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Turnco Project

Part A

Brief Report

Introduction

Markov Decision Process (MDP) is a mathematical method that models the optimal decision process in dynamic problems. As for this Turnco Engineering case, since this engineering machine shop operating in different conditions is subject to deterioration and failure, we need to make decisions concerning each condition as whether to keep operating or not. Therefore, based on the transition probabilities among conditions, due to the different operation profits and possible to return to the excellent condition by preventive maintenance or manufacturer repair, we can solve this Turnco operation case as a MDP problem.

We need to solve the following issues:

1. the optimal policy for the operation of the grinding equipment
2. how robust the findings are to changes in the problem parameters
3. the proportion of time the equipment is operated and the proportion of time it is out of use in the long run

Before solving the issues, we firstly make assumptions and basic explanations on gaining results.

Assumption

Our basic assumption is - Turnco can only be repaired by the manufacturer when it is below the standard required for normal operation. That means, only the result of the inspection is below the required standard, we need to contact the manufacturer to repair. If Turnco is about to operate normally tomorrow, no matter in condition 1 or in condition 5, we will not get it repaired tomorrow.

As for the profit from the operation of Turnco for one day, we find the great differences between Scenario A & Scenario B concerning condition 2, condition 3 & condition 4 (scenario A & B generate the same profits in conditions 1 & 5). Hence, we set the profit generated in such three conditions as a linear combination of Scenarios A & B when modeling the problem. Initially, we set the 'new profits in Scenario C' as 'half of Scenario A + half of Scenario B'. The values can be shown as follows,

reward condition 1 A	ra_1	5000
reward condition 2 A	ra_2	4200
reward condition 3 A	ra_3	3400
reward condition 4 A	ra_4	2600
reward condition 5 A	ra_5	1800
reward condition 1 B	rb_1	5000
reward condition 2 B	rb_2	4800
reward condition 3 B	rb_3	4200
reward condition 4 B	rb_4	3200
reward condition 5 B	rb_5	1800
reward condition 1 C	rc_1	5000
reward condition 2 C	rc_2	4500
reward condition 3 C	rc_3	3800
reward condition 4 C	rc_4	2900
reward condition 5 C	rc_5	1800

After gaining the optimal policy, we will change the coefficients of this linear combination in Scenario Summary for Sensitivity Analysis.

Another assumption is - setting the probability of successful preventive maintenance as 0.9. We use the higher value (rather than 0.7, probability of successful manufacturer repair) because we consider that preventive maintenance is much easier to be applied than manufacturer repair. What' more, we use all data from the problem sheet rather than creating values on our own.

The aim is to maximize the expected discounted reward over n steps (n iterations), as we know, the given discount factor is 0.95, we could not directly set the number of steps, but we can restrict the error (the value at the nth iteration should be within 0.01). Detailed explanations of iterations and error-control can be seen in the Appendix.

Basic explanations

Generally, the expected total reward is not suitable as a decision criterion. To achieve optimal policy for Turnco operations, we consider the expected discount reward concerning each initial state to avoid the infinite profit under any other policy. If not, we will not get relatively stable optimal rewards in the long run, and also, it will be difficult and inaccurate to determine the optimal policy. Hence the discount factor is very important, and it is not robust to changes, the reasons and corresponding results will be shown later.

After entering all the parameters, we need to create functions used for further iterations. Hence, it is important to know the middle process - setting states and using proper parameters for each condition of Turnco. As we know, we make decisions depending on the result of the inspection at the end of each day. Based on that, at the start of the next day, we could choose to keep operating (kp), get preventive maintenance (pm) if Turnco is reported in condition 1, condition 2,...... condition 5. Additionally, Turnco will be repaired by the manufacturer (mr) when it is below the required standard.

Hence, we set six states depending on the uncertain actions and reported conditions, and the optimal policy is based on the reward (profit) value for each state initially. Our target is to determine the optimal action to achieve the maximum profit in the long run when we find Turnco is initially at state 1 (condition1), state 2 (condition),..... state f (below the required standard).

The detailed explanations for using parameters and creating formulations on achieving results will be shown in detail in the Appendix. Here, we just give you the general view of that.

Optimal Policy

The following figures illustrate the last few iterations to determine the optimal policy,

294	291	90892.530	85539.350	80481.926	77043.640	77097.575	83277.666	kp	pm	pm	kp	kp	mr
295	292	90892.531	85539.351	80481.927	77043.641	77097.576	83277.667	kp	pm	pm	kp	kp	mr
296	293	90892.533	85539.353	80481.929	77043.642	77097.578	83277.669	kp	pm	pm	kp	kp	mr
297	294	90892.534	85539.354	80481.930	77043.644	77097.579	83277.670	kp	pm	pm	kp	kp	mr
298	295	90892.535	85539.355	80481.931	77043.645	77097.580	83277.671	kp	pm	pm	kp	kp	mr
299	296	90892.537	85539.357	80481.932	77043.646	77097.581	83277.672	kp	pm	pm	kp	kp	mr
300	297	90892.538	85539.358	80481.933	77043.647	77097.583	83277.673	kp	pm	pm	kp	kp	mr
301	298	90892.539	85539.359	80481.934	77043.648	77097.584	83277.675	kp	pm	pm	kp	kp	mr
302	299	90892.540	85539.360	80481.935	77043.649	77097.585	83277.676	kp	pm	pm	kp	kp	mr
303	300	90892.541	85539.361	80481.936	77043.650	77097.586	83277.677	kp	pm	pm	kp	kp	mr
304	301	90892.542	85539.362	80481.937	77043.651	77097.587	83277.677	kp	pm	pm	kp	kp	mr
305	302	90892.543	85539.363	80481.938	77043.652	77097.587	83277.678	kp	pm	pm	kp	kp	mr
306	303	90892.543	85539.363	80481.939	77043.653	77097.588	83277.679	kp	pm	pm	kp	kp	mr
307	304	90892.544	85539.364	80481.940	77043.654	77097.589	83277.680	kp	pm	pm	kp	kp	mr
308	305	90892.545	85539.365	80481.941	77043.655	77097.590	83277.681	kp	pm	pm	kp	kp	mr
309	306	90892.546	85539.366	80481.941	77043.655	77097.591	83277.681	kp	pm	pm	kp	kp	mr
310	307	90892.546	85539.366	80481.942	77043.656	77097.591	83277.682	kp	pm	pm	kp	kp	mr
311	308	90892.547	85539.367	80481.943	77043.657	77097.592	83277.683	kp	pm	pm	kp	kp	mr
312	309	90892.548	85539.368	80481.943	77043.657	77097.592	83277.683	kp	pm	pm	kp	kp	mr
313	310	90892.548	85539.368	80481.944	77043.658	77097.593	83277.684	kp	pm	pm	kp	kp	mr
314	311	90892.549	85539.369	80481.944	77043.658	77097.594	83277.684	kp	pm	pm	kp	kp	mr

294	90892.530	-300	85237.566	85539.350	80065.526	80481.926	77043.640	75831.191	77097.575	72896.580	83277.666	0.002	0.002	0.002	0.002	0.002	0.002	0.029
295	90892.531	-300	85237.567	85539.351	80065.527	80481.927	77043.641	75831.192	77097.576	72896.581	83277.667	0.001	0.001	0.001	0.001	0.001	0.001	0.028
296	90892.533	-300	85237.569	85539.353	80065.528	80481.929	77043.642	75831.194	77097.578	72896.583	83277.669	0.001	0.001	0.001	0.001	0.001	0.001	0.026
297	90892.534	-300	85237.570	85539.354	80065.530	80481.930	77043.644	75831.195	77097.579	72896.584	83277.670	0.001	0.001	0.001	0.001	0.001	0.001	0.025
298	90892.535	-300	85237.571	85539.355	80065.531	80481.931	77043.645	75831.196	77097.580	72896.585	83277.671	0.001	0.001	0.001	0.001	0.001	0.001	0.024
299	90892.537	-300	85237.573	85539.357	80065.532	80481.932	77043.646	75831.197	77097.581	72896.587	83277.672	0.001	0.001	0.001	0.001	0.001	0.001	0.023
300	90892.538	-300	85237.574	85539.358	80065.533	80481.933	77043.647	75831.198	77097.583	72896.588	83277.673	0.001	0.001	0.001	0.001	0.001	0.001	0.021
301	90892.539	-300	85237.575	85539.359	80065.534	80481.934	77043.648	75831.200	77097.584	72896.589	83277.675	0.001	0.001	0.001	0.001	0.001	0.001	0.020
302	90892.540	-300	85237.576	85539.360	80065.535	80481.935	77043.649	75831.201	77097.585	72896.590	83277.676	0.001	0.001	0.001	0.001	0.001	0.001	0.019
303	90892.541	-300	85237.577	85539.361	80065.536	80481.936	77043.650	75831.202	77097.586	72896.591	83277.677	0.001	0.001	0.001	0.001	0.001	0.001	0.018
304	90892.542	-300	85237.578	85539.362	80065.537	80481.937	77043.651	75831.202	77097.587	72896.592	83277.677	0.001	0.001	0.001	0.001	0.001	0.001	0.017
305	90892.543	-300	85237.579	85539.363	80065.538	80481.938	77043.652	75831.203	77097.587	72896.593	83277.678	0.001	0.001	0.001	0.001	0.001	0.001	0.017
306	90892.543	-300	85237.579	85539.363	80065.539	80481.939	77043.653	75831.204	77097.588	72896.593	83277.679	0.001	0.001	0.001	0.001	0.001	0.001	0.016
307	90892.544	-300	85237.580	85539.364	80065.540	80481.940	77043.654	75831.205	77097.589	72896.594	83277.680	0.001	0.001	0.001	0.001	0.001	0.001	0.015
308	90892.545	-300	85237.581	85539.365	80065.541	80481.941	77043.655	75831.206	77097.590	72896.595	83277.681	0.001	0.001	0.001	0.001	0.001	0.001	0.014
309	90892.546	-300	85237.582	85539.366	80065.541	80481.941	77043.655	75831.206	77097.591	72896.596	83277.681	0.001	0.001	0.001	0.001	0.001	0.001	0.014
310	90892.546	-300	85237.582	85539.366	80065.542	80481.942	77043.656	75831.207	77097.591	72896.596	83277.682	0.001	0.001	0.001	0.001	0.001	0.001	0.013
311	90892.547	-300	85237.583	85539.367	80065.543	80481.943	77043.657	75831.208	77097.592	72896.597	83277.683	0.001	0.001	0.001	0.001	0.001	0.001	0.012
312	90892.548	-300	85237.584	85539.368	80065.543	80481.943	77043.657	75831.208	77097.592	72896.598	83277.683	0.001	0.001	0.001	0.001	0.001	0.001	0.012
313	90892.548	-300	85237.584	85539.368	80065.544	80481.944	77043.658	75831.209	77097.593	72896.598	83277.684	0.001	0.001	0.001	0.001	0.001	0.001	0.011
314	90892.549	-300	85237.585	85539.369	80065.544	80481.944	77043.658	75831.209	77097.594	72896.599	83277.684	0.001	0.001	0.001	0.001	0.001	0.001	0.010

Hence, we can gain the optimal policy, since the still actions are 'kp', 'pm', 'pm', 'kp', 'kp', 'mr' respectively. Additionally, we find the values of $v_j^n - v_j^{n-1}$ are all equal to 0.001, which control the error and indicate the convergence of profit.

Therefore, the final optimal policy under our assumption and initial parameters are as follows:

When Turnco is reported as being in condition 1, keep the operation tomorrow.
When Turnco is reported as being in condition 2, give it preventive maintenance tomorrow.
When Turnco is reported as being in condition 3, give it preventive maintenance tomorrow.
When Turnco is reported as being in condition 4, keep the operation tomorrow.
When Turnco is reported as being in condition 5, keep the operation tomorrow.

When Turnco is reported as being below the required standard, contact the manufacturer to repair tomorrow.

Finally, we can see the expected profits in each condition initially. They are 90892.549, 85539.396, 80481.944, 77043.658, 77097.594, 83277.684. Overall, we find our optimal policy seems rational, since if Turnco is in condition 2 or 3, we should apply pm to avoid transiting to worse conditions, and if Turnco is in any worse condition, we had better keep the operations to make it below the required standard quickly to get repaired.

Sensitivity Analysis

The MDP formulation and calculations are immediately updated to reflect the changes in parameter values due to the developments of iterations. That makes the sensitivity analysis quite essential which targets to investigate how robust the optimal policy gained before is to changes in one or more parameters. If we test several parameters one by one, we just need to use Data Table from the What-If Analysis in Excel. Yet a more complex and common situation is that changing several parameters at the same time, and then the Data Table technique is not suitable, we use the Scenario Summary from the What-If Analysis in Excel.

