THE UNIVERSITY OF QUEENSLAND
AUSTRALIA

This exam paper must not be removed from the venue

Venue ____________________

Seat Number ________

Student Number |__|__|__|__|__|__|__|__|

Family Name _____________________

First Name _____________________

School of Mathematics & Physics

EXAMINATION

Semester Two Final Examinations, 2019

STAT7203 Applied Probability and Statistics

This paper is for St Lucia Campus students.

Examination Duration: 120 minutes

Reading Time: 10 minutes

Exam Conditions:

This is a Central Examination

This is a Closed Book Examination - specified materials permitted

During reading time - write only on the rough paper provided

This examination paper will be released to the Library

Materials Permitted In The Exam Venue:

(No electronic aids are permitted e.g. laptops, phones)

Calculators - Casio FX82 series or UQ approved (labelled)

One A4 sheet of handwritten notes double sided is permitted

Materials To Be Supplied To Students:

None

Instructions To Students:

Additional exam materials (eg. rough paper) will be provided upon request.

There are 50 marks available on this exam from 5 questions.

Write your answers in the spaces provided on pages 2 – 13 of this examination paper. Show your working and state conclusions where appropriate. Pages 14 – 18 give formulas and statistical tables. Those pages will not be marked.

For Examiner Use Only

Question	Mark

Total ________

1. [10 marks] A pair of random variables (X, Y) has a joint probability distribution in which $X \sim \mathsf{Uniform}(0, 1)$, and $(Y \mid X = x) \sim \mathsf{Uniform}(-x, 2x)$; that is, the conditional distribution of Y given $\{X = x\}$ is uniform on the interval $(-x, 2x)$.

   (a) Write down the joint probability density function of (X, Y), clearly specifying the support of the distribution. [1 mark]

   (b) Determine the marginal probability density function of Y. [3 marks]

(c) Using the formula $\mathbb{E}Y = \mathbb{E}\left[\mathbb{E}\left[Y \mid X\right]\right]$, find the expectation of Y. [3 marks]

(d) Compute $\mathrm{Cov}(X, Y)$. [3 marks]

2. [6 marks] Let X be the continuous random variable with probability density function

$$f_X(x) = \begin{cases} -\log(x), & x \in (0,1) \\ 0, & \text{otherwise.} \end{cases}$$

Define the random variable $Y = -\log(X)$.

(a) Find the probability density function of Y. [3 marks]

(b) The moment generating function of Y is

$$M_Y(t) = \mathbb{E}(e^{tY}) = (1-t)^{-2}, \quad t < 1.$$

Determine the expected value and variance of Y. [3 marks]

3. [10 marks] A study compared the short term effect of playing computer games and watching television on young children's aggression. In the study 28 children aged 4 years to 6 years were randomly assigned to one of two groups with 14 children being assigned to each group. The children were monitored before and after these activities and the level of aggressive behaviour rated on a scale from 1 (no aggressive behaviour) to 100 (very aggressive behaviour).

   (a) The first group of children, those who watched television, experienced an average increase of 5.88 in their aggressive behaviour rating with a sample standard deviation of 6.44. Construct a 95% confidence interval for the mean change in aggressive behaviour rating for children after watching television. What does this confidence interval imply in terms of the mean change in aggressive behaviour rating? [4 marks]

   (b) If the number of children in the study was doubled, but all other sample statistics remained the same, what would happen to the width of the confidence interval? [1 mark]

(c) The children in the second group, those who played a computer game, experienced an average increase of 3.50 in their aggressive behaviour rating with a sample standard deviation of 8.00. Is there any evidence of a difference in the mean change in aggression rating between the two groups? State the null and alternative hypotheses, and use an appropriate test statistic to determine the P-value. What do you conclude? [5 marks]

4. [10 marks] A study was conducted in Germany on the demographics of the online gaming population. Part of this study involved interviewing 688 adult volunteers about their online gaming experience.

