STAT2203/7203: Week 6 Practical Questions

1. Let $(a_1, \ldots, a_n)$ be a random permutation of the integers $\{1, 2, \ldots, n\}$ with all permuations equally likely. An inversion in a permutation is an ordered pair (i, j) such that $i < j$ and $a_i > a_j$. For example, $(1, 3, 2, 4)$ has one inversion while $(1, 4, 2, 3)$ two inversions.

   For $i < j$, let X_{ij} be the random variable such that

   $$X_{ij} = \begin{cases} 1, & a_i > a_j \\ 0, & a_i < a_j \end{cases}$$

   where $(a_1, \ldots, a_n)$ is a random permutation.

   Aside: The sorting algorithm Bubblesort sorts a list by resolving inversions one by one. The above analysis essentially determines the expected number of swaps performed by Bubblesort.

   (a) What is the expected value of X_{ij}?

   (b) What is the expected number of inversions in a random permutation?

2. Binomial distribution:

   (a) A couple decides that they will have four children. Let X be the number of girls they will have. Assuming that the probability of a girl is 0.50, independent across births, what is the distribution of X?

   (b) Let X be the number of towns in which it will rain tomorrow among five neighbouring towns. Is X a Binomial random variable?

   (c) Suppose 10% of people are left-handed and let X be the number of left-handed people in sample of 20 individuals. What is the probability of at least one left-handed person in the sample?

   (d) Suppose a drug has a 20% chance of making a person drowsy. Out of a sample of 80 people who each take the drug, what is the probability that no more than 10 of them experience drowsiness?

   (e) An expensive piece of equipment in a laboratory is starting to show signs of age. Let X be the number of weekdays in any week that the equipment is working and suppose that X has a Binomial distribution with expected value and standard deviation to be 2.92 and 1.102, respectively. What are the parameters of the Binomial distribution?