In sensitivity analysis, we will test how robust the changes to findings are. As the optimal policy is based on the maximum of discounted reward over an infinite horizon, it is necessary to consider the changes to discount factors (initial $\beta = 0.95$). Besides the 'new profits in Scenario C' in Scenario summary before, we consider the other two important parameters, 'probability of successful preventive maintenance (we set the initial value as 0.90)' and 'probability of successful manufacturer repair (initial value is 0.70)'.

To start with, consider the three parameters above, probability of successful manufacturer repair (psr), probability of successful preventive maintenance (psm), and discount factor (dff). We can get the following results,

As for psr ranging from 0.95 to 0.50 (step = 0.05), the expected discounted rewards all decrease, and the optimal policy changes when psr is less than or equal to 0.55 - the optimal action for state 4 changes to 'pm'. The error is relatively stable.

As for psm ranging from 0.99 to 0.70 (step = 0.01), the expected discounted rewards all decrease, and the optimal policy changes when psr is less than or equal to 0.74 - the optimal action for state 3 changes to 'kp'. The error is relatively stable.

As for dff ranging from 0.99 to 0.85 (step = 0.01), the expected discounted rewards all decrease (relatively fast because dff directly influence the value and speed of the convergence for expected discounted rewards), and the optimal policy changes when psr is less and equal than 0.90 - both of optimal actions for state 2 and state 3 change to 'kp', and the optimal policy also changes when psr is equal to 0.91 - the optimal action for state 3 changes to 'kp'. Surprisingly, the error dramatically decreases (close to 0 when $df < 0.94$). That indicates when the discount factor is a little bit smaller, it greatly contributes to the expected discounted rewards and error, to some extent, not robust.

Then we consider changing several parameters simultaneously. It is reasonable for changes to profits in condition 2, 3, and 4 because in the previous assumption we set the coefficients of Scenario A & B as 0.5 & 0.5. Here we change the coefficients of such a linear combination to test how robust the changes to profits are. Here we use Scenario Summary to accomplish our goal. We create 21 scenarios, with coefficients ranging from (1,0), (0.95,0.05),... to (0.05,0.95), (0,1). The results are shown as in the following image,

Scenario Summary	A	0.95A+0.05B	0.90A+0.10B	0.85A+0.15B	0.80A+0.20B	0.75A+0.25B	0.70A+0.30B
Changing Cells:							
reward condition 2	4200	4230	4260	4290	4320	4350	4380
reward condition 3	3400	3440	3480	3520	3560	3600	3640
reward condition 4	2600	2630	2660	2690	2720	2750	2780
Result Cells:							
Value when in condition 1 initially	90714.909	90732.673	90750.437	90768.201	90785.965	90803.729	90821.493
Value when in condition 2 initially	85371.543	85388.326	85405.108	85421.891	85438.673	85455.456	85472.238
Value when in condition 3 initially	80323.391	80339.246	80355.102	80370.957	80386.812	80402.668	80418.523
Value when in condition 4 initially	75870.119	75987.473	76104.827	76222.181	76339.535	76456.888	76574.242
Value when in condition 5 initially	76953.973	76968.335	76982.697	76997.059	77011.421	77025.783	77040.145
Value when below standard initially	83112.467	83128.989	83145.51	83162.032	83178.554	83195.076	83211.597
Decision when in condition1	kp	kp	kp	kp	kp	kp	kp
Decision when in condition1	pm	pm	pm	pm	pm	pm	pm
Decision when in condition1	pm	pm	pm	pm	pm	pm	pm
Decision when in condition1	kp	kp	kp	kp	kp	kp	kp
Decision when in condition1	kp	kp	kp	kp	kp	kp	kp
Decision when below standard	mr	mr	mr	mr	mr	mr	mr
Error	0.0104813	0.0104798	0.0104781	0.0104764	0.0104746	0.0104719	0.0104692

Scenario Summary	0.65A+0.35B	0.60A+0.40B	0.55A+0.45B	0.50A+0.50B	0.45A+0.55B	0.40A+0.60B	0.35A+0.65B
Changing Cells:							
reward condition 2	4410	4440	4470	4500	4530	4560	4590
reward condition 3	3680	3720	3760	3800	3840	3880	3920
reward condition 4	2810	2840	2870	2900	2930	2960	2990
Result Cells:							
Value when in condition 1 initially	90839.257	90857.021	90874.785	90892.549	90910.313	90928.077	90945.841
Value when in condition 2 initially	85489.021	85505.803	85522.586	85539.369	85556.151	85572.934	85589.716
Value when in condition 3 initially	80434.378	80450.234	80466.089	80481.944	80497.8	80513.655	80529.51
Value when in condition 4 initially	76691.596	76808.95	76926.304	77043.658	77161.012	77278.366	77395.72
Value when in condition 5 initially	77054.507	77068.869	77083.232	77097.594	77111.956	77126.318	77140.68
Value when below standard initially	83228.119	83244.641	83261.163	83277.684	83294.206	83310.728	83327.25
Decision when in condition1	kp	kp	kp	kp	kp	kp	kp
Decision when in condition1	pm	pm	pm	pm	pm	pm	pm
Decision when in condition1	pm	pm	pm	pm	pm	pm	pm
Decision when in condition1	kp	kp	kp	kp	kp	kp	kp
Decision when in condition1	kp	kp	kp	kp	kp	kp	kp
Decision when below standard	mr	mr	mr	mr	mr	mr	mr
Error	0.010466	0.0104622	0.010458	0.0104527	0.0104469	0.0104402	0.0104318

Scenario Summary	0.30A+0.70B	0.25A+0.75B	0.20A+0.80B	0.15A+0.85B	0.10A+0.90B	0.05A+0.95B	B
Changing Cells:							
reward condition 2	4620	4650	4680	4710	4740	4770	4800
reward condition 3	3960	4000	4040	4080	4120	4160	4200
reward condition 4	3020	3050	3080	3110	3140	3170	3200
Result Cells:							
Value when in condition 1 initially	90963.605	90981.369	90999.133	91016.897	91034.661	91052.425	91150.962
Value when in condition 2 initially	85606.499	85623.282	85640.064	85656.847	85673.629	85690.412	85874.919
Value when in condition 3 initially	80545.366	80561.221	80577.077	80592.932	80608.787	80624.643	80912.821
Value when in condition 4 initially	77513.074	77630.428	77747.782	77865.136	77982.49	78099.844	78276.337
Value when in condition 5 initially	77155.042	77169.404	77183.766	77198.128	77212.49	77226.852	77306.519
Value when below standard initially	83343.772	83360.293	83376.815	83393.337	83409.859	83426.38	83518.027
Decision when in condition1	kp	kp	kp	kp	kp	kp	kp
Decision when in condition1	pm	pm	pm	pm	pm	pm	kp
Decision when in condition1	pm	pm	pm	pm	pm	pm	kp
Decision when in condition1	kp	kp	kp	kp	kp	kp	kp
Decision when in condition1	kp	kp	kp	kp	kp	kp	kp
Decision when below standard	mr	mr	mr	mr	mr	mr	mr
Error	0.0104222	0.0104108	0.0103967	0.0103793	0.0103563	0.0103216	0.0103061

It is clear that the expected discounted rewards all increase due to increasing profits for each state and the error is kind of still. A much more surprising result is - As we totally apply Scenario B for Turnco operations, the optimal policy change greatly, from previous 'kp', 'pm', 'pm', 'kp', 'kp', 'mr' to 'kp', 'kp', 'kp', 'kp', 'kp', 'mr' respectively. That means Turnco should keep operations at any condition because profits are large enough that preventive maintenance could be ignored.

Therefore, it is natural to consider the changes to preventive maintenance cost (pmc) and manufacturer repair cost (mrc) to deeply seek the relations between profits and costs. We apply Sensitivity Analysis for pmc and mrc, and we can get the results below,

As for pmc ranging from 150 to 1300 (step = 50), according to the optimal decisions, the breakpoint will belong to (1150 , 1200) because action for state 3 changes to 'kp' when 'pmc=1200', which means we determine to keep operations when Turnco is found in condition 3. For values of profits, it is obvious that they will go down as pmc increases. To sum up, both profits and errors are stable.

As for mrc ranging from 450 to 1800 (step = 50), according to the optimal decisions, the breakpoint will belong to (1550 , 1600) because action for state 4 changes to 'pm' when 'mrc=1600', which means we determine to use preventive maintenance when Turnco is found in condition 4. For values of profits, it is obvious that they will go down as mrc increases. To sum up, both profits and errors are stable.

Proportion of time

As for the proportion of time for normal operations and out of use, we need to consider the proportion of time in each state firstly, and then, combined with the optimal policy we gained before, we add the proportion of state 1, state 4, and state 5 for the proportion of operations time together. Since both preventive maintenance and manufacturer repair needs one day out of use, we add the proportion of time of the remained states together. The calculations for each proportion will be shown in the Appendix.

Overall speaking, we get the following proportions approximately, 0.8815, 0.0392, 0.0098, 0.0353, 0.0091 and 0.0252 for the six states respectively. After additive calculation, we get the proportion of normal operations is 0.9259, and the proportion of out of use is 0.0741, which means under such optimal policy, Turnco will operate for approximately 92.59% of the total time, and the remained time (7.41%) is out of use due to preventive maintenance or manufacturer repair.

From our perspective, this proportion is reasonable and not bad because it is larger than 90%. As the proportion of normal operations of Turnco implies the efficiency of its working to some extent, we hope to increase that to more than 95% without losing profits. If we want to achieve the target, base on Sensitivity Analysis, we could increase psr or psm to save time on out of use (this way will not change the optimal policy but will increase the optimal rewards), or fully apply Scenario B or greater Scenario to increase both proportions and profits without changing the transition matrix.

Appendix

How to use the data from the description

Based on the general description from the Brief Report, we will use these figures to achieve the optimal policy,

Values of profits and costs,

Profit in condition 1 under Scenario A : 5000 (A_1)
Profit in condition 2 under Scenario A : 4200 (A_2)
Profit in condition 3 under Scenario A : 3400 (A_3)
Profit in condition 4 under Scenario A : 2600 (A_4)
Profit in condition 5 under Scenario A : 1800 (A_5)
Profit in condition 1 under Scenario B : 5000 (B_1)
Profit in condition 2 under Scenario B : 4800 (B_2)
Profit in condition 3 under Scenario B : 4200 (B_3)
Profit in condition 4 under Scenario B : 3200 (B_4)
Profit in condition 5 under Scenario B : 1800 (B_5)
Profit in condition 1 under applied Scenario C : 5000 (C_1)
Profit in condition 2 under applied Scenario C : 4500 ($C_2 = 0.5A_2 + 0.5B_2$)
Profit in condition 3 under applied Scenario C : 3800 ($C_3 = 0.5A_3 + 0.5B_3$)
Profit in condition 4 under applied Scenario C : 2900 ($C_4 = 0.5A_4 + 0.5B_4$)
Profit in condition 5 under applied Scenario C : 1800 (C_5)
Cost of preventive maintenance : pmc = 300
Cost of manufacturer repair : mrc = 900

Probabilities of condition transitions (based on Table 1 and Table 2 in problem s)

Probability of condition 1 to condition 1 : ($P_{1,1} = 0.94$)
Probability of condition 1 to condition 2 : ($P_{1,2} = 0.03$)
Probability of condition 1 to condition 3 : ($P_{1,3} = 0.01$)
Probability of condition 1 to condition 4 : ($P_{1,4} = 0.01$)
Probability of condition 1 to condition 5 : ($P_{1,5} = 0$)
Probability of condition 1 to condition below required standard : ($P_{1,f} = 0.01$)
Probability of condition 2 to condition 1 : ($P_{2,1} = 0$)
Probability of condition 2 to condition 2 : ($P_{2,2} = 0.90$)
Probability of condition 2 to condition 3 : ($P_{2,3} = 0.05$)
Probability of condition 2 to condition 4 : ($P_{2,4} = 0.02$)
Probability of condition 2 to condition 5 : ($P_{2,5} = 0.01$)
Probability of condition 2 to condition below required standard : ($P_{2,f} = 0.02$)
Probability of condition 3 to condition 1 : ($P_{3,1} = 0$)
Probability of condition 3 to condition 2 : ($P_{3,2} = 0$)
Probability of condition 3 to condition 3 : ($P_{3,3} = 0.85$)
Probability of condition 3 to condition 4 : ($P_{3,4} = 0.07$)
Probability of condition 3 to condition 5 : ($P_{3,5} = 0.03$)
Probability of condition 3 to condition below required standard : ($P_{1,f} = 0.05$)
Probability of condition 4 to condition 1 : ($P_{4,1} = 0$)
Probability of condition 4 to condition 2 : ($P_{4,2} = 0$)
Probability of condition 4 to condition 3 : ($P_{4,3} = 0$)
Probability of condition 4 to condition 4 : ($P_{4,4} = 0.75$)
Probability of condition 4 to condition 5 : ($P_{4,5} = 0.09$)
Probability of condition 4 to condition below required standard : ($P_{4,f} = 0.16$)