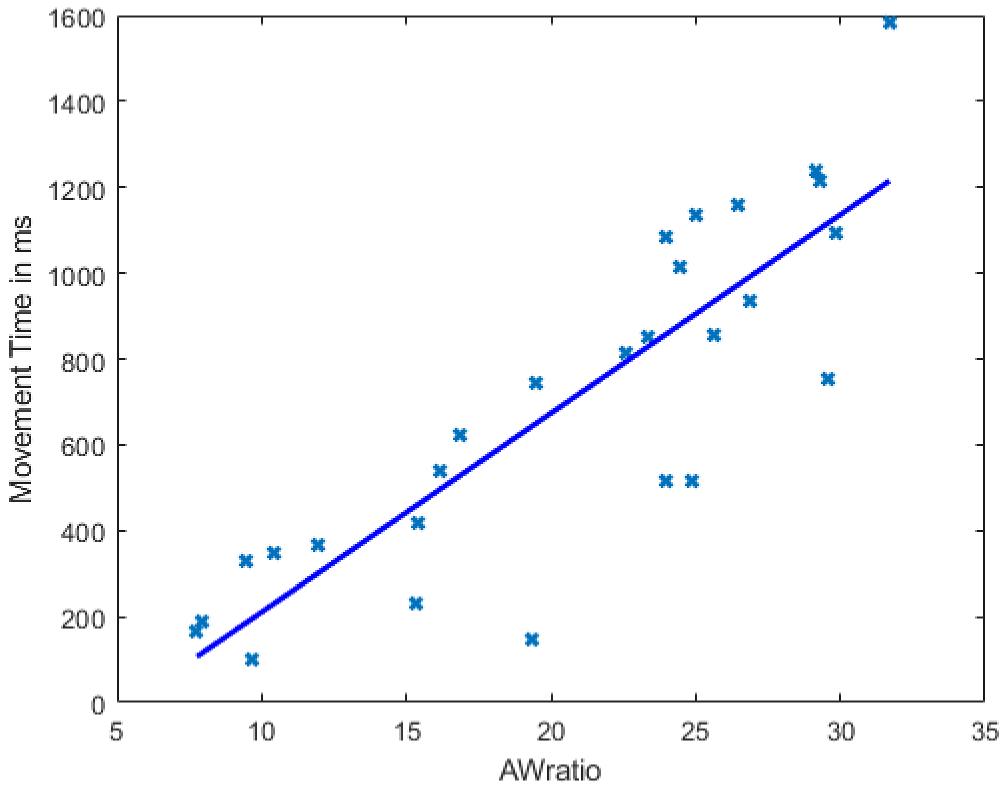
   (a) The volunteers were asked what type of games they played online (Role-playing, Action, Strategy, Sports and Racing). Of the 464 men interviewed, 325 said they play role-playing games while of the 224 women interviewed 178 said they play role-playing games. Is there evidence of a difference in the proportion of men and women who play role playing games online? State the null and alternative hypotheses, and use an appropriate test statistic to determine the P-value. What do you conclude? [4 marks]

(b) The volunteers were also asked how many years have they been playing online games. The responses are summarised in the table below.

Sex	**Online gaming experience (years)**		
	< 3 years	3 – 5 years	> 5 years
Men	223	120	121
Women	132	55	37

Based on this table, is there evidence of an association between sex and years of experience in online gaming? [6 marks]

5. [14 marks] Twenty seven right-handed university students were recruited into a study to investigate the time taken for a person to move a cursor on a computer screen along a specified path of length A and width W using a mouse. Each student was given a single path along which they were to move the cursor and the time (in ms) taken to complete the task was recorded. The data are displayed in the figure below together with the fitted least squares line.



The output on the next page shows the results of a linear regression fit in MATLAB for the relationship between the time taken to complete the task (`MovementTime`) and the ratio A/W (`AWratio`).

```
Linear regression model:
    MovementTime ~ 1 + AWratio

Estimated Coefficients:
                   Estimate      SE        tStat       pValue
                   ________    ______    _______    _________

    (Intercept)    -252.36     120.79    -2.0892     0.047029
    AWratio         46.299     5.5179     8.3906    9.747e-09


Number of observations: 27, Error degrees of freedom: 25
Root Mean Squared Error: 211
R-squared: 0.738,  Adjusted R-Squared 0.727
F-statistic vs. constant model: 70.4, p-value = 9.75e-09
```

(a) Briefly interpret the value 46.299 in the regression output. [1 mark]

(b) Briefly interpret the value 211 in the regression output. [1 mark]

