1.

(a)

$f(z) = f(-1, 1) = [-1 \quad -2]^T$ #

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(b)

by (a) we can get $f(z) = [-1 \ -2]^T$

$$g(z) = \begin{bmatrix} -1 \\ -2 \end{bmatrix} [2 \quad 1] = \begin{bmatrix} -2 & -1 \\ -4 & -2 \end{bmatrix}$$ #

(c)

$$g(z)\, z = \begin{bmatrix} -2 & -1 \\ -4 & -2 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\| g(z) z \| = \sqrt{1^2 + 2^2} = \sqrt{5}$$ #

(d)

$[-2 \ -4]^T$, $[-1 \ -2]^T$ inner product $\Rightarrow (-2 \times -1) + (-4 \times -2) = 10$ #

(e) $g(x) = \begin{bmatrix} u^3 v \\ uv^2 - e^{u+v} \end{bmatrix} [2 \quad 1] = \begin{bmatrix} 2u^3 v & u^3 v \\ 2uv^2 - 2e^{u+v} & uv^2 - e^{u+v} \end{bmatrix}$

$\det(g(x)) = 0 \Rightarrow$ So the answer does not depend on x #.

(f) $\nabla f(u, v) = \begin{bmatrix} 3u^2 v, & u^3 \\ v^2 - e^{u+v}, & 2uv - e^{u+v} \end{bmatrix}$ #

(g)

$$\hat{f}(x) = \begin{bmatrix} -1 \\ -2 \end{bmatrix} + \begin{bmatrix} 3 & -1 \\ 0 & -3 \end{bmatrix}(x - z)$$

$$\hat{f}(x^0) = 0 \Rightarrow \begin{bmatrix} -1 \\ -2 \end{bmatrix} + \begin{bmatrix} 3 & -1 \\ 0 & -3 \end{bmatrix} [x_1+1 \quad x_2-1]$$

$$\Rightarrow \begin{bmatrix} -1 \\ -2 \end{bmatrix} + \begin{bmatrix} 3x_1+4-x_2 \\ -3x_2+3 \end{bmatrix}$$

$$\begin{cases} 3x_1+4-x_2 = 1 \\ -3x_2+3 = 2 \end{cases} \Rightarrow \begin{cases} x_1 = -\frac{11}{9} \\ x_2 = \frac{1}{3} \end{cases}$$

So we can find $x^0 = [-\frac{11}{9} \quad \frac{1}{3}]$ #

small calculation error

-0.5

2.

proof: $\because$ A and B is lower triangular matrix.

$e_1 = [1, 0 \cdots 0]^T$

$e_1^T A = [1\ 0 \cdots 0] \cdot [A_{11} \quad 0 \quad \cdots \quad 0]$

$Be_1 = [B_{11} \ 0 \cdots \ 0]^T \cdot [1\ 0\ 0 \cdots 0]$

So we can get $e_1^T A B e_1 = [1\ 0 \cdots 0][A_{11}\ 0 \cdots 0][B_{11} \cdots 0]^T[1\ 0 \cdots 0]$

$\Rightarrow A_{11} \cdot B_{11}$ #

3.

$$\|u+v\|^2 = \|u\|^2 + 2u^Tv + \|v\|^2$$

$\because u^Tv = 0$

$\therefore \|u+v\|^2 = \|u\|^2 + \|v\|^2$ why? -1

So $\|u+v\|^2 - \|u\|^2 - \|v\|^2 = 0$ #.