Probability of condition 5 to condition 1 : ($P_{5,1} = 0$)
Probability of condition 5 to condition 2 : ($P_{5,2} = 0$)
Probability of condition 5 to condition 3 : ($P_{5,3} = 0$)
Probability of condition 5 to condition 4 : ($P_{5,4} = 0$)
Probability of condition 5 to condition 5 : ($P_{5,5} = 0.65$)
Probability of condition 5 to condition below required standard : ($P_{5,f} = 0.35$)
Probability of below required condition to condition 1 : ($P_{f,1} = 0.70$)
Probability of below required condition to condition 2 : ($P_{f,2} = 0$)
Probability of below required condition to condition 3 : ($P_{f,3} = 0$)
Probability of below required condition to condition 4 : ($P_{f,4} = 0$)
Probability of below required condition to condition 5 : ($P_{f,5} = 0$)
Probability of below required condition to itself : ($P_{f,f} = 0.30$)

Hence we can find there are six states, S_1 for condition 1, S_2 for condition 2, S_3 for condition 3, S_4 for condition 4, S_5 for condition 5, S_f for below-required standard. The reason for $P_{f,i}$ when $i = 2, 3, 4, 5$ is because $P_{f,1}$ means Turnco is repaired successfully to condition1, and $P_{f,f}$ means manufacturer repair fails. Additionally, the discount factor is given as $\beta = 0.95$.

Enter the above data on the spreadsheet

To begin with, we enter all the data above. Here, we discard all 0 values and set the probability of successful preventive maintenance as 0.9. and $pfr = 1 - psr$, $pfm = 1 - psm$.

The column headed “Label” defines a name to be given to each parameter to make it easier to enter, check, and understand formulae relating to the parameters. Select the columns of "Label" & "Value". In the Defined Names area on the Formulas tab, select Create from Selection to display the Create Names from Selection dialog box. Ensure the “Left column” option is selected and click “OK”. Now we can use the replacement cost in a formula, for example, the name “pmc”, "mrc", "rc_1" can be used.

The image below illustrates the results, (in Sheet 1)

Problem Parameters	Label	Value
preventive maintenance cost	pmc	300
manufacturer repair cost	mrc	900
reward condition 1 A	ra_1	5000
reward condition 2 A	ra_2	4200
reward condition 3 A	ra_3	3400
reward condition 4 A	ra_4	2600
reward condition 5 A	ra_5	1800
reward condition 1 B	rb_1	5000
reward condition 2 B	rb_2	4800
reward condition 3 B	rb_3	4200
reward condition 4 B	rb_4	3200
reward condition 5 B	rb_5	1800
reward condition 1 C	rc_1	5000
reward condition 2 C	rc_2	4500
reward condition 3 C	rc_3	3800
reward condition 4 C	rc_4	2900
reward condition 5 C	rc_5	1800
Probability of 1 to 1 if operated	p_11	0.94
Probability of 1 to 2 if operated	p_12	0.03
Probability of 1 to 3 if operated	p_13	0.01
Probability of 1 to 4 if operated	p_14	0.01
Probability of 1 to failure if operated	p_1f	0.01
Probability of 2 to 2 if operated	p_22	0.9
Probability of 2 to 3 if operated	p_23	0.05
Probability of 2 to 4 if operated	p_24	0.02
Probability of 2 to 5 if operated	p_25	0.01
Probability of 2 to failure if operated	p_2f	0.02
Probability of 3 to 3 if operated	p_33	0.85
Probability of 3 to 4 if operated	p_34	0.07
Probability of 3 to 5 if operated	p_35	0.03
Probability of 3 to failure if operated	p_3f	0.05
Probability of 4 to 4 if operated	p_44	0.75
Probability of 4 to 5 if operated	p_45	0.09
Probability of 4 to f if operated	p_4f	0.16
Probability of 5 to 5 if operated	p_55	0.65
Probability of 5 to failure if operated	p_5f	0.35
Probability of successful repair	psr	0.7
Probability of failed repair	pfr	0.3
Probability of successful maintenance	psm	0.9
Probability of failed maintenance	pfm	0.1
discount factor	dff	0.95

Explanation for MDP

As we know, a finite state, finite action, stationary Markov decision process is characterized by :

State description : $S = \{1, 2, 3, 4, 5, f\}$, the set of six possible states.

Action description : for each $i \in S$, K_i is the set of possible actions (decisions), here we have three decisions, kp (keep operating), pm (preventive maintenance), and mr (manufacturer repair).

Immediate reward : for each $i \in S$, $k \in K_i$, r_i^k is the reward received when the process is in state **i** and action k is chosen.

State transitions : for each $i \in S$ & $k \in K_i$, $p_{i,j}^k$ is the probability of process making a transition to state **j** when the process is in state **i** and action **k** is chosen.

The optimality equation completes the formulation. The form of the optimality equation depends on the decision criterion and planning horizon chosen. Additionally, our final target is to gain optimal policy. As we know, a policy for a Markov decision process is a rule that prescribes the action to take in each state at each step. In this Turnco case, since we want to get the maximum expected discounted reward, we consider the stationary optimal policy δ (there is always a stationary optimal policy with an expected discounted reward).

For example, we can get such an optimal policy like
$\delta_1 = kp, \delta_2 = kp, \delta_3 = kp, \delta_4 = pm, \delta_5 = pm, \delta_f = mr.$

After setting the parameters, The next step is to arrange the data for the MDP formulation in a systematic way so that it can be easily referenced by the formulae that will be entered to perform the calculations required by value iteration. One convenient format is as follows. List all possible combinations of **State i** and **Action k** in the first two columns. This can be done by first listing all possible actions in state 1, then all possible actions in state 2 and so on. List the immediate rewards r_i^k for each combination of state and action in a further column. List the probabilities of a transition to **State 1** ($p_{i,1}^k$) for each combination of state and action in another column; the probabilities of a transition to **State 2** ($p_{i,2}^k$) for each combination of state and action in another column; and so on. Labels are used for the immediate reward and the set of transition probabilities for each combination of state and action. Following a similar approach to the above, the names to be used are listed in a column immediately to the left of the corresponding data. An example of the general format is shown below where the columns labeled “R Label” and “P Label” contain the names to be given to the immediate rewards and the transition probabilities respectively. The last column "check" means we want to confirm that most sum of probabilities is equal to 1, only when choosing action pm in state 1, the value in check is 0 (no need to apply preventive maintenance)

	A	B	C	D	E	F	G	H	I	J	K	L
1	i	k	R_label	r_i,k	P_label	p_i,k,1	p_i,k,2	p_i,k,3	p_i,k,4	p_i,k,5	p_i,k,f	check
2	1	kp	r_1,k	=rc_1	p_1,k	=p_11	=p_12	=p_13	=p_14		=p_1f	=SUM(F2:K2)
3	1	pm	r_1,p	=-pmc	p_1,p							=SUM(F3:K3)
4	2	kp	r_2,k	=rc_2	p_2,k		=p_22	=p_23	=p_24	=p_25	=p_2f	=SUM(F4:K4)
5	2	pm	r_2,p	=-pmc	p_2,p	=psm	=pfm					=SUM(F5:K5)
6	3	kp	r_3,k	=rc_3	p_3,k			=p_33	=p_34	=p_35	=p_3f	=SUM(F6:K6)
7	3	pm	r_3,p	=-pmc	p_3,p		=psm	=pfm				=SUM(F7:K7)
8	4	kp	r_4,k	=rc_4	p_4,k				=p_44	=p_45	=p_4f	=SUM(F8:K8)
9	4	pm	r_4,p	=-pmc	p_4,p			=psm	=pfm			=SUM(F9:K9)
10	5	kp	r_5,k	=rc_5	p_5,k					=p_55	=p_5f	=SUM(F10:K10)
11	5	pm	r_5,p	=-pmc	p_5,p				=psm	=pfm		=SUM(F11:K11)
12	f	mr	r_f,m	=-mrc	p_f,m	=psr					=pfr	=SUM(F12:K12)

In the table above (in Sheet 2),

Columns A and B list all possible combinations of state and action by listing all allowable actions in state 1, then all allowable actions in state 2 and so on.

Column C lists the labels to be used for the immediate reward for each combination of state and action.

Column D lists the immediate reward for each combination of state and action.

Column E lists the labels to be used for the transition probabilities for each combination of state and action.

Column F lists the probability of a transition to state 1 for each combination of state and action,

Column G lists the probability of a transition to state 2 for each combination of state and action, and so on.

After entering formulas in the table above, which means if any of the data is changed, the problem parameters are immediately updated. In this case, we can get the following values for further iterations,

	A	B	C	D	E	F	G	H	I	J	K	L
1	i	k	R_label	r_i,k	P_label	p_i,k,1	p_i,k,2	p_i,k,3	p_i,k,4	p_i,k,5	p_i,k,f	check
2	1	kp	r_1,k	5000	p_1,k	0.94	0.03	0.01	0.01		0.01	1
3	1	pm	r_1,p	-300	p_1,p							0
4	2	kp	r_2,k	4500	p_2,k		0.9	0.05	0.02	0.01	0.02	1
5	2	pm	r_2,p	-300	p_2,p	0.9	0.1					1
6	3	kp	r_3,k	3800	p_3,k			0.85	0.07	0.03	0.05	1
7	3	pm	r_3,p	-300	p_3,p		0.9	0.1				1
8	4	kp	r_4,k	2900	p_4,k				0.75	0.09	0.16	1
9	4	pm	r_4,p	-300	p_4,p			0.9	0.1			1
10	5	kp	r_5,k	1800	p_5,k					0.65	0.35	1
11	5	pm	r_5,p	-300	p_5,p				0.9	0.1		1
12	f	mr	r_f,m	-900	p_f,m	0.7					0.3	1

A similar approach to before is used to name the data. Select the range C2:D12. In the Defined Names area on the Formulas tab, select Create from Selection to display the Create Names from Selection dialog box. Ensure the “Left column” option is selected and click “OK”. Select the range E2:K12. In the Defined Names area on the Formulas tab, select Create from Selection to display the Create Names from Selection dialog box. Ensure the “Left column” option is selected and click “OK”. Now the immediate reward when action “kp” is chosen in state 1 can be referred to using the name ”r_1k” and the corresponding transition probabilities can be referred to using the name “p_1k”.

Value Iteration Accomplishment

Formulation

The problem is to maximize the expected total reward over **n** steps. Let v_i^n = maximum expected total reward over **n** steps when the process is in state **i** initially. We can get the optimal equation as follows:

$$v_i^n = \max_{k \in K_i}\{r_i^k + \beta\,(p_{i,1}^k v_1^{n-1} + p_{i,2}^k v_2^{n-1} + p_{i,3}^k v_3^{n-1} + p_{i,4}^k v_4^{n-1} + p_{i,5}^k v_5^{n-1} + p_{i,f}^k v_f^{n-1})\}$$

Later on, we set 0 values for v^0 and use optimality equations to find $v^1, v^2, \dots v^n$.

In Excel, it is easier to calculate the term in brackets for each decision in separate cells and then perform the MAX operation. Define

$$v_{i,k}^n = r_i^k + \beta\,(p_{i,1}^k v_1^{n-1} + p_{i,2}^k v_2^{n-1} + p_{i,3}^k v_3^{n-1} + p_{i,4}^k v_4^{n-1} + p_{i,5}^k v_5^{n-1} + p_{i,f}^k v_f^{n-1})$$

and we can calculate v_i^n by $v_i^n = \max_{k \in K_i}\{v_{i,k}^n\}$

To find where we stop the iteration, we use the following criterion to achieve the **Error** after **n** iterations, and apply 'when $Error \le 0.01$, we think the results are accurate enough that we can stop the iteration and determine the optimal policy',

$$E^n = \frac{\beta}{1-\beta} \max_{i \in S} |v_i^n - v_i^{n-1}|$$

Again in Excel, it is easier to calculate the absolute difference for each state in separate cells and then perform the MAX operation.