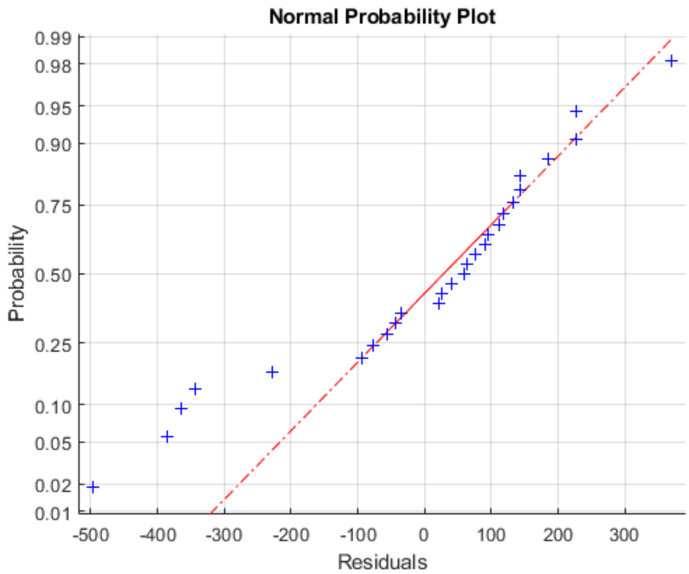
(c) Give a 95% confidence interval for the underlying slope of the linear relationship between the time taken to complete the task and the ratio A/W. [2 marks]

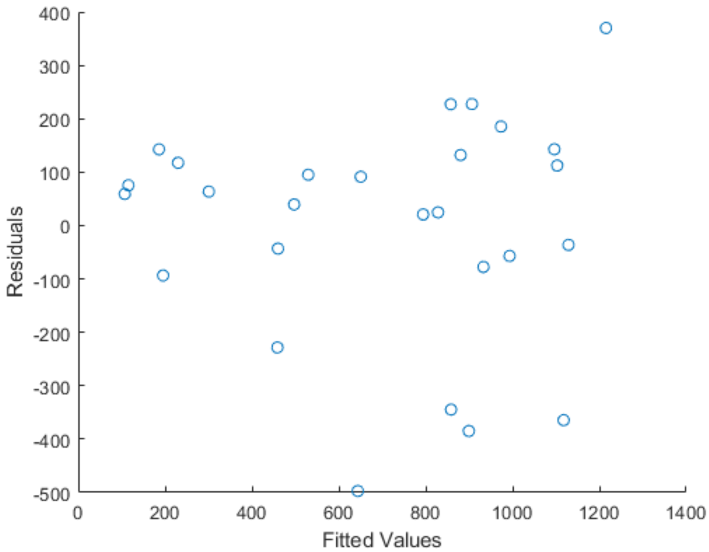
(d) Drury's law suggests that the time taken to complete the task will depend on the ratio A/W in the following way:

$$\text{Movement Time} = \text{Constant} \times (A/W)\,.$$

Does the data provide evidence that when the ratio A/W is zero, the time taken to complete the task will also be zero? State the null and alternative hypotheses, and report the appropriate test statistic and P-value from the output. What do you conclude? [3 marks]

(e) The following figures were generated in MATLAB to help check the assumptions underlying the linear regression. Comment on the validity of the assumptions underlying linear regression for this data with reference to these figures and the figure on page 9. [3 marks]





(additional space for answer to part (e))

(f) The covariance matrix for the estimator of (intercept, slope) is

$$\begin{bmatrix} 14591 & -628 \\ -628 & 30 \end{bmatrix}.$$

Construct a 95% confidence interval for the expected time taken to complete the task when the ratio A/W is 20. [4 marks]

END OF EXAMINATION

Formula Sheet

Elementary probability

- **Sum rule:** For disjoint $A_1, A_2, \ldots$: $\mathbb{P}(\bigcup_i A_i) = \sum_i \mathbb{P}(A_i)$.
- $\mathbb{P}(A^c) = 1 - \mathbb{P}(A)$.
- $\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cap B)$.
- **Conditional probability:** $\mathbb{P}(A \mid B) = \frac{\mathbb{P}(A\cap B)}{\mathbb{P}(B)}$.
- **Law of total probability**: $\mathbb{P}(A) = \sum_{i=1}^n \mathbb{P}(A \mid B_i)\, \mathbb{P}(B_i)$, where $B_1, B_2, \ldots, B_n$ is a partition of Ω.
- **Bayes' Rule:** $\mathbb{P}(B_j|A) = \frac{\mathbb{P}(B_j)\,\mathbb{P}(A|B_j)}{\sum_{i=1}^n \mathbb{P}(B_i)\,\mathbb{P}(A|B_i)}$.
- **Independent events**: $\mathbb{P}(A \cap B) = \mathbb{P}(A)\,\mathbb{P}(B)$.

