A scientific calculator is permitted. Using a calculator app on the phone is fine. Any other software is not allowed.
Recommended materials	Ensure the following materials are available during the exam: Scientific calculator; bilingual dictionary; phone/camera/scanner
Instructions	<p>The exam procedures are very similar to Quiz 1 and Quiz 2 which took place during the semester. The only difference is that there is more time allocated and a doublesided summary sheet is allowed.</p> <ul style="list-style-type: none"> You should join the normal class Zoom session just before 12:00PM (BNE time). The course coordinator will then share a URL with you where you can see the exam questions. It is a PDF file. There will then be a 10-minute reading time slot for the exam question where you should read and make sure all questions are clear. At the end of the reading time, you should ask if anything is not clear and the coordinator will answer. You can then start to answer the exam questions by handwriting the solution on your own paper. Make sure to write your name and student number at the top of the paper. If you use multiple pages for the solution, make sure to number the pages. Answer the questions sequentially in order, and when you finish the exam, take photos or scan your paper. Then upload it to a clean organized single PDF file, not exceeding 8MB as specified in the hand-in instructions (on the course website). If the file is larger than 8MB due to high resolution images, use some form of compression utility (either on the images or the PDF file). In any case, make sure your answers are legible. Typing up your answers is not banned – however it is highly recommended that you simply use handwriting due to the time limit.

	<ul style="list-style-type: none"> Beyond the 10 minute reading time, you have 90 minutes for the exam + 20 additional minutes to prepare your hand-in. This includes scanning your answers, formatting them, and preparing a voice recording (audio clip). You are not banned from writing during any time within this 120 minute window. Submit your exam (single PDF file) and the voice recording via the course e-mail address, as specified in the course website. You must submit prior to 2:00PM BNE time. Submissions without a voice recording will not be accepted. Late submissions will be heavily penalized and submissions that are more than 15 minutes late will not be accepted. In the voice recording, just briefly mention your experience in the exam and if valid, state that you followed all the instructions, all the work is yours, and you didn't use any materials that are not allowed. You are allowed (and encouraged to prepare) a double sided summary (formula) sheet. The summary sheet can be handwritten or typed, but needs to be your own. This summary sheet should be appended into your submission PDF as the last two pages. If you do not have a summary sheet, you must explicitly state this in the last page of your submission. Points will be deducted for failure to submit a formula sheet, or failure to state that you do not have one. (It is highly recommended to make one). <p>What is NOT-allowed / allowed in the exam?</p> <ul style="list-style-type: none"> You are NOT allowed to use: <ul style="list-style-type: none"> Any material from the Internet. Any material from the course website. Any files from your computer. Any written notes of your own with the exception of a double page (two sides) summary sheet which you can prepare beforehand (and need to upload as part of the PDF submission – read above). Any communication with anyone outside your household. Mathematica or any other software that computes. You are allowed to use: <ul style="list-style-type: none"> The double page summary sheet. A scientific calculator. Note that using the calculator on your phone is allowed.
Who to contact	The course coordinator will be available during the exam via Zoom.
Late or incomplete submissions	<p>In the event of a late submission, you will be required to submit evidence that you completed the assessment in the time allowed. This will also apply if there is an error in your submission (e.g. corrupt file, missing pages, poor quality scan). We strongly recommend you use a phone camera to take time-stamped photos (or a video) of every page of your paper during the time allowed (even if you submit on time).</p> <p>If you submit your paper after the due time, then you should send details to exam submission e-mail address as soon as possible after the end of the time allowed. Include an explanation of why you submitted late (with any evidence of technical</p>

	issues) AND time-stamped images of every page of your paper (eg screen shot from your phone showing both the image and the time at which it was taken).
Important exam condition information	<p>Academic integrity is a core value of the UQ community and as such the highest standards of academic integrity apply to all examinations, whether undertaken in-person or online.</p> <p>This means:</p> <ul style="list-style-type: none"> You are permitted to refer to the allowed resources for this exam, but you cannot cut-and-paste material other than your own work as answers. You are not permitted to consult any other person – whether directly, online, or through any other means – about any aspect of this examination during the period that it is available. If it is found that you have given or sought outside assistance with this examination, then that will be deemed to be cheating. You must submit a voice recording stating a truthful statement in the voice recording. <p>If you submit your online exam after the end of your specified reading time, duration, and 15 minutes submission time, the following penalties will be applied to your final examination score for late submission:</p> <ul style="list-style-type: none"> Less than 5 minutes – 5% penalty From 5 minutes to less than 15 minutes – 20% penalty More than 15 minutes – 100% penalty <p>These penalties will be applied to all online exams unless there is sufficient evidence of problems with the system and/or process that were beyond your control.</p> <p>Undertaking this online exam deems your commitment to UQ's academic integrity pledge as summarised in the following declaration:</p> <p><i>"I certify that I have completed this examination in an honest, fair and trustworthy manner, that my submitted answers are entirely my own work, and that I have neither given nor received any unauthorised assistance on this examination".</i></p>

The exam has 4 questions, each with three items. Each item is worth 8 points, except for items 4(ii), and 4(iii), worth 10 points each. There are 100 points total.

Question 1: Derive the Taylor series for the following functions and in each case determine the radius of convergence:

- (i) **(8pts)** $f(x) = 1 + x^2 - x^3$ about $x = 1$.
- (ii) **(8pts)** $f(x) = \ln(e + x)$ about $x = 0$.
- (iii) **(8pts)** $f(x) = \frac{1}{5+x}$ about $x = 1$.

Question 2: Consider the function $f(x, y) = \frac{1}{1+x^2+y^2}$.

- (i) **(8pts)** For any given constant $C > 0$, determine the contour $f(x, y) = C$. Represent it as a subset of \mathbb{R}^2 using set notation. What is it geometrically?
- (ii) **(8pts)** Calculate the gradient ∇f .
- (iii) **(8pts)** For which value of (x, y) is $\|\nabla f\|$ the largest?

Hint: it might be helpful to introduce the new variable $R = x^2 + y^2$ so that $\|\nabla f\|$ is a function of R only.

Question 3: Consider the probability density function $f(x) = 2e^{-2x}$ defined for $x \geq 0$.

- (i) **(8pts)** Show that $\int_0^\infty f(x) dx = 1$.
- (ii) **(8pts)** Consider $p_n = \int_n^{n+1} f(x) dx$ for $n = 0, 1, 2, \dots$. Compute an expression for p_n .
- (iii) **(8pts)** Consider the series $\sum_{i=0}^\infty p_n$. Using the expression from (ii) determine if the series converges, and in case it does evaluate its value.

Question 4: Consider the three functions of three variables $x(r, \theta, \varphi)$, $y(r, \theta, \varphi)$ and $z(r, \theta, \varphi)$ with

$$x = r \cos(\varphi) \sin(\theta), \quad y = r \sin(\varphi) \sin(\theta), \quad z = r \cos(\theta).$$

- (i) **(8pts)** Evaluate the matrix

$$A = \begin{bmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} & \frac{\partial x}{\partial \varphi} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} & \frac{\partial y}{\partial \varphi} \\ \frac{\partial z}{\partial r} & \frac{\partial z}{\partial \theta} & \frac{\partial z}{\partial \varphi} \end{bmatrix}$$

- (ii) **(10pts)** What is the determinant of A ?
- (iii) **(10pts)** Are there any values of r , θ and φ for which A^{-1} does not exist? If so what are these values?

END OF EXAMINATION