Calculation (in Sheet 3)

The calculation for the replacement problem can be tabulated as the following table:

	A	B	C	D	E	F	G	H	I	J	K	L	M
1													
2	n	v_n,1	v_n,2	v_n,3	v_n,4	v_n,5	v_n,f	d_n,1	d_n,2	d_n,3	d_n,4	d_n,5	d_n,f
3	0	0	0	0	0	0	0						
4	1	5000.000	4500.000	3800.000	2900.000	1800.000	-900.000	kp	kp	kp	kp	kp	mr
5	2	9648.350	8583.100	7069.900	4983.350	2612.250	2168.500	kp	kp	kp	kp	kp	mr
6	3	13995.702	12335.072	10017.790	7003.596	4208.928	6134.175	kp	kp	kp	kp	pm	mr
7	4	18069.689	15811.934	12766.432	9182.320	6438.626	10155.381	kp	kp	kp	kp	kp	mr
8	5	21891.862	19054.193	15385.400	11536.524	9152.516	14010.627	kp	kp	kp	kp	kp	mr
9	6	25481.336	22094.486	17917.241	14031.929	12110.212	17651.117	kp	kp	kp	kp	kp	mr
10	7	28855.729	24958.879	20384.864	16616.142	15147.052	21075.657	kp	kp	kp	kp	kp	mr
11	8	32031.222	27668.164	22976.404	19237.574	18160.961	24295.622	kp	kp	pm	kp	kp	mr
12	9	35024.265	30247.319	25539.039	21852.468	21092.687	27325.015	kp	kp	pm	kp	kp	mr
13	10	37848.524	32709.315	27987.667	24426.711	23910.302	30178.766	kp	kp	pm	kp	kp	mr
14	11	40515.583	35167.873	30325.293	26935.534	26599.051	32870.217	kp	pm	pm	kp	kp	mr
15	12	43038.945	37681.771	32649.434	29362.060	29154.261	35410.874	kp	pm	pm	kp	kp	mr
16	13	45433.220	40078.066	35019.611	31695.610	31576.872	37812.997	kp	pm	pm	kp	kp	mr
17	14	47707.109	42352.820	37293.609	33930.520	33871.540	40089.796	kp	pm	pm	kp	kp	mr
18	15	49866.986	44513.096	39454.554	36065.161	36045.533	42250.819	kp	pm	pm	kp	kp	mr

	N	O	P	Q	R	S	T	U	V	W	X
1											
2	v_n,1,k	v_n,1,p	v_n,2,k	v_n,2,p	v_n,3,k	v_n,3,p	v_n,4,k	v_n,4,p	v_n,5,k	v_n,5,p	v_n,f,m
3											
4	5000.000	-300	4500.000	-300.000	3800.000	-300.000	2900.000	-300.000	1800.000	-300.000	-900.000
5	9648.350	-300	8583.100	4402.500	7069.900	3908.500	4983.350	3224.500	2612.250	2350.500	2168.500
6	13995.702	-300	12335.072	8764.734	10017.790	7710.191	7003.596	6218.183	4134.091	4208.928	6134.175
7	18069.689	-300	15811.934	12838.157	12766.432	11198.177	9182.320	8930.552	6438.626	6087.923	10155.381
8	21891.862	-300	19054.193	16651.718	15385.400	14432.015	11536.524	11487.620	9152.516	8162.553	14010.627
9	25481.336	-300	22094.486	20227.690	17917.241	17452.948	14031.929	13950.487	12110.212	10433.217	17651.117
10	28855.729	-300	24958.879	23585.519	20384.864	20292.923	16616.142	16352.274	15147.052	12847.769	21075.657
11	32031.222	-300	27668.164	26742.742	22798.536	22976.404	19237.574	18707.592	18160.961	15345.771	24295.622
12	35024.265	-300	30247.319	29715.170	25304.374	25539.039	21852.468	21172.395	21092.687	17873.417	27325.015
13	37848.524	-300	32709.315	32519.242	27775.043	27987.667	24426.711	23611.862	23910.302	20387.665	30178.766
14	40515.583	-300	35060.530	35167.873	30139.352	30325.293	26935.534	25949.992	26599.051	22856.316	32870.217
15	43038.945	-300	37397.983	37681.771	32398.295	32649.434	29362.060	28187.001	29154.261	25256.792	35410.874
16	45433.220	-300	39776.414	40078.066	34629.908	35019.611	31695.610	30404.662	31576.872	27574.216	37812.997
17	47707.109	-300	42050.822	42352.820	36882.152	37293.609	33930.520	32652.850	33871.540	29799.549	40089.796
18	49866.986	-300	44211.273	44513.096	39040.573	39454.554	36065.161	34809.435	36045.533	31928.391	42250.819

	Y	Z	AA	AB	AC	AD	AE
1	|v_n,j-v_n-1,j|						
2	j=1	j=2	j=3	j=4	j=5	j=f	E_n
3							
4	5000.000	4500.000	3800.000	2900.000	1800.000	900.000	95000.000
5	4648.350	4083.100	3269.900	2083.350	812.250	3068.500	88318.650
6	4347.352	3751.972	2947.890	2020.246	1596.678	3965.675	82599.679
7	4073.987	3476.862	2748.642	2178.724	2229.698	4021.206	77405.759
8	3822.173	3242.258	2618.968	2354.203	2713.890	3855.245	73249.662
9	3589.475	3040.293	2531.841	2495.405	2957.696	3640.490	69169.306
10	3374.393	2864.393	2467.624	2584.213	3036.840	3424.540	65066.263
11	3175.493	2709.285	2591.540	2621.432	3013.908	3219.965	61179.335
12	2993.043	2579.155	2562.635	2614.894	2931.727	3029.393	57558.470
13	2824.259	2461.996	2448.628	2574.242	2817.615	2853.751	54221.264
14	2667.058	2458.558	2337.626	2508.824	2688.749	2691.451	51137.575
15	2523.362	2513.898	2324.142	2426.526	2555.210	2540.657	48548.992
16	2394.276	2396.295	2370.176	2333.550	2422.611	2402.123	46029.605
17	2273.888	2274.754	2273.999	2234.910	2294.668	2276.799	43598.694
18	2159.877	2160.276	2160.944	2134.641	2173.993	2161.023	41305.868

Column A shows the iteration number, **n**.
Columns B, C, D, E, F, G show $v_1^n, v_2^n, v_3^n, v_4^n, v_5^n, v_f^n$ respectively.
Columns from H to M show $\delta_1^n, \delta_2^n, \delta_3^n, \delta_4^n, \delta_5^n, \delta_f^n$ respectively.
Columns N and O show $v_{1,kp}^n \ and \ v_{1,pm}^n$ respectively. Columns P and Q show $v_{2,kp}^n \ and \ v_{2,pm}^n$ respectively.
Columns R and S show $v_{3,kp}^n \ and \ v_{3,pm}^n$ respectively. Columns T and U show $v_{4,kp}^n \ and \ v_{4,pm}^n$ respectively.
Columns V and W show $v_{5,kp}^n \ and \ v_{5,pm}^n$ respectively. Columns X show $v_{f,mr}^n$ respectively.
Columns from Y to AD shows
$|v_1^n - v_1^{n-1}|, |v_2^n - v_2^{n-1}|, |v_4^n - v_1^{n-1}|, |v_4^n - v_4^{n-1}|, |v_5^n - v_5^{n-1}|, |v_f^n - v_f^{n-1}|$ respectively.
Column AE shows E^n.

At iteration 0, $v_1^0 = v_2^0 = v_3^0 = v_4^0 = v_5^0 = v_f^0$, and none of the other entities have any meaning. Enter 0 from cells A3 tp G3 to represent this.

The calculations required for value iteration can be performed in Excel using basic arithmetic operators (+, –, *, /) and the functions SUMPRODUCT, MAX, IF, and ABS. By carefully entering formulae to perform the calculation at the first iteration, the calculation required for further iterations can be performed using the “fill handle” to copy the formula into subsequent rows. For the calculation required at iteration 1:

The formula =A3+1 is entered in cell A4 to update the iteration number.

The formula =r_1_k+dff*SUMPRODUCT(p_1_k, $\$B3:\$G3$) is entered in N4 to calculate $v_{1,kp}^n$.

The formula =r_1_p+dff*SUMPRODUCT(p_1_p, $\$B3:\$G3$) is entered in O4 to calculate $v_{1,pm}^n$.

As the column references are absolute, a convenient way to do this might be to copy the formula in cell N4 & O4 to the following cells from P4 & Q4 and so on until V4 & W4 and then update the names referring to different immediate profits and transition probabilities. As for X4, just enter '=r_f_m + dff*SUMPRODUCT(p_f_m, $\$B3:\$G3$)'.

The formula = MAX(N4 : O4) is entered in B4 to calculate v_1^n.

The formula =IFS(B4=N4,"kp",B4=O4,"pm") is entered in H4 to calculate δ_1^n.

The formula = ABS(B4 - B3) us entered in Y4 to calculate $|v_1^n - v_1^{n-1}|$.

The calculations for other 5 states are similar to the procedures above.

Finally, the formula =dff*MAX($Y4$:AD4)/(1-dff) is entered in AE4 to calculate E^n.

After getting results when n = 1, to apply for further iterations,
Select the cells in the range A4 to AE4.
Position the mouse pointer over the bottom right-hand corner of the selection so that the fill handle (a black cross) appears.
Click the left-hand mouse button and drag the mouse down over several rows.
When you release the button the calculation for further iterations will fill in — one iteration for each row.

We have restricted the error to stop iteration as $E^n \leq 0.01$, we think 0.01 is sufficiently small compared to thousands of profits. Therefore, we find when n = 311(number of iterations), we think we can stop the iterations and gain final results.

Final results and conclusions

We finally get the expected discounted profits and optimal policy based on the previous process.

Results	
Number of iterations	311
Value when in condition 1 initially	90892.549
Value when in condition 2 initially	85539.369
Value when in condition 3 initially	80481.944
Value when in condition 4 initially	77043.658
Value when in condition 5 initially	77097.594
Value when below standard initially	83277.684
Decision when in condition1	kp
Decision when in condition1	pm
Decision when in condition1	pm
Decision when in condition1	kp
Decision when in condition1	kp
Decision when below standard	mr
Error	0.010

However, the table above is just for a single situation. To some extent, we want to find how results will change under different parameter settings. Hence, we use Sensitivity Analysis to test how robust the findings are to changes in the problem parameters.

Excerpts from our spreadsheet

From previous parts, we have shown how the data is organized and used for formulations in spreadsheets (sheet 1 and sheet2). Here, we add more excerpts for sheet 3 to show value iteration calculation in detail. The results are shown in the images below,