Random variables

- **Cdf** of X: $F(x) = \mathbb{P}(X \leqslant x),\ x \in \mathbb{R}$.
- **Pmf** of X: (discrete r.v.) $f(x) = \mathbb{P}(X = x)$.
- **Pdf** of X: (continuous r.v.) $f(x) = F'(x)$.
- For a discrete r.v. X: $\mathbb{P}(X \in B) = \sum_{x\in B} \mathbb{P}(X = x)$.
- For a continuous r.v. X with pdf f: $\mathbb{P}(X \in B) = \int_B f(x)\, dx$.
- In particular (continuous), $F(x) = \int_{-\infty}^x f(u)\, du$.
- **Important discrete distributions**:

Distr.	pmf	suppport
$\mathsf{Ber}(p)$	$p^x(1-p)^{1-x}$	$\{0,1\}$
$\mathsf{Bin}(n,p)$	$\binom{n}{x} p^x(1-p)^{n-x}$	$\{0,1,\ldots,n\}$
$\mathsf{Poi}(\lambda)$	$\mathrm{e}^{-\lambda}\,\frac{\lambda^x}{x!}$	$\{0,1,\ldots\}$
$\mathsf{Geom}(p)$	$p(1-p)^{x-1}$	$\{1,2,\ldots\}$

- **Important continuous distributions**:

Distr.	pdf	$x \in$
$\mathsf{U}[a,b]$	$\frac{1}{b-a}$	$[a,b]$
$\mathsf{Exp}(\lambda)$	$\lambda\,\mathrm{e}^{-\lambda x}$	$\mathbb{R}_+$
$\mathsf{N}(\mu,\sigma^2)$	$\frac{1}{\sigma\sqrt{2\pi}}\,\mathrm{e}^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$	$\mathbb{R}$

- **Expectation** (discr.): $\mathbb{E}X = \sum_x x\,\mathbb{P}(X = x)$.
- (of function) $\mathbb{E}\,g(X) = \sum_x g(x)\,\mathbb{P}(X = x)$.
- **Expectation** (cont.): $\mathbb{E}X = \int x\,f(x)\,dx$.
- (of function) $\mathbb{E}\,g(X) = \int g(x)f(x)\,dx$,
- $\mathbb{E}X$ **and** $\mathbf{Var}(X)$ **for discrete distributions**:

	$\mathbb{E}X$	$\mathrm{Var}(X)$
$\mathsf{Ber}(p)$	p	$p(1-p)$
$\mathsf{Bin}(n,p)$	np	$np(1-p)$
$\mathsf{Geom}(p)$	$\frac{1}{p}$	$\frac{1-p}{p^2}$
$\mathsf{Poi}(\lambda)$	λ	λ

- $\mathbb{E}X$ **and** $\mathbf{Var}(X)$ **for continuous distributions**:

	$\mathbb{E}X$	$\mathrm{Var}(X)$
$\mathsf{U}(a,b)$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
$\mathsf{Exp}(\lambda)$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$
$\mathsf{N}(\mu,\sigma^2)$	μ	σ^2

Multiple random variables

- **Joint distribution:** $\mathbb{P}((X,Y) \in B) = \iint_B f_{X,Y}(x,y)\,dx\,dy$.
- **Marginal pdf:** $f_X(x) = \int f_{X,Y}(x,y)\,dy$.
- **Independent r.v.'s:** $f_{X_1,\ldots,X_n}(x_1,\ldots,x_n) = \prod_{k=1}^n f_{X_k}(x_k)$.
- **Expected sum :** $\mathbb{E}(aX + bY) = a\,\mathbb{E}X + b\,\mathbb{E}Y$.
- **Expected product** (if X, Y independent): $\mathbb{E}[X\,Y] = \mathbb{E}X\,\mathbb{E}Y$.
- **Markov inequality:** $\mathbb{P}(X \geqslant x) \leqslant \frac{\mathbb{E}X}{x}$.
- **Covariance:** $\mathrm{cov}(X,Y) = \mathbb{E}(X - \mathbb{E}X)(Y - \mathbb{E}Y)$.
- **Properties of Var and Cov**:

$$
\boxed{\begin{array}{l}
\mathrm{Var}(X) = \mathbb{E}X^2 - (\mathbb{E}X)^2. \\
\mathrm{Var}(aX+b) = a^2\mathrm{Var}(X). \\
\mathrm{cov}(X,Y) = \mathbb{E}XY - \mathbb{E}X\mathbb{E}Y. \\
\mathrm{cov}(X,Y) = \mathrm{cov}(Y,X). \\
\mathrm{cov}(aX+bY,Z) = a\,\mathrm{cov}(X,Z) + b\,\mathrm{cov}(Y,Z). \\
\mathrm{cov}(X,X) = \mathrm{Var}(X). \\
\mathrm{Var}(X+Y) = \mathrm{Var}(X) + \mathrm{Var}(Y) + 2\,\mathrm{cov}(X,Y). \\
X \text{ and } Y \text{ independent} \Longrightarrow \mathrm{cov}(X,Y) = 0.
\end{array}}
$$

