STAT2203/7203: Week 4 Practical Questions

1. A person flips a coin until both a Heads and a Tails has been observed or the coin has been flipped three times. Let the random variable X be the number of Heads resulting from this approach. Assuming that flips are independent and equally likely to be Heads or Tails, an incomplete probability function for X is as follows:

x	0	1	2	3
$P(X = x)$		0.625	0.125	

   (a) Calculate the missing values in this table.

   (b) Based on these probabilities, what is the expected number of Heads obtained following this approach.

   (c) Based on these probabilities, what is the standard deviation of the number of Heads obtained following this approach.

2. Particle size distribution is an important characteristic of microstructures in multiphase materials. The diameter (in μm) of particles in a certain material has the following pdf

$$f(x) = c\,x \exp\left(-\frac{x^2}{200}\right), \quad x \geq 0,$$

   for some constant c, and $f(x) = 0$ for $x < 0$.

   (a) Determine the value of c which makes f a valid pdf.

   (b) Determine the cdf of particle diameter.

   (c) What is the probability that a particle's diameter is between 10 μm and 20 μm ?

   (d) Determine the quantile function of particle diameter.

   (e) Determine the pdf of (particle diameter)^2.

3. The logarithmic series distribution has pmf

$$f(x) = \frac{-1}{\ln(1-p)}\frac{p^x}{x}, \quad x = 1, 2, 3, \ldots,$$

   where $p \in (0, 1)$ is a parameter. Recall the Taylor series

$$-\ln(1-z) = z + \frac{z^2}{2} + \frac{z^3}{3} + \cdots,$$

   which converges for $|z| < 1$.

   (a) Find the moment generating function of this distribution.

   (b) Determine the mean.

4. Suppose X has pdf

$$f(x) = 12x(1-x)^2, \quad x \in (0, 1)$$

   and $f(x) = 0$ for $x \notin (0, 1)$. Determine the mean and variance of X.