Functions for optimal rewards

	A	B	C	D	E	F	G
2	n	v_n,1	v_n,2	v_n,3	v_n,4	v_n,5	v_n,f
3	0	0	0	0	0	0	0
4	=A3+1	=MAX(N4:O4)	=MAX(P4:Q4)	=MAX(R4:S4)	=MAX(T4:U4)	=MAX(V4:W4)	=MAX(X4)
5	=A4+1	=MAX(N5:O5)	=MAX(P5:Q5)	=MAX(R5:S5)	=MAX(T5:U5)	=MAX(V5:W5)	=MAX(X5)
6	=A5+1	=MAX(N6:O6)	=MAX(P6:Q6)	=MAX(R6:S6)	=MAX(T6:U6)	=MAX(V6:W6)	=MAX(X6)
7	=A6+1	=MAX(N7:O7)	=MAX(P7:Q7)	=MAX(R7:S7)	=MAX(T7:U7)	=MAX(V7:W7)	=MAX(X7)
8	=A7+1	=MAX(N8:O8)	=MAX(P8:Q8)	=MAX(R8:S8)	=MAX(T8:U8)	=MAX(V8:W8)	=MAX(X8)
9	=A8+1	=MAX(N9:O9)	=MAX(P9:Q9)	=MAX(R9:S9)	=MAX(T9:U9)	=MAX(V9:W9)	=MAX(X9)
10	=A9+1	=MAX(N10:O10)	=MAX(P10:Q10)	=MAX(R10:S10)	=MAX(T10:U10)	=MAX(V10:W10)	=MAX(X10)
11	=A10+1	=MAX(N11:O11)	=MAX(P11:Q11)	=MAX(R11:S11)	=MAX(T11:U11)	=MAX(V11:W11)	=MAX(X11)
12	=A11+1	=MAX(N12:O12)	=MAX(P12:Q12)	=MAX(R12:S12)	=MAX(T12:U12)	=MAX(V12:W12)	=MAX(X12)
13	=A12+1	=MAX(N13:O13)	=MAX(P13:Q13)	=MAX(R13:S13)	=MAX(T13:U13)	=MAX(V13:W13)	=MAX(X13)
14	=A13+1	=MAX(N14:O14)	=MAX(P14:Q14)	=MAX(R14:S14)	=MAX(T14:U14)	=MAX(V14:W14)	=MAX(X14)
15	=A14+1	=MAX(N15:O15)	=MAX(P15:Q15)	=MAX(R15:S15)	=MAX(T15:U15)	=MAX(V15:W15)	=MAX(X15)
16	=A15+1	=MAX(N16:O16)	=MAX(P16:Q16)	=MAX(R16:S16)	=MAX(T16:U16)	=MAX(V16:W16)	=MAX(X16)
17	=A16+1	=MAX(N17:O17)	=MAX(P17:Q17)	=MAX(R17:S17)	=MAX(T17:U17)	=MAX(V17:W17)	=MAX(X17)
18	=A17+1	=MAX(N18:O18)	=MAX(P18:Q18)	=MAX(R18:S18)	=MAX(T18:U18)	=MAX(V18:W18)	=MAX(X18)
19	=A18+1	=MAX(N19:O19)	=MAX(P19:Q19)	=MAX(R19:S19)	=MAX(T19:U19)	=MAX(V19:W19)	=MAX(X19)
20	=A19+1	=MAX(N20:O20)	=MAX(P20:Q20)	=MAX(R20:S20)	=MAX(T20:U20)	=MAX(V20:W20)	=MAX(X20)
21	=A20+1	=MAX(N21:O21)	=MAX(P21:Q21)	=MAX(R21:S21)	=MAX(T21:U21)	=MAX(V21:W21)	=MAX(X21)
22	=A21+1	=MAX(N22:O22)	=MAX(P22:Q22)	=MAX(R22:S22)	=MAX(T22:U22)	=MAX(V22:W22)	=MAX(X22)
23	=A22+1	=MAX(N23:O23)	=MAX(P23:Q23)	=MAX(R23:S23)	=MAX(T23:U23)	=MAX(V23:W23)	=MAX(X23)

Functions for taking decisions

H	I	J	K	L	M
d_n,1	d_n,2	d_n,3	d_n,4	d_n,5	d_n,f
=IFS(B4=N4,"kp",B4=O4,"pm")	=IFS(C4=P4,"kp",C4=Q4,"pm")	=IFS(D4=R4,"kp",D4=S4,"pm")	=IFS(E4=T4,"kp",E4=U4,"pm")	=IFS(F4=V4,"kp",F4=W4,"pm")	mr
=IFS(B5=N5,"kp",B5=O5,"pm")	=IFS(C5=P5,"kp",C5=Q5,"pm")	=IFS(D5=R5,"kp",D5=S5,"pm")	=IFS(E5=T5,"kp",E5=U5,"pm")	=IFS(F5=V5,"kp",F5=W5,"pm")	mr
=IFS(B6=N6,"kp",B6=O6,"pm")	=IFS(C6=P6,"kp",C6=Q6,"pm")	=IFS(D6=R6,"kp",D6=S6,"pm")	=IFS(E6=T6,"kp",E6=U6,"pm")	=IFS(F6=V6,"kp",F6=W6,"pm")	mr
=IFS(B7=N7,"kp",B7=O7,"pm")	=IFS(C7=P7,"kp",C7=Q7,"pm")	=IFS(D7=R7,"kp",D7=S7,"pm")	=IFS(E7=T7,"kp",E7=U7,"pm")	=IFS(F7=V7,"kp",F7=W7,"pm")	mr
=IFS(B8=N8,"kp",B8=O8,"pm")	=IFS(C8=P8,"kp",C8=Q8,"pm")	=IFS(D8=R8,"kp",D8=S8,"pm")	=IFS(E8=T8,"kp",E8=U8,"pm")	=IFS(F8=V8,"kp",F8=W8,"pm")	mr
=IFS(B9=N9,"kp",B9=O9,"pm")	=IFS(C9=P9,"kp",C9=Q9,"pm")	=IFS(D9=R9,"kp",D9=S9,"pm")	=IFS(E9=T9,"kp",E9=U9,"pm")	=IFS(F9=V9,"kp",F9=W9,"pm")	mr
=IFS(B10=N10,"kp",B10=O10,"pm")	=IFS(C10=P10,"kp",C10=Q10,"pm")	=IFS(D10=R10,"kp",D10=S10,"pm")	=IFS(E10=T10,"kp",E10=U10,"pm")	=IFS(F10=V10,"kp",F10=W10,"pm")	mr
=IFS(B11=N11,"kp",B11=O11,"pm")	=IFS(C11=P11,"kp",C11=Q11,"pm")	=IFS(D11=R11,"kp",D11=S11,"pm")	=IFS(E11=T11,"kp",E11=U11,"pm")	=IFS(F11=V11,"kp",F11=W11,"pm")	mr
=IFS(B12=N12,"kp",B12=O12,"pm")	=IFS(C12=P12,"kp",C12=Q12,"pm")	=IFS(D12=R12,"kp",D12=S12,"pm")	=IFS(E12=T12,"kp",E12=U12,"pm")	=IFS(F12=V12,"kp",F12=W12,"pm")	mr
=IFS(B13=N13,"kp",B13=O13,"pm")	=IFS(C13=P13,"kp",C13=Q13,"pm")	=IFS(D13=R13,"kp",D13=S13,"pm")	=IFS(E13=T13,"kp",E13=U13,"pm")	=IFS(F13=V13,"kp",F13=W13,"pm")	mr
=IFS(B14=N14,"kp",B14=O14,"pm")	=IFS(C14=P14,"kp",C14=Q14,"pm")	=IFS(D14=R14,"kp",D14=S14,"pm")	=IFS(E14=T14,"kp",E14=U14,"pm")	=IFS(F14=V14,"kp",F14=W14,"pm")	mr
=IFS(B15=N15,"kp",B15=O15,"pm")	=IFS(C15=P15,"kp",C15=Q15,"pm")	=IFS(D15=R15,"kp",D15=S15,"pm")	=IFS(E15=T15,"kp",E15=U15,"pm")	=IFS(F15=V15,"kp",F15=W15,"pm")	mr
=IFS(B16=N16,"kp",B16=O16,"pm")	=IFS(C16=P16,"kp",C16=Q16,"pm")	=IFS(D16=R16,"kp",D16=S16,"pm")	=IFS(E16=T16,"kp",E16=U16,"pm")	=IFS(F16=V16,"kp",F16=W16,"pm")	mr
=IFS(B17=N17,"kp",B17=O17,"pm")	=IFS(C17=P17,"kp",C17=Q17,"pm")	=IFS(D17=R17,"kp",D17=S17,"pm")	=IFS(E17=T17,"kp",E17=U17,"pm")	=IFS(F17=V17,"kp",F17=W17,"pm")	mr
=IFS(B18=N18,"kp",B18=O18,"pm")	=IFS(C18=P18,"kp",C18=Q18,"pm")	=IFS(D18=R18,"kp",D18=S18,"pm")	=IFS(E18=T18,"kp",E18=U18,"pm")	=IFS(F18=V18,"kp",F18=W18,"pm")	mr
=IFS(B19=N19,"kp",B19=O19,"pm")	=IFS(C19=P19,"kp",C19=Q19,"pm")	=IFS(D19=R19,"kp",D19=S19,"pm")	=IFS(E19=T19,"kp",E19=U19,"pm")	=IFS(F19=V19,"kp",F19=W19,"pm")	mr
=IFS(B20=N20,"kp",B20=O20,"pm")	=IFS(C20=P20,"kp",C20=Q20,"pm")	=IFS(D20=R20,"kp",D20=S20,"pm")	=IFS(E20=T20,"kp",E20=U20,"pm")	=IFS(F20=V20,"kp",F20=W20,"pm")	mr
=IFS(B21=N21,"kp",B21=O21,"pm")	=IFS(C21=P21,"kp",C21=Q21,"pm")	=IFS(D21=R21,"kp",D21=S21,"pm")	=IFS(E21=T21,"kp",E21=U21,"pm")	=IFS(F21=V21,"kp",F21=W21,"pm")	mr
=IFS(B22=N22,"kp",B22=O22,"pm")	=IFS(C22=P22,"kp",C22=Q22,"pm")	=IFS(D22=R22,"kp",D22=S22,"pm")	=IFS(E22=T22,"kp",E22=U22,"pm")	=IFS(F22=V22,"kp",F22=W22,"pm")	mr
=IFS(B23=N23,"kp",B23=O23,"pm")	=IFS(C23=P23,"kp",C23=Q23,"pm")	=IFS(D23=R23,"kp",D23=S23,"pm")	=IFS(E23=T23,"kp",E23=U23,"pm")	=IFS(F23=V23,"kp",F23=W23,"pm")	mr