- **Conditional pdf:** If $f_X(x) > 0$, $f_{Y\,|\,X}(y\,|\,x) := \frac{f_{X,Y}(x,y)}{f_X(x)}, \quad y \in \mathbb{R}$.
- The corresponding **conditional expectation**: $\mathbb{E}[Y \mid X = x] = \int y\, f_{Y\,|\,X}(y\,|\,x)dy$.
- $\mathbb{E}[Y] = \mathbb{E}[\mathbb{E}[Y \mid X]]$
- **Moment Generating Function (MGF)**: When it exists, for $t \in I \subset \mathbb{R}$, $M(t) = \mathbb{E}\,\mathrm{e}^{tX} = \int_{-\infty}^{\infty} \mathrm{e}^{tx}\,f(x)\,dx$.

- **MGFs for various distributions:**

$\mathsf{U}(a,b)$	$\frac{e^{bt}-e^{at}}{t(b-a)}$
$\mathsf{Exp}(\lambda)$	$\left(\frac{\lambda}{\lambda-s}\right)$
$\mathsf{N}(\mu,\sigma^2)$	$e^{t\mu+\sigma^2t^2/2}$

- **Moment property**: $\mathbb{E}X^n = M^{(n)}(0)$.

- $M_{X+Y}(t) = M_X(t)\,M_Y(t)$, $\forall t$, if X, Y independent.

- If $X_i \sim \mathsf{N}(\mu_i, \sigma_i^2)$ (independent), then $a + \sum_{i=1}^n b_i\,X_i \sim \mathsf{N}\left(a + \sum_{i=1}^n b_i\,\mu_i,\ \sum_{i=1}^n b_i^2\,\sigma_i^2\right)$.

- Pdf of the **multivariate Normal** distribution:

$$f_{\boldsymbol{Z}}(\boldsymbol{z}) = \frac{1}{\sqrt{(2\pi)^n\,|\Sigma|}}\,e^{-\frac{1}{2}(\boldsymbol{z}-\boldsymbol{\mu})^T\Sigma^{-1}(\boldsymbol{z}-\boldsymbol{\mu})}\,.$$

  Σ is the covariance matrix, and $\boldsymbol{\mu}$ the mean vector.

- If $\mathbf{X}$ has a multivariate Normal distribution $\mathsf{N}(\boldsymbol{\mu}, \Sigma)$ (dimension n) and $\mathbf{Y} = \boldsymbol{a} + B\mathbf{X}$ (dimension $m \leqslant n$), then $\mathbf{Y} \sim \mathsf{N}(\boldsymbol{a} + B\boldsymbol{\mu}, B^T\Sigma B)$.

- **Central Limit Theorem**:

$$\lim_{n\to\infty} \mathbb{P}\left(\frac{S_n - n\mu}{\sigma\sqrt{n}} \leqslant x\right) = \Phi(x),$$

  where Φ is the cdf of the standard Normal distribution.

- **Normal Approximation to Binomial:** If $X \sim \mathsf{Bin}(n,p)$, then, for large n, $\mathbb{P}(X \leqslant k) \approx \mathbb{P}(Y \leqslant k)$, where $Y \sim \mathsf{N}(np, np(1-p))$.

Statistics

Tests and Confidence Intervals Based on Standard Errors

- Test statistic: $\frac{\text{estimate}-\text{hypothesised}}{\text{se(estimate)}}$.

- Confidence interval: estimate $\pm$ (critical value) $\times$ se(estimate).

- $se(\bar{x}) = \frac{s}{\sqrt{n}}$

- $se(\bar{x}-\bar{y}) = s_p\sqrt{\frac{1}{n_x}+\frac{1}{n_y}}$

- (pooled sample variance) $s_p^2 = \frac{(n_x-1)s_x^2+(n_y-1)s_y^2}{n_x+n_y-2}$.

- $se(\widehat{p}) = \sqrt{\frac{\widehat{p}(1-\widehat{p})}{n}}$

- $se(\widehat{p}_x - \widehat{p}_y) = \sqrt{\frac{\widehat{p}_x(1-\widehat{p}_x)}{n_x}+\frac{\widehat{p}_y(1-\widehat{p}_y)}{n_y}}$

- Use t-distribution for means, correlation and regression. Use normal distribution for proportions.

Chi-squared test

- expected count $= \frac{\text{(row total)}\times\text{(column total)}}{\text{overall total}}$.

