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Exam information

Course code and title	MATH7501 Mathematics for Data Science 1
Semester	Semester 1, 2023
Exam type	Online, non-invigilated, end-of-semester examination
Exam technology	File upload to Blackboard Assignment
Exam date and time	Your examination will begin at the time specified in your personal examination timetable. If you commence your examination after this time, the end for your examination does NOT change. The total time for your examination from the scheduled starting time will be: 100 minutes (including 10 minutes reading time during which you should read the exam paper and plan your responses to the questions). A 15-minute submission period is available for submitting your examination after the allowed time shown above. If your examination is submitted after this period, late penalties will be applied unless you can demonstrate that there were problems with the system and/or process that were beyond your control.
Exam window	You must commence your exam at the time listed in your personalised timetable. You have from the start date/time to the end date/time listed in which you must complete your exam.
Permitted materials	Closed Book - no materials permitted
Recommended materials	Ensure the following materials are available during the exam: bilingual dictionary; phone/camera/scanner
Instructions	You will need to download the question paper included within the Blackboard Test. Once you have completed the exam, upload the completed exam answers file to the TurnItIn submission link. You may submit multiple times, but only the last uploaded file will be graded. You can print the question paper and write on that paper or write your answers on blank paper (clearly label your solutions so that it is clear which problem it is a solution to) or annotate an electronic file on a suitable device.
Who to contact	Given the nature of this examination, responding to student queries and/or relaying corrections to exam content during the exam may not be feasible. If you have any concerns or queries about a particular question or need to make any assumptions to answer the question, state these at the start of your solution to that question. You may also include queries you may have made with respect to a particular question, should you have been able to 'raise your hand' in an examination-type setting. If you experience any interruptions to your examination, please collect evidence of the interruption (e.g. photographs, screenshots or emails). If you experience any issues during the examination, contact ONLY the Library AskUs service for advice as soon as practicable:

	Chat: support.my.uq.edu.au/app/chat/chat_launch_lib **Phone:** +61 7 3335 7047 **Email:** examsupport@library.uq.edu.au You should also ask for an email documenting the advice provided so you can provide this as evidence for a late submission.
Late or incomplete submissions	In the event of a **late submission**, you will be required to submit evidence that you completed the assessment in the time allowed. This will also apply if there is an **error in your submission** (e.g. corrupt file, missing pages, poor quality scan). We **strongly recommend** you use a phone camera to take time-stamped photos (or a video) of every page of your paper during the time allowed (even if you submit on time). If you submit your paper after the due time, then you should send details to SMP Exams (exams.smp@uq.edu.au) as soon as possible after the end of the time allowed. Include an explanation of why you submitted late (with any evidence of technical issues) AND time-stamped images of every page of your paper (eg screen shot from your phone showing both the image and the time at which it was taken).
Important exam condition information	You are responsible for managing your multi-factor authentication in this examination. Please check the guidance on How do I MFA before an online exam? Academic integrity is a core value of the UQ community and as such the highest standards of academic integrity apply to all examinations, whether undertaken in-person or online. This means: • You are permitted to refer to the allowed resources for this exam, but you cannot cut-and-paste material other than your own work as answers. • You are not permitted to consult any other person – whether directly, online, or through any other means – about any aspect of this examination during the period that it is available. • If it is found that you have given or sought outside assistance with this examination, then that will be deemed to be cheating. If you submit your online exam after the end of your specified reading time, duration, and 15 minutes submission time, the following penalties will be applied to your final examination score for late submission: • Less than 5 minutes – 5% penalty • From 5 minutes to less than 15 minutes – 20% penalty • More than 15 minutes – 100% penalty These penalties will be applied to all online exams unless there is sufficient evidence of problems with the system and/or process that were beyond your control. Undertaking this online exam deems your commitment to UQ's academic integrity pledge as summarised in the following declaration: *"I certify that I have completed this examination in an honest, fair and trustworthy manner, that my submitted answers are entirely my own work, and that I have neither given nor received any unauthorised assistance on this examination".*

1. [5 marks] Let x be a real number, and let

$$A = \begin{pmatrix} 1 & x & 0 \\ 0 & 1 & x \\ 0 & 0 & 1 \end{pmatrix}.$$

(a) Find A^{-1}. [1 marks]

(b) Find

$$\lim_{x \to \infty} \det \left(A^{-1} \right).$$

[2 marks]

(c) Prove that for any integer $n \geq 1$,

$$A^n = \begin{pmatrix} 1 & nx & \frac{n(n-1)x^2}{2} \\ 0 & 1 & nx \\ 0 & 0 & 1 \end{pmatrix}$$

where

$$A^n = \underbrace{A \times A \times \cdots \times A}_{n \text{ times}},$$

and $\times$ denotes the usual operation of matrix multiplication. [2 mark]

2. [5 marks] Recall the binomial expansion

$$(1+x)^n = \sum_{k=0}^{n} \binom{n}{k} x^k,$$

for any integer $n \geq 0$, and any real number x, noting that we use $0^0 := 1$.

(a) Find $\binom{100}{2}$. [1 marks]

(b) Show that for any integer k, such that $0 \leq k \leq n$, we have

$$\binom{n}{k} = \binom{n}{n-k}.$$

[2 mark]

(c) Find

$$\lim_{n\to\infty} n\left(\left(1+\frac{1}{n}\right)^{100} - 1\right).$$

[2 marks]

3. [5 marks] Find the following limits:

(a)

$$\lim_{x \to 0} \frac{\sin(\tan x)}{\sin x}.$$

[2 mark]

(b)

$$\lim_{x \to \infty} \frac{x^2 + 4x + 4}{x^2 + x - 2}.$$

[1 mark]

(c)

$$\lim_{x \to 1} \; (x-1)^2 \sin\left(x^2\right) \cos\left(1 + e^x\right).$$

[2 mark]

4. [5 marks] Let $f(x) = \ln(1 + x^2) + 4x^2$.

   (a) Show that $f(x)$ has a critical point on the domain $\mathbb{R}$ at $x = 0$. [1 mark]

   (b) Determine whether the critical point $x = 0$ is a local maximum or minimum. Justify your answer. [2 mark]

   (c) Determine whether the critical point $x = 0$ is a global maximum or minimum. Justify your answer. [2 mark]

5. [5 marks] Let

$$f(x, y) = e^x + e^y \ln\left(1 + x^2\right).$$

(a) Compute the partial derivatives $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$. [2 mark]

(b) Construct a linear approximation of f at the point $(1, 1)$. [2 mark]

(c) Using the linear approximation of f, provide an estimate for the change in f:

$$\Delta f = f(1.05, 1.05) - f(1, 1).$$

[1 mark]

6. [5 marks] Find the following integrals:

(a)

$$\int \left(e^{3x} + \cos(2x)\right) \mathrm{d}x.$$

[1 mark]

(b)

$$\int x^2 (1+x)^{100} \, \mathrm{d}x.$$

[2 mark]

(c)

$$\int (\sin x)^2 \, \mathrm{d}x.$$

[2 mark]

7. [5 marks] Let $f(x, y) = 10 - \ln(1 + x^2 + y^2)$.

   (a) Find the gradient ∇f. [1 mark]

   (b) Compute the directional derivative of f at $(1, 1)$ in the direction $\boldsymbol{u} = (-1, -1)$.

   [2 mark]

   (c) What is the direction and magnitude of the steepest slope at $(-1, -1)$? [2 mark]

END OF EXAMINATION