Functions for profits with each decision

	N	O	P	Q	R	S
2	v_n,1,k	v_n,1,p	v_n,2,k	v_n,2,p	v_n,3,k	v_n,3,p
3						
4	=r_1_k+dff*SUMPRODUCT(p_1_k, $B3:$G3)	=r_1_p+dff*SUMPRODUCT(p_1_p, $B3:$G3)	=r_2_k+dff*SUMPRODUCT(p_2_k, $B3:$G3)	=r_2_p+dff*SUMPRODUCT(p_2_p,$B3:$G3)	=r_3_k+dff*SUMPRODUCT(p_3_k,$B3:$G3)	=r_3_p+dff*SUMPRODUCT(p_3_p,$B3:$G3)
5	=r_1_k+dff*SUMPRODUCT(p_1_k, $B4:$G4)	=r_1_p+dff*SUMPRODUCT(p_1_p, $B4:$G4)	=r_2_k+dff*SUMPRODUCT(p_2_k, $B4:$G4)	=r_2_p+dff*SUMPRODUCT(p_2_p,$B4:$G4)	=r_3_k+dff*SUMPRODUCT(p_3_k,$B4:$G4)	=r_3_p+dff*SUMPRODUCT(p_3_p,$B4:$G4)
6	=r_1_k+dff*SUMPRODUCT(p_1_k, $B5:$G5)	=r_1_p+dff*SUMPRODUCT(p_1_p, $B5:$G5)	=r_2_k+dff*SUMPRODUCT(p_2_k, $B5:$G5)	=r_2_p+dff*SUMPRODUCT(p_2_p,$B5:$G5)	=r_3_k+dff*SUMPRODUCT(p_3_k,$B5:$G5)	=r_3_p+dff*SUMPRODUCT(p_3_p,$B5:$G5)
7	=r_1_k+dff*SUMPRODUCT(p_1_k, $B6:$G6)	=r_1_p+dff*SUMPRODUCT(p_1_p, $B6:$G6)	=r_2_k+dff*SUMPRODUCT(p_2_k, $B6:$G6)	=r_2_p+dff*SUMPRODUCT(p_2_p,$B6:$G6)	=r_3_k+dff*SUMPRODUCT(p_3_k,$B6:$G6)	=r_3_p+dff*SUMPRODUCT(p_3_p,$B6:$G6)
8	=r_1_k+dff*SUMPRODUCT(p_1_k, $B7:$G7)	=r_1_p+dff*SUMPRODUCT(p_1_p, $B7:$G7)	=r_2_k+dff*SUMPRODUCT(p_2_k, $B7:$G7)	=r_2_p+dff*SUMPRODUCT(p_2_p,$B7:$G7)	=r_3_k+dff*SUMPRODUCT(p_3_k,$B7:$G7)	=r_3_p+dff*SUMPRODUCT(p_3_p,$B7:$G7)
9	=r_1_k+dff*SUMPRODUCT(p_1_k, $B8:$G8)	=r_1_p+dff*SUMPRODUCT(p_1_p, $B8:$G8)	=r_2_k+dff*SUMPRODUCT(p_2_k, $B8:$G8)	=r_2_p+dff*SUMPRODUCT(p_2_p,$B8:$G8)	=r_3_k+dff*SUMPRODUCT(p_3_k,$B8:$G8)	=r_3_p+dff*SUMPRODUCT(p_3_p,$B8:$G8)
10	=r_1_k+dff*SUMPRODUCT(p_1_k, $B9:$G9)	=r_1_p+dff*SUMPRODUCT(p_1_p, $B9:$G9)	=r_2_k+dff*SUMPRODUCT(p_2_k, $B9:$G9)	=r_2_p+dff*SUMPRODUCT(p_2_p,$B9:$G9)	=r_3_k+dff*SUMPRODUCT(p_3_k,$B9:$G9)	=r_3_p+dff*SUMPRODUCT(p_3_p,$B9:$G9)
11	=r_1_k+dff*SUMPRODUCT(p_1_k, $B10:$G10)	=r_1_p+dff*SUMPRODUCT(p_1_p, $B10:$G10)	=r_2_k+dff*SUMPRODUCT(p_2_k, $B10:$G10)	=r_2_p+dff*SUMPRODUCT(p_2_p,$B10:$G10)	=r_3_k+dff*SUMPRODUCT(p_3_k,$B10:$G10)	=r_3_p+dff*SUMPRODUCT(p_3_p,$B10:$G10)
12	=r_1_k+dff*SUMPRODUCT(p_1_k, $B11:$G11)	=r_1_p+dff*SUMPRODUCT(p_1_p, $B11:$G11)	=r_2_k+dff*SUMPRODUCT(p_2_k, $B11:$G11)	=r_2_p+dff*SUMPRODUCT(p_2_p,$B11:$G11)	=r_3_k+dff*SUMPRODUCT(p_3_k,$B11:$G11)	=r_3_p+dff*SUMPRODUCT(p_3_p,$B11:$G11)
13	=r_1_k+dff*SUMPRODUCT(p_1_k, $B12:$G12)	=r_1_p+dff*SUMPRODUCT(p_1_p, $B12:$G12)	=r_2_k+dff*SUMPRODUCT(p_2_k, $B12:$G12)	=r_2_p+dff*SUMPRODUCT(p_2_p,$B12:$G12)	=r_3_k+dff*SUMPRODUCT(p_3_k,$B12:$G12)	=r_3_p+dff*SUMPRODUCT(p_3_p,$B12:$G12)
14	=r_1_k+dff*SUMPRODUCT(p_1_k, $B13:$G13)	=r_1_p+dff*SUMPRODUCT(p_1_p, $B13:$G13)	=r_2_k+dff*SUMPRODUCT(p_2_k, $B13:$G13)	=r_2_p+dff*SUMPRODUCT(p_2_p,$B13:$G13)	=r_3_k+dff*SUMPRODUCT(p_3_k,$B13:$G13)	=r_3_p+dff*SUMPRODUCT(p_3_p,$B13:$G13)
15	=r_1_k+dff*SUMPRODUCT(p_1_k, $B14:$G14)	=r_1_p+dff*SUMPRODUCT(p_1_p, $B14:$G14)	=r_2_k+dff*SUMPRODUCT(p_2_k, $B14:$G14)	=r_2_p+dff*SUMPRODUCT(p_2_p,$B14:$G14)	=r_3_k+dff*SUMPRODUCT(p_3_k,$B14:$G14)	=r_3_p+dff*SUMPRODUCT(p_3_p,$B14:$G14)
16	=r_1_k+dff*SUMPRODUCT(p_1_k, $B15:$G15)	=r_1_p+dff*SUMPRODUCT(p_1_p, $B15:$G15)	=r_2_k+dff*SUMPRODUCT(p_2_k, $B15:$G15)	=r_2_p+dff*SUMPRODUCT(p_2_p,$B15:$G15)	=r_3_k+dff*SUMPRODUCT(p_3_k,$B15:$G15)	=r_3_p+dff*SUMPRODUCT(p_3_p,$B15:$G15)
17	=r_1_k+dff*SUMPRODUCT(p_1_k, $B16:$G16)	=r_1_p+dff*SUMPRODUCT(p_1_p, $B16:$G16)	=r_2_k+dff*SUMPRODUCT(p_2_k, $B16:$G16)	=r_2_p+dff*SUMPRODUCT(p_2_p,$B16:$G16)	=r_3_k+dff*SUMPRODUCT(p_3_k,$B16:$G16)	=r_3_p+dff*SUMPRODUCT(p_3_p,$B16:$G16)
18	=r_1_k+dff*SUMPRODUCT(p_1_k, $B17:$G17)	=r_1_p+dff*SUMPRODUCT(p_1_p, $B17:$G17)	=r_2_k+dff*SUMPRODUCT(p_2_k, $B17:$G17)	=r_2_p+dff*SUMPRODUCT(p_2_p,$B17:$G17)	=r_3_k+dff*SUMPRODUCT(p_3_k,$B17:$G17)	=r_3_p+dff*SUMPRODUCT(p_3_p,$B17:$G17)
19	=r_1_k+dff*SUMPRODUCT(p_1_k, $B18:$G18)	=r_1_p+dff*SUMPRODUCT(p_1_p, $B18:$G18)	=r_2_k+dff*SUMPRODUCT(p_2_k, $B18:$G18)	=r_2_p+dff*SUMPRODUCT(p_2_p,$B18:$G18)	=r_3_k+dff*SUMPRODUCT(p_3_k,$B18:$G18)	=r_3_p+dff*SUMPRODUCT(p_3_p,$B18:$G18)
20	=r_1_k+dff*SUMPRODUCT(p_1_k, $B19:$G19)	=r_1_p+dff*SUMPRODUCT(p_1_p, $B19:$G19)	=r_2_k+dff*SUMPRODUCT(p_2_k, $B19:$G19)	=r_2_p+dff*SUMPRODUCT(p_2_p,$B19:$G19)	=r_3_k+dff*SUMPRODUCT(p_3_k,$B19:$G19)	=r_3_p+dff*SUMPRODUCT(p_3_p,$B19:$G19)
21	=r_1_k+dff*SUMPRODUCT(p_1_k, $B20:$G20)	=r_1_p+dff*SUMPRODUCT(p_1_p, $B20:$G20)	=r_2_k+dff*SUMPRODUCT(p_2_k, $B20:$G20)	=r_2_p+dff*SUMPRODUCT(p_2_p,$B20:$G20)	=r_3_k+dff*SUMPRODUCT(p_3_k,$B20:$G20)	=r_3_p+dff*SUMPRODUCT(p_3_p,$B20:$G20)
22	=r_1_k+dff*SUMPRODUCT(p_1_k, $B21:$G21)	=r_1_p+dff*SUMPRODUCT(p_1_p, $B21:$G21)	=r_2_k+dff*SUMPRODUCT(p_2_k, $B21:$G21)	=r_2_p+dff*SUMPRODUCT(p_2_p,$B21:$G21)	=r_3_k+dff*SUMPRODUCT(p_3_k,$B21:$G21)	=r_3_p+dff*SUMPRODUCT(p_3_p,$B21:$G21)
23	=r_1_k+dff*SUMPRODUCT(p_1_k, $B22:$G22)	=r_1_p+dff*SUMPRODUCT(p_1_p, $B22:$G22)	=r_2_k+dff*SUMPRODUCT(p_2_k, $B22:$G22)	=r_2_p+dff*SUMPRODUCT(p_2_p,$B22:$G22)	=r_3_k+dff*SUMPRODUCT(p_3_k,$B22:$G22)	=r_3_p+dff*SUMPRODUCT(p_3_p,$B22:$G22)

	T	U	V	W	X
2	v_n,4,k	v_n,4,p	v_n,5,k	v_n,5,p	v_n,f,m
3					
4	=r_4_k+dff*SUMPRODUCT(p_4_k,$B3:$G3)	=r_4_p+dff*SUMPRODUCT(p_4_p,$B3:$G3)	=r_5_k+dff*SUMPRODUCT(p_5_k,$B3:$G3)	=r_5_p+dff*SUMPRODUCT(p_5_p,$B3:$G3)	=r_f_m+dff*SUMPRODUCT(p_f_m,$B3:$G3)
5	=r_4_k+dff*SUMPRODUCT(p_4_k,$B4:$G4)	=r_4_p+dff*SUMPRODUCT(p_4_p,$B4:$G4)	=r_5_k+dff*SUMPRODUCT(p_5_k,$B4:$G4)	=r_5_p+dff*SUMPRODUCT(p_5_p,$B4:$G4)	=r_f_m+dff*SUMPRODUCT(p_f_m,$B4:$G4)
6	=r_4_k+dff*SUMPRODUCT(p_4_k,$B5:$G5)	=r_4_p+dff*SUMPRODUCT(p_4_p,$B5:$G5)	=r_5_k+dff*SUMPRODUCT(p_5_k,$B5:$G5)	=r_5_p+dff*SUMPRODUCT(p_5_p,$B5:$G5)	=r_f_m+dff*SUMPRODUCT(p_f_m,$B5:$G5)
7	=r_4_k+dff*SUMPRODUCT(p_4_k,$B6:$G6)	=r_4_p+dff*SUMPRODUCT(p_4_p,$B6:$G6)	=r_5_k+dff*SUMPRODUCT(p_5_k,$B6:$G6)	=r_5_p+dff*SUMPRODUCT(p_5_p,$B6:$G6)	=r_f_m+dff*SUMPRODUCT(p_f_m,$B6:$G6)
8	=r_4_k+dff*SUMPRODUCT(p_4_k,$B7:$G7)	=r_4_p+dff*SUMPRODUCT(p_4_p,$B7:$G7)	=r_5_k+dff*SUMPRODUCT(p_5_k,$B7:$G7)	=r_5_p+dff*SUMPRODUCT(p_5_p,$B7:$G7)	=r_f_m+dff*SUMPRODUCT(p_f_m,$B7:$G7)
9	=r_4_k+dff*SUMPRODUCT(p_4_k,$B8:$G8)	=r_4_p+dff*SUMPRODUCT(p_4_p,$B8:$G8)	=r_5_k+dff*SUMPRODUCT(p_5_k,$B8:$G8)	=r_5_p+dff*SUMPRODUCT(p_5_p,$B8:$G8)	=r_f_m+dff*SUMPRODUCT(p_f_m,$B8:$G8)
10	=r_4_k+dff*SUMPRODUCT(p_4_k,$B9:$G9)	=r_4_p+dff*SUMPRODUCT(p_4_p,$B9:$G9)	=r_5_k+dff*SUMPRODUCT(p_5_k,$B9:$G9)	=r_5_p+dff*SUMPRODUCT(p_5_p,$B9:$G9)	=r_f_m+dff*SUMPRODUCT(p_f_m,$B9:$G9)
11	=r_4_k+dff*SUMPRODUCT(p_4_k,$B10:$G10)	=r_4_p+dff*SUMPRODUCT(p_4_p,$B10:$G10)	=r_5_k+dff*SUMPRODUCT(p_5_k,$B10:$G10)	=r_5_p+dff*SUMPRODUCT(p_5_p,$B10:$G10)	=r_f_m+dff*SUMPRODUCT(p_f_m,$B10:$G10)
12	=r_4_k+dff*SUMPRODUCT(p_4_k,$B11:$G11)	=r_4_p+dff*SUMPRODUCT(p_4_p,$B11:$G11)	=r_5_k+dff*SUMPRODUCT(p_5_k,$B11:$G11)	=r_5_p+dff*SUMPRODUCT(p_5_p,$B11:$G11)	=r_f_m+dff*SUMPRODUCT(p_f_m,$B11:$G11)
13	=r_4_k+dff*SUMPRODUCT(p_4_k,$B12:$G12)	=r_4_p+dff*SUMPRODUCT(p_4_p,$B12:$G12)	=r_5_k+dff*SUMPRODUCT(p_5_k,$B12:$G12)	=r_5_p+dff*SUMPRODUCT(p_5_p,$B12:$G12)	=r_f_m+dff*SUMPRODUCT(p_f_m,$B12:$G12)
14	=r_4_k+dff*SUMPRODUCT(p_4_k,$B13:$G13)	=r_4_p+dff*SUMPRODUCT(p_4_p,$B13:$G13)	=r_5_k+dff*SUMPRODUCT(p_5_k,$B13:$G13)	=r_5_p+dff*SUMPRODUCT(p_5_p,$B13:$G13)	=r_f_m+dff*SUMPRODUCT(p_f_m,$B13:$G13)
15	=r_4_k+dff*SUMPRODUCT(p_4_k,$B14:$G14)	=r_4_p+dff*SUMPRODUCT(p_4_p,$B14:$G14)	=r_5_k+dff*SUMPRODUCT(p_5_k,$B14:$G14)	=r_5_p+dff*SUMPRODUCT(p_5_p,$B14:$G14)	=r_f_m+dff*SUMPRODUCT(p_f_m,$B14:$G14)
16	=r_4_k+dff*SUMPRODUCT(p_4_k,$B15:$G15)	=r_4_p+dff*SUMPRODUCT(p_4_p,$B15:$G15)	=r_5_k+dff*SUMPRODUCT(p_5_k,$B15:$G15)	=r_5_p+dff*SUMPRODUCT(p_5_p,$B15:$G15)	=r_f_m+dff*SUMPRODUCT(p_f_m,$B15:$G15)
17	=r_4_k+dff*SUMPRODUCT(p_4_k,$B16:$G16)	=r_4_p+dff*SUMPRODUCT(p_4_p,$B16:$G16)	=r_5_k+dff*SUMPRODUCT(p_5_k,$B16:$G16)	=r_5_p+dff*SUMPRODUCT(p_5_p,$B16:$G16)	=r_f_m+dff*SUMPRODUCT(p_f_m,$B16:$G16)
18	=r_4_k+dff*SUMPRODUCT(p_4_k,$B17:$G17)	=r_4_p+dff*SUMPRODUCT(p_4_p,$B17:$G17)	=r_5_k+dff*SUMPRODUCT(p_5_k,$B17:$G17)	=r_5_p+dff*SUMPRODUCT(p_5_p,$B17:$G17)	=r_f_m+dff*SUMPRODUCT(p_f_m,$B17:$G17)
19	=r_4_k+dff*SUMPRODUCT(p_4_k,$B18:$G18)	=r_4_p+dff*SUMPRODUCT(p_4_p,$B18:$G18)	=r_5_k+dff*SUMPRODUCT(p_5_k,$B18:$G18)	=r_5_p+dff*SUMPRODUCT(p_5_p,$B18:$G18)	=r_f_m+dff*SUMPRODUCT(p_f_m,$B18:$G18)
20	=r_4_k+dff*SUMPRODUCT(p_4_k,$B19:$G19)	=r_4_p+dff*SUMPRODUCT(p_4_p,$B19:$G19)	=r_5_k+dff*SUMPRODUCT(p_5_k,$B19:$G19)	=r_5_p+dff*SUMPRODUCT(p_5_p,$B19:$G19)	=r_f_m+dff*SUMPRODUCT(p_f_m,$B19:$G19)
21	=r_4_k+dff*SUMPRODUCT(p_4_k,$B20:$G20)	=r_4_p+dff*SUMPRODUCT(p_4_p,$B20:$G20)	=r_5_k+dff*SUMPRODUCT(p_5_k,$B20:$G20)	=r_5_p+dff*SUMPRODUCT(p_5_p,$B20:$G20)	=r_f_m+dff*SUMPRODUCT(p_f_m,$B20:$G20)
22	=r_4_k+dff*SUMPRODUCT(p_4_k,$B21:$G21)	=r_4_p+dff*SUMPRODUCT(p_4_p,$B21:$G21)	=r_5_k+dff*SUMPRODUCT(p_5_k,$B21:$G21)	=r_5_p+dff*SUMPRODUCT(p_5_p,$B21:$G21)	=r_f_m+dff*SUMPRODUCT(p_f_m,$B21:$G21)
23	=r_4_k+dff*SUMPRODUCT(p_4_k,$B22:$G22)	=r_4_p+dff*SUMPRODUCT(p_4_p,$B22:$G22)	=r_5_k+dff*SUMPRODUCT(p_5_k,$B22:$G22)	=r_5_p+dff*SUMPRODUCT(p_5_p,$B22:$G22)	=r_f_m+dff*SUMPRODUCT(p_f_m,$B22:$G22)