- $X^2 = \sum \frac{(\text{observed}-\text{expected})^2}{\text{expected}}$

- degrees of freedom = (#rows − 1) × (#columns − 1).

Linear regression

- $\mathbf{Y} \sim \mathsf{N}(\mathbf{X}\beta, \sigma^2 I)$

- estimator $\widehat{\beta} = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{Y}$

- $\mathrm{Cov}(\widehat{\beta}) = \sigma^2(\mathbf{X}^T\mathbf{X})^{-1}$

- $s^2 = \frac{(\mathbf{Y}-\mathbf{X}\widehat{\beta})^T(\mathbf{Y}-\mathbf{X}\widehat{\beta})}{n-p}$

Other Mathematical Formulas

- Factorial. $n! = n\,(n-1)\,(n-2)\cdots 1$. Gives the number of *permutations* (orderings) of $\{1,\ldots,n\}$.

- Binomial coefficient. $\binom{n}{k} = \frac{n!}{k!\,(n-k)!}$. Gives the number *combinations* (no order) of k different numbers from $\{1,\ldots,n\}$.

- Newton's binomial theorem: $(a+b)^n = \sum_{k=0}^n a^k\,b^{n-k}$.

- Geometric sum: $1 + a + a^2 + \cdots + a^n = \frac{1-a^{n+1}}{1-a}$ $(a \neq 1)$.
  If $|a| < 1$ then $1 + a + a^2 + \cdots = \frac{1}{1-a}$.

- Logarithms:

  1. $\log(x\,y) = \log x + \log y$.
  2. $e^{\log x} = x$.

- Exponential:

  1. $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$.
  2. $e^x = \lim_{n\to\infty}\left(1+\frac{x}{n}\right)^n$.
  3. $e^{x+y} = e^x\,e^y$.

- Differentiation:

  1. $(f+g)' = f' + g'$
  2. $(fg)' = f'g + fg'$
  3. $\left(\frac{f}{g}\right)' = \frac{f'g-fg'}{g^2}$
  4. $\frac{d}{dx}x^n = n\,x^{n-1}$
  5. $\frac{d}{dx}e^x = e^x$
  6. $\frac{d}{dx}\log(x) = \frac{1}{x}$

- Chain rule: $(f(g(x)))' = f'(g(x))\,g'(x)$.

- Integration: $\int_a^b f(x)\,dx = [F(x)]_a^b = F(b) - F(a)$, where $F' = f$.

- Integration by parts: $\int_a^b f(x)\,G(x)\,dx = [F(x)\,G(x)]_a^b - \int_a^b F(x)\,g(x)\,dx$. (Here $F' = f$ and $G' = f$.)

Table 12.1: Standard Normal distribution

	Second decimal place of z									
z	0	1	2	3	4	5	6	7	8	9
0.0	0.500	0.496	0.492	0.488	0.484	0.480	0.476	0.472	0.468	0.464
0.1	0.460	0.456	0.452	0.448	0.444	0.440	0.436	0.433	0.429	0.425
0.2	0.421	0.417	0.413	0.409	0.405	0.401	0.397	0.394	0.390	0.386
0.3	0.382	0.378	0.374	0.371	0.367	0.363	0.359	0.356	0.352	0.348
0.4	0.345	0.341	0.337	0.334	0.330	0.326	0.323	0.319	0.316	0.312
0.5	0.309	0.305	0.302	0.298	0.295	0.291	0.288	0.284	0.281	0.278
0.6	0.274	0.271	0.268	0.264	0.261	0.258	0.255	0.251	0.248	0.245
0.7	0.242	0.239	0.236	0.233	0.230	0.227	0.224	0.221	0.218	0.215
0.8	0.212	0.209	0.206	0.203	0.200	0.198	0.195	0.192	0.189	0.187
0.9	0.184	0.181	0.179	0.176	0.174	0.171	0.169	0.166	0.164	0.161
1.0	0.159	0.156	0.154	0.152	0.149	0.147	0.145	0.142	0.140	0.138
1.1	0.136	0.133	0.131	0.129	0.127	0.125	0.123	0.121	0.119	0.117
1.2	0.115	0.113	0.111	0.109	0.107	0.106	0.104	0.102	0.100	0.099
1.3	0.097	0.095	0.093	0.092	0.090	0.089	0.087	0.085	0.084	0.082
1.4	0.081	0.079	0.078	0.076	0.075	0.074	0.072	0.071	0.069	0.068
1.5	0.067	0.066	0.064	0.063	0.062	0.061	0.059	0.058	0.057	0.056
1.6	0.055	0.054	0.053	0.052	0.051	0.049	0.048	0.047	0.046	0.046
1.7	0.045	0.044	0.043	0.042	0.041	0.040	0.039	0.038	0.038	0.037
1.8	0.036	0.035	0.034	0.034	0.033	0.032	0.031	0.031	0.030	0.029
1.9	0.029	0.028	0.027	0.027	0.026	0.026	0.025	0.024	0.024	0.023
2.0	0.023	0.022	0.022	0.021	0.021	0.020	0.020	0.019	0.019	0.018
2.1	0.018	0.017	0.017	0.017	0.016	0.016	0.015	0.015	0.015	0.014
2.2	0.014	0.014	0.013	0.013	0.013	0.012	0.012	0.012	0.011	0.011
2.3	0.011	0.010	0.010	0.010	0.010	0.009	0.009	0.009	0.009	0.008
2.4	0.008	0.008	0.008	0.008	0.007	0.007	0.007	0.007	0.007	0.006
2.5	0.006	0.006	0.006	0.006	0.006	0.005	0.005	0.005	0.005	0.005
2.6	0.005	0.005	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004
2.7	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003
2.8	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002
2.9	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.001	0.001	0.001
3.0	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
3.1	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
3.2	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
3.3										

