Questions: 45 marks in total. Weight: 15%.
Important points about submission of this assignment:

- No coversheet is needed. However, by submitting your assignment on Blackboard (using the link under Assessment), we understand you to be making the following statement:

  I hereby state that the work contained in this assignment has not previously been submitted for assessment, either in whole or in part, by either myself or any other student at either The University of Queensland or at any other tertiary institution except where explicitly acknowledged. To the best of my knowledge and belief, the assignment contains no material that has been previously published or written by another person except where due reference is made. I make this Statement in full knowledge of an understanding that, should it be found to be false, I will be subject to disciplinary action under Student Integrity and Misconduct Policy 3.60.04 of the University of Queensland.

- Submit a single PDF file, with pages of uniform size, and a file size that does not exceed 8MB (you can use a pdf compression utility if needed). The name of the PDF file should be `FFFF_LLLL_SN-ProblemSet2.pdf` where FFFF is your first name, LLLL, is your last name, and SN is your student number.

- Do not submit code files - instead format your code into the PDF file. Both handwritten notes and typed notes are acceptable. A combination of handwritten and typed notes is acceptable as long as it is formatted nicely and continuously in a single PDF file. All graphs, plots, source code, and other figures must be clearly labeled. All questions/items must appear in order.

- You may use mathematica as an aid for the computations, however make sure to do the hand calculations where suitable as well. Ensure that your working is set out clearly and neatly, that you state any assumptions and define notation. If rounding a numerical answer (at the end of a question), three significant figures is enough.

- Be aware of UQ's academic integrity and plagiarism policy - see `https://my.uq.edu.au/information-and-services/manage-my-program/student-integrity-and-conduct/academic-integrity-and-student-conduct`.

1. Let $a_n = n^n/n!$ be a sequence for each $n \geq 1$.

   (a) Show that $a_n \geq n$, for each $n \geq 1$. [2 marks]

   (b) Via definitions from Chapter 6 of the Course Reader, prove that

   $$\lim_{n\to\infty} a_n = \infty$$

   [2 marks]

   (c) Evaluate the following limit:

   $$\lim_{n\to\infty} \frac{a_{n+1}}{a_n}.$$

   [2 marks]

   (d) Evaluate the following limit:

   $$\lim_{n\to\infty} \frac{n!}{n^n}.$$

   [2 marks]

   (e) For all k and n, such that $1 \leq k \leq n$, prove that

   $$a_n \geq \frac{n^k}{k!}.$$

   [2 marks]

   (f) Determine whether or not the following series converges:

   $$\sum_{n=1}^{\infty} \frac{n!}{n^n}.$$

   [2 marks]

2. Determine whether or not the following series converges:

   $$\sum_{n=5}^{\infty} \frac{6}{7^n}.$$

   If the series converges, also determine its value. [3 marks]

3. Determine the values of each of the following limits:

(a)

$$\lim_{x\to\infty} \frac{x^2+4x-21}{x^2-5x+6}.$$

[2 marks]

(b)

$$\lim_{x\to\infty} \frac{x^2-3x+2}{x^2+2x-3}.$$

[2 marks]

(c)

$$\lim_{x\to 1} \frac{\sin x}{x}.$$

[2 marks]

(d)

$$\lim_{x\to\infty} \ln\left(\frac{x^2+2x+1}{x^2-1}\right).$$

[2 marks]

(e)

$$\lim_{x\to\infty} \frac{x^2\left(\sin x+(\cos x)^3\right)}{(x^2+1)(x-3)}.$$

[2 marks]

4. Let f be a continuous real valued function. If f is one-to-one, show that f must either be strictly increasing or strictly decreasing. [3 marks]

5. Suppose that the real valued function f satisfies

$$\lim_{h\to 0} \left(f(x+h)-f(x-h)\right)=0$$

for each $x\in\mathbb{R}$. Determine whether or not f is continuous. [3 marks]

6. Obtain the derivatives of the following functions:

(a)

$$f(x)=x^2\cos x.$$

[2 marks]

(b)

$$f(x)=\frac{2x+1}{3x-2}.$$

[2 marks]

(c)

$$f(x) = \cos(\sin x).$$

[2 marks]

7. Determine the value of the following limit

$$\lim_{x \to \frac{\pi}{4}} \frac{\sqrt{2}\cos x - 1}{1 - (\tan x)^2}.$$

[2 marks]

8. Locate the critical points of the function

$$f(x) = \frac{1}{1+|x|} + \frac{1}{1+|x-2|},$$

and determine the value of f at its global maximum/maxima, on the real domain. [3 marks]

9. For any $a > 0$ and $b > 0$, determine the global maximum of

$$f(x) = x^a (1-x)^b$$

on the domain $[0, 1]$. [3 marks]