Functions for Error calculation

Y	Z	AA	AB	AC	AD	AE
j=1	**j=2**	**j=3**	**j=4**	**j=5**	**j=f**	**E_n**
=ABS(B4-B3)	=ABS(C4-C3)	=ABS(D4-D3)	=ABS(E4-E3)	=ABS(F4-F3)	=ABS(G4-G3)	=dff*MAX($Y4:$AD4)/(1-dff)
=ABS(B5-B4)	=ABS(C5-C4)	=ABS(D5-D4)	=ABS(E5-E4)	=ABS(F5-F4)	=ABS(G5-G4)	=dff*MAX($Y5:$AD5)/(1-dff)
=ABS(B6-B5)	=ABS(C6-C5)	=ABS(D6-D5)	=ABS(E6-E5)	=ABS(F6-F5)	=ABS(G6-G5)	=dff*MAX($Y6:$AD6)/(1-dff)
=ABS(B7-B6)	=ABS(C7-C6)	=ABS(D7-D6)	=ABS(E7-E6)	=ABS(F7-F6)	=ABS(G7-G6)	=dff*MAX($Y7:$AD7)/(1-dff)
=ABS(B8-B7)	=ABS(C8-C7)	=ABS(D8-D7)	=ABS(E8-E7)	=ABS(F8-F7)	=ABS(G8-G7)	=dff*MAX($Y8:$AD8)/(1-dff)
=ABS(B9-B8)	=ABS(C9-C8)	=ABS(D9-D8)	=ABS(E9-E8)	=ABS(F9-F8)	=ABS(G9-G8)	=dff*MAX($Y9:$AD9)/(1-dff)
=ABS(B10-B9)	=ABS(C10-C9)	=ABS(D10-D9)	=ABS(E10-E9)	=ABS(F10-F9)	=ABS(G10-G9)	=dff*MAX($Y10:$AD10)/(1-dff)
=ABS(B11-B10)	=ABS(C11-C10)	=ABS(D11-D10)	=ABS(E11-E10)	=ABS(F11-F10)	=ABS(G11-G10)	=dff*MAX($Y11:$AD11)/(1-dff)
=ABS(B12-B11)	=ABS(C12-C11)	=ABS(D12-D11)	=ABS(E12-E11)	=ABS(F12-F11)	=ABS(G12-G11)	=dff*MAX($Y12:$AD12)/(1-dff)
=ABS(B13-B12)	=ABS(C13-C12)	=ABS(D13-D12)	=ABS(E13-E12)	=ABS(F13-F12)	=ABS(G13-G12)	=dff*MAX($Y13:$AD13)/(1-dff)
=ABS(B14-B13)	=ABS(C14-C13)	=ABS(D14-D13)	=ABS(E14-E13)	=ABS(F14-F13)	=ABS(G14-G13)	=dff*MAX($Y14:$AD14)/(1-dff)
=ABS(B15-B14)	=ABS(C15-C14)	=ABS(D15-D14)	=ABS(E15-E14)	=ABS(F15-F14)	=ABS(G15-G14)	=dff*MAX($Y15:$AD15)/(1-dff)
=ABS(B16-B15)	=ABS(C16-C15)	=ABS(D16-D15)	=ABS(E16-E15)	=ABS(F16-F15)	=ABS(G16-G15)	=dff*MAX($Y16:$AD16)/(1-dff)
=ABS(B17-B16)	=ABS(C17-C16)	=ABS(D17-D16)	=ABS(E17-E16)	=ABS(F17-F16)	=ABS(G17-G16)	=dff*MAX($Y17:$AD17)/(1-dff)
=ABS(B18-B17)	=ABS(C18-C17)	=ABS(D18-D17)	=ABS(E18-E17)	=ABS(F18-F17)	=ABS(G18-G17)	=dff*MAX($Y18:$AD18)/(1-dff)
=ABS(B19-B18)	=ABS(C19-C18)	=ABS(D19-D18)	=ABS(E19-E18)	=ABS(F19-F18)	=ABS(G19-G18)	=dff*MAX($Y19:$AD19)/(1-dff)
=ABS(B20-B19)	=ABS(C20-C19)	=ABS(D20-D19)	=ABS(E20-E19)	=ABS(F20-F19)	=ABS(G20-G19)	=dff*MAX($Y20:$AD20)/(1-dff)
=ABS(B21-B20)	=ABS(C21-C20)	=ABS(D21-D20)	=ABS(E21-E20)	=ABS(F21-F20)	=ABS(G21-G20)	=dff*MAX($Y21:$AD21)/(1-dff)
=ABS(B22-B21)	=ABS(C22-C21)	=ABS(D22-D21)	=ABS(E22-E21)	=ABS(F22-F21)	=ABS(G22-G21)	=dff*MAX($Y22:$AD22)/(1-dff)
=ABS(B23-B22)	=ABS(C23-C22)	=ABS(D23-D22)	=ABS(E23-E22)	=ABS(F23-F22)	=ABS(G23-G22)	=dff*MAX($Y23:$AD23)/(1-dff)

First 20 iterations results

2	n	v_n,1	v_n,2	v_n,3	v_n,4	v_n,5	v_n,f	d_n,1	d_n,2	d_n,3	d_n,4	d_n,5	d_n,f	v_n,1,k	v_n,1,p	v_n,2,k
3	0	0	0	0	0	0	0									
4	1	5000.000	4500.000	3800.000	2900.000	1800.000	-900.000	kp	kp	kp	kp	kp	mr	5000.000	-300	4500.000
5	2	9648.350	8583.100	7069.900	4983.350	2612.250	2168.500	kp	kp	kp	kp	kp	mr	9648.350	-300	8583.100
6	3	13995.702	12335.072	10017.790	7003.596	4208.928	6134.175	kp	kp	kp	kp	pm	mr	13995.702	-300	12335.072
7	4	18069.689	15811.934	12766.432	9182.320	6438.626	10155.381	kp	kp	kp	kp	kp	mr	18069.689	-300	15811.934
8	5	21891.862	19054.193	15385.400	11536.524	9152.516	14010.627	kp	kp	kp	kp	kp	mr	21891.862	-300	19054.193
9	6	25481.336	22094.486	17917.241	14031.929	12110.212	17651.117	kp	kp	kp	kp	kp	mr	25481.336	-300	22094.486
10	7	28855.729	24958.879	20384.864	16616.142	15147.052	21075.657	kp	kp	kp	kp	kp	mr	28855.729	-300	24958.879
11	8	32031.222	27668.164	22976.404	19237.574	18160.961	24295.622	kp	kp	pm	kp	kp	mr	32031.222	-300	27668.164
12	9	35024.265	30247.319	25539.039	21852.468	21092.687	27325.015	kp	kp	pm	kp	kp	mr	35024.265	-300	30247.319
13	10	37848.524	32709.315	27987.667	24426.711	23910.302	30178.766	kp	kp	pm	kp	kp	mr	37848.524	-300	32709.315
14	11	40515.583	35167.873	30325.293	26935.534	26599.051	32870.217	kp	pm	pm	kp	kp	mr	40515.583	-300	35060.530
15	12	43038.945	37681.771	32649.434	29362.060	29154.261	35410.874	kp	pm	pm	kp	kp	mr	43038.945	-300	37397.983
16	13	45433.220	40078.066	35019.611	31695.610	31576.872	37812.997	kp	pm	pm	kp	kp	mr	45433.220	-300	39776.414
17	14	47707.109	42352.820	37293.609	33930.520	33871.540	40089.796	kp	pm	pm	kp	kp	mr	47707.109	-300	42050.822
18	15	49866.986	44513.096	39454.554	36065.161	36045.533	42250.819	kp	pm	pm	kp	kp	mr	49866.986	-300	44211.273
19	16	51918.662	46565.017	41506.880	38100.445	38106.514	44303.029	kp	pm	pm	kp	kp	mr	51918.662	-300	46263.225
20	17	53867.616	48514.132	43456.243	40038.734	40061.530	46252.273	kp	pm	pm	kp	kp	mr	53867.616	-300	48212.344
21	18	55719.033	50365.654	45307.926	41883.205	41916.875	48103.863	kp	pm	pm	kp	kp	mr	55719.033	-300	50063.868
22	19	57477.820	52124.510	47066.888	43637.463	43678.205	49862.758	kp	pm	pm	kp	kp	mr	57477.820	-300	51822.726
23	20	59148.629	53795.365	48737.811	45305.318	45350.658	51533.636	kp	pm	pm	kp	kp	mr	59148.629	-300	53493.581