This table gives $P(Z \geq z)$ for $Z \sim \text{Normal}(0,1)$. Critical values of the Normal distribution, the z^* values such that $P(Z \geq z^*) = p$ for a particular p, can be found from the ∞ row of Table 14.2.

Table 14.2: Critical values of Student's T distribution

	Probability p								
df	0.25	0.10	0.05	0.025	0.01	0.005	0.001	0.0005	0.0001
1	1.000	3.078	6.314	12.71	31.82	63.66	318.3	636.6	3183.1
2	0.816	1.886	2.920	4.303	6.965	9.925	22.33	31.60	70.70
3	0.765	1.638	2.353	3.182	4.541	5.841	10.21	12.92	22.20
4	0.741	1.533	2.132	2.776	3.747	4.604	7.173	8.610	13.03
5	0.727	1.476	2.015	2.571	3.365	4.032	5.893	6.869	9.678
6	0.718	1.440	1.943	2.447	3.143	3.707	5.208	5.959	8.025
7	0.711	1.415	1.895	2.365	2.998	3.499	4.785	5.408	7.063
8	0.706	1.397	1.860	2.306	2.896	3.355	4.501	5.041	6.442
9	0.703	1.383	1.833	2.262	2.821	3.250	4.297	4.781	6.010
10	0.700	1.372	1.812	2.228	2.764	3.169	4.144	4.587	5.694
11	0.697	1.363	1.796	2.201	2.718	3.106	4.025	4.437	5.453
12	0.695	1.356	1.782	2.179	2.681	3.055	3.930	4.318	5.263
13	0.694	1.350	1.771	2.160	2.650	3.012	3.852	4.221	5.111
14	0.692	1.345	1.761	2.145	2.624	2.977	3.787	4.140	4.985
15	0.691	1.341	1.753	2.131	2.602	2.947	3.733	4.073	4.880
16	0.690	1.337	1.746	2.120	2.583	2.921	3.686	4.015	4.791
17	0.689	1.333	1.740	2.110	2.567	2.898	3.646	3.965	4.714
18	0.688	1.330	1.734	2.101	2.552	2.878	3.610	3.922	4.648
19	0.688	1.328	1.729	2.093	2.539	2.861	3.579	3.883	4.590
20	0.687	1.325	1.725	2.086	2.528	2.845	3.552	3.850	4.539
21	0.686	1.323	1.721	2.080	2.518	2.831	3.527	3.819	4.493
22	0.686	1.321	1.717	2.074	2.508	2.819	3.505	3.792	4.452
23	0.685	1.319	1.714	2.069	2.500	2.807	3.485	3.768	4.415
24	0.685	1.318	1.711	2.064	2.492	2.797	3.467	3.745	4.382
25	0.684	1.316	1.708	2.060	2.485	2.787	3.450	3.725	4.352
26	0.684	1.315	1.706	2.056	2.479	2.779	3.435	3.707	4.324
27	0.684	1.314	1.703	2.052	2.473	2.771	3.421	3.690	4.299
28	0.683	1.313	1.701	2.048	2.467	2.763	3.408	3.674	4.275
29	0.683	1.311	1.699	2.045	2.462	2.756	3.396	3.659	4.254
30	0.683	1.310	1.697	2.042	2.457	2.750	3.385	3.646	4.234
40	0.681	1.303	1.684	2.021	2.423	2.704	3.307	3.551	4.094
50	0.679	1.299	1.676	2.009	2.403	2.678	3.261	3.496	4.014
60	0.679	1.296	1.671	2.000	2.390	2.660	3.232	3.460	3.962
70	0.678	1.294	1.667	1.994	2.381	2.648	3.211	3.435	3.926
80	0.678	1.292	1.664	1.990	2.374	2.639	3.195	3.416	3.899
90	0.677	1.291	1.662	1.987	2.368	2.632	3.183	3.402	3.878
100	0.677	1.290	1.660	1.984	2.364	2.626	3.174	3.390	3.862
∞	0.674	1.282	1.645	1.960	2.326	2.576	3.090	3.291	3.719

This table gives t^* such that $P(T \geq t^*) = p$, where $T \sim$ Student(df).

Table 22.4: χ^2 distribution

	Probability p								
df	0.975	0.95	0.25	0.10	0.05	0.025	0.01	0.005	0.001
1	0.001	0.004	1.323	2.706	3.841	5.024	6.635	7.879	10.83
2	0.051	0.103	2.773	4.605	5.991	7.378	9.210	10.60	13.82
3	0.216	0.352	4.108	6.251	7.815	9.348	11.34	12.84	16.27
4	0.484	0.711	5.385	7.779	9.488	11.14	13.28	14.86	18.47
5	0.831	1.145	6.626	9.236	11.07	12.83	15.09	16.75	20.52
6	1.237	1.635	7.841	10.64	12.59	14.45	16.81	18.55	22.46
7	1.690	2.167	9.037	12.02	14.07	16.01	18.48	20.28	24.32
8	2.180	2.733	10.22	13.36	15.51	17.53	20.09	21.95	26.12
9	2.700	3.325	11.39	14.68	16.92	19.02	21.67	23.59	27.88
10	3.247	3.940	12.55	15.99	18.31	20.48	23.21	25.19	29.59
11	3.816	4.575	13.70	17.28	19.68	21.92	24.72	26.76	31.26
12	4.404	5.226	14.85	18.55	21.03	23.34	26.22	28.30	32.91
13	5.009	5.892	15.98	19.81	22.36	24.74	27.69	29.82	34.53
14	5.629	6.571	17.12	21.06	23.68	26.12	29.14	31.32	36.12
15	6.262	7.261	18.25	22.31	25.00	27.49	30.58	32.80	37.70
16	6.908	7.962	19.37	23.54	26.30	28.85	32.00	34.27	39.25
17	7.564	8.672	20.49	24.77	27.59	30.19	33.41	35.72	40.79
18	8.231	9.390	21.60	25.99	28.87	31.53	34.81	37.16	42.31
19	8.907	10.12	22.72	27.20	30.14	32.85	36.19	38.58	43.82
20	9.591	10.85	23.83	28.41	31.41	34.17	37.57	40.00	45.31
21	10.28	11.59	24.93	29.62	32.67	35.48	38.93	41.40	46.80
22	10.98	12.34	26.04	30.81	33.92	36.78	40.29	42.80	48.27
23	11.69	13.09	27.14	32.01	35.17	38.08	41.64	44.18	49.73
24	12.40	13.85	28.24	33.20	36.42	39.36	42.98	45.56	51.18
25	13.12	14.61	29.34	34.38	37.65	40.65	44.31	46.93	52.62
26	13.84	15.38	30.43	35.56	38.89	41.92	45.64	48.29	54.05
27	14.57	16.15	31.53	36.74	40.11	43.19	46.96	49.64	55.48
28	15.31	16.93	32.62	37.92	41.34	44.46	48.28	50.99	56.89
29	16.05	17.71	33.71	39.09	42.56	45.72	49.59	52.34	58.30
30	16.79	18.49	34.80	40.26	43.77	46.98	50.89	53.67	59.70
40	24.43	26.51	45.62	51.81	55.76	59.34	63.69	66.77	73.40
50	32.36	34.76	56.33	63.17	67.50	71.42	76.15	79.49	86.66
60	40.48	43.19	66.98	74.40	79.08	83.30	88.38	91.95	99.61
70	48.76	51.74	77.58	85.53	90.53	95.02	100.4	104.2	112.3
80	57.15	60.39	88.13	96.58	101.9	106.6	112.3	116.3	124.8
90	65.65	69.13	98.65	107.6	113.1	118.1	124.1	128.3	137.2
100	74.22	77.93	109.1	118.5	124.3	129.6	135.8	140.2	149.4

This table gives x^* such that $P(X^2 \geq x^*) = p$, where $X^2 \sim \chi^2(\text{df})$.