2	v_n,2,p	v_n,3,k	v_n,3,p	v_n,4,k	v_n,4,p	v_n,5,k	v_n,5,p	v_n,f,m	j=1	j=2	j=3	j=4	j=5	j=f	E_n
3															
4	-300.000	3800.000	-300.000	2900.000	-300.000	1800.000	-300.000	-900.000	5000.000	4500.000	3800.000	2900.000	1800.000	900.000	95000.000
5	4402.500	7069.900	3908.500	4983.350	3224.500	2612.250	2350.500	2168.500	4648.350	4083.100	3269.900	2083.350	812.250	3068.500	88318.650
6	8764.734	10017.790	7710.191	7003.596	6218.183	4134.091	4208.928	6134.175	4347.352	3751.972	2947.890	2020.246	1596.678	3965.675	82599.679
7	12838.157	12766.432	11198.177	9182.320	8930.552	6438.626	6087.923	10155.381	4073.987	3476.862	2748.642	2178.724	2229.698	4021.206	77405.759
8	16651.718	15385.400	14432.015	11536.524	11487.620	9152.516	8162.553	14010.627	3822.173	3242.258	2618.968	2354.203	2713.890	3855.245	73249.662
9	20227.690	17917.241	17452.948	14031.929	13950.487	12110.212	10433.217	17651.117	3589.475	3040.293	2531.841	2495.405	2957.696	3640.490	69169.306
10	23585.519	20384.864	20292.923	16616.142	16352.274	15147.052	12847.769	21075.657	3374.393	2864.393	2467.624	2584.213	3036.840	3424.540	65066.263
11	26742.742	22798.536	22976.404	19237.574	18707.592	18160.961	15345.771	24295.622	3175.493	2709.285	2591.540	2621.432	3013.908	3219.965	61179.335
12	29715.170	25304.374	25539.039	21852.468	21172.395	21092.687	17873.417	27325.015	2993.043	2579.155	2562.635	2614.894	2931.727	3029.393	57558.470
13	32519.242	27775.043	27987.667	24426.711	23611.862	23910.302	20387.665	30178.766	2824.259	2461.996	2448.628	2574.242	2817.615	2853.751	54221.264
14	35167.873	30139.352	30325.293	26935.534	25949.992	26599.051	22856.316	32870.217	2667.058	2458.558	2337.626	2508.824	2688.749	2691.451	51137.575
15	37681.771	32398.295	32649.434	29362.060	28187.001	29154.261	25256.792	35410.874	2523.362	2513.898	2324.142	2426.526	2555.210	2540.657	48548.992
16	40078.066	34629.908	35019.611	31695.610	30404.662	31576.872	27574.216	37812.997	2394.276	2396.295	2370.176	2333.550	2422.611	2402.123	46029.605
17	42352.820	36882.152	37293.609	33930.520	32652.850	33871.540	29799.549	40089.796	2273.888	2274.754	2273.999	2234.910	2294.668	2276.799	43598.694
18	44513.096	39040.573	39454.554	36065.161	34809.435	36045.533	31928.391	42250.819	2159.877	2160.276	2160.944	2134.641	2173.993	2161.023	41305.868
19	46565.017	41092.097	41506.880	38100.445	36859.834	38106.514	33960.039	44303.029	2051.676	2051.921	2052.326	2035.284	2060.981	2052.210	39158.639
20	48514.132	43040.914	43456.243	40038.734	38807.924	40061.530	35895.999	46252.273	1948.955	1949.115	1949.363	1938.289	1955.016	1949.244	37145.295
21	50365.654	44892.229	45307.926	41883.205	40658.768	41916.875	37738.963	48103.863	1851.417	1851.522	1851.683	1844.470	1855.346	1851.589	35251.570
22	52124.510	46650.948	47066.888	43637.463	42417.181	43678.205	39492.243	49862.758	1758.787	1758.856	1758.961	1754.259	1761.330	1758.895	33465.261
23	53795.365	48321.713	48737.811	45305.318	44087.748	45350.658	41159.461	51533.636	1670.809	1670.854	1670.923	1667.855	1672.454	1670.879	31776.618

Last 20 iterations results

293	290	90892.528	85539.348	80481.924	77043.638	77097.573	83277.664	kp	pm	pm	kp	kp	mr	90892.528	-300	85237.564
294	291	90892.530	85539.350	80481.926	77043.640	77097.575	83277.666	kp	pm	pm	kp	kp	mr	90892.530	-300	85237.566
295	292	90892.531	85539.351	80481.927	77043.641	77097.576	83277.667	kp	pm	pm	kp	kp	mr	90892.531	-300	85237.567
296	293	90892.533	85539.353	80481.929	77043.642	77097.578	83277.669	kp	pm	pm	kp	kp	mr	90892.533	-300	85237.569
297	294	90892.534	85539.354	80481.930	77043.644	77097.579	83277.670	kp	pm	pm	kp	kp	mr	90892.534	-300	85237.570
298	295	90892.535	85539.355	80481.931	77043.645	77097.580	83277.671	kp	pm	pm	kp	kp	mr	90892.535	-300	85237.571
299	296	90892.537	85539.357	80481.932	77043.646	77097.581	83277.672	kp	pm	pm	kp	kp	mr	90892.537	-300	85237.573
300	297	90892.538	85539.358	80481.933	77043.647	77097.583	83277.673	kp	pm	pm	kp	kp	mr	90892.538	-300	85237.574
301	298	90892.539	85539.359	80481.934	77043.648	77097.584	83277.675	kp	pm	pm	kp	kp	mr	90892.539	-300	85237.575
302	299	90892.540	85539.360	80481.935	77043.649	77097.585	83277.676	kp	pm	pm	kp	kp	mr	90892.540	-300	85237.576
303	300	90892.541	85539.361	80481.936	77043.650	77097.586	83277.677	kp	pm	pm	kp	kp	mr	90892.541	-300	85237.577
304	301	90892.542	85539.362	80481.937	77043.651	77097.587	83277.677	kp	pm	pm	kp	kp	mr	90892.542	-300	85237.578
305	302	90892.543	85539.363	80481.938	77043.652	77097.587	83277.678	kp	pm	pm	kp	kp	mr	90892.543	-300	85237.579
306	303	90892.543	85539.363	80481.939	77043.653	77097.588	83277.679	kp	pm	pm	kp	kp	mr	90892.543	-300	85237.579
307	304	90892.544	85539.364	80481.940	77043.654	77097.589	83277.680	kp	pm	pm	kp	kp	mr	90892.544	-300	85237.580
308	305	90892.545	85539.365	80481.941	77043.655	77097.590	83277.681	kp	pm	pm	kp	kp	mr	90892.545	-300	85237.581
309	306	90892.546	85539.366	80481.941	77043.655	77097.591	83277.681	kp	pm	pm	kp	kp	mr	90892.546	-300	85237.582
310	307	90892.546	85539.366	80481.942	77043.656	77097.591	83277.682	kp	pm	pm	kp	kp	mr	90892.546	-300	85237.582
311	308	90892.547	85539.367	80481.943	77043.657	77097.592	83277.683	kp	pm	pm	kp	kp	mr	90892.547	-300	85237.583
312	309	90892.548	85539.368	80481.943	77043.657	77097.592	83277.683	kp	pm	pm	kp	kp	mr	90892.548	-300	85237.584
313	310	90892.548	85539.368	80481.944	77043.658	77097.593	83277.684	kp	pm	pm	kp	kp	mr	90892.548	-300	85237.584
314	311	90892.549	85539.369	80481.944	77043.658	77097.594	83277.684	kp	pm	pm	kp	kp	mr	90892.549	-300	85237.585

293	85539.348	80065.524	80481.924	77043.638	75831.189	77097.573	72896.578	83277.664	0.002	0.002	0.002	0.002	0.002	0.002	0.031
294	85539.350	80065.526	80481.926	77043.640	75831.191	77097.575	72896.580	83277.666	0.002	0.002	0.002	0.002	0.002	0.002	0.029
295	85539.351	80065.527	80481.927	77043.641	75831.192	77097.576	72896.581	83277.667	0.001	0.001	0.001	0.001	0.001	0.001	0.028
296	85539.353	80065.528	80481.929	77043.642	75831.194	77097.578	72896.583	83277.669	0.001	0.001	0.001	0.001	0.001	0.001	0.026
297	85539.354	80065.530	80481.930	77043.644	75831.195	77097.579	72896.584	83277.670	0.001	0.001	0.001	0.001	0.001	0.001	0.025
298	85539.355	80065.531	80481.931	77043.645	75831.196	77097.580	72896.585	83277.671	0.001	0.001	0.001	0.001	0.001	0.001	0.024
299	85539.357	80065.532	80481.932	77043.646	75831.197	77097.581	72896.587	83277.672	0.001	0.001	0.001	0.001	0.001	0.001	0.023
300	85539.358	80065.533	80481.933	77043.647	75831.198	77097.583	72896.588	83277.673	0.001	0.001	0.001	0.001	0.001	0.001	0.021
301	85539.359	80065.534	80481.934	77043.648	75831.200	77097.584	72896.589	83277.675	0.001	0.001	0.001	0.001	0.001	0.001	0.020
302	85539.360	80065.535	80481.935	77043.649	75831.201	77097.585	72896.590	83277.676	0.001	0.001	0.001	0.001	0.001	0.001	0.019
303	85539.361	80065.536	80481.936	77043.650	75831.202	77097.586	72896.591	83277.677	0.001	0.001	0.001	0.001	0.001	0.001	0.018
304	85539.362	80065.537	80481.937	77043.651	75831.202	77097.587	72896.592	83277.677	0.001	0.001	0.001	0.001	0.001	0.001	0.017
305	85539.363	80065.538	80481.938	77043.652	75831.203	77097.587	72896.593	83277.678	0.001	0.001	0.001	0.001	0.001	0.001	0.017
306	85539.363	80065.539	80481.939	77043.653	75831.204	77097.588	72896.593	83277.679	0.001	0.001	0.001	0.001	0.001	0.001	0.016
307	85539.364	80065.540	80481.940	77043.654	75831.205	77097.589	72896.594	83277.680	0.001	0.001	0.001	0.001	0.001	0.001	0.015
308	85539.365	80065.541	80481.941	77043.655	75831.206	77097.590	72896.595	83277.681	0.001	0.001	0.001	0.001	0.001	0.001	0.014
309	85539.366	80065.541	80481.941	77043.655	75831.206	77097.591	72896.596	83277.681	0.001	0.001	0.001	0.001	0.001	0.001	0.014
310	85539.366	80065.542	80481.942	77043.656	75831.207	77097.591	72896.596	83277.682	0.001	0.001	0.001	0.001	0.001	0.001	0.013
311	85539.367	80065.543	80481.943	77043.657	75831.208	77097.592	72896.597	83277.683	0.001	0.001	0.001	0.001	0.001	0.001	0.012
312	85539.368	80065.543	80481.943	77043.657	75831.208	77097.592	72896.598	83277.683	0.001	0.001	0.001	0.001	0.001	0.001	0.012
313	85539.368	80065.544	80481.944	77043.658	75831.209	77097.593	72896.598	83277.684	0.001	0.001	0.001	0.001	0.001	0.001	0.011
314	85539.369	80065.544	80481.944	77043.658	75831.209	77097.594	72896.599	83277.684	0.001	0.001	0.001	0.001	0.001	0.001	0.010

Sensitivity Analysis

Due to the construction of the spreadsheet above, if a parameter value is changed in Sheet1, the MDP formulation and calculations are immediately updated to reflect the change. This makes the spreadsheet very convenient for Sensitivity Analysis which aims to investigate how robust the optimal policy is to changes in one or perhaps more of the problem parameters. This is an important step in the modeling process as one might hesitate to implement a policy that depends critically on precise estimates of the problem parameters.

Excel has functions that make it easy to perform sensitivity analysis by systematically changing a problem parameter or more parameters simultaneously and noting the effect on the optimal policy. The following steps can be used to perform sensitivity analysis on the probability, values, and discount factor of the Turnco operations,

Changes of probabilities and discount factor

Firstly, we consider the changes to psr (probability of successful manufacturer repair), psm (probability of successful preventive maintenance), and df (discount factor) one by one.

To say in detail, we set the range of psr as (0.95 , 0.5) with 'step=0.05', and the range of psm (0.99 , 0.70) with 'step=0.01', and the range of dff (discount factor) with 'step=0.01'. Initially, we enter the first column with the value '0.7', '0.9', '0.95' respectively since they are the initial values used for our model above. Secondly, enter the optimal results for each label on the left-hand side. Enter psr (application below) as the "Row input cell" and click “OK”. Finally, get Excel filled the table. We choose all the cells and select Data Table from the What-If Analysis drop down menu in the Data Tools area on the Data tab & select New Rule from the Conditional Formatting drop down menu in the Styles area on the Home tab to accomplish Sensitivity Analysis.

New Rule is the technique to highlight the changes of optimal decisions. We select all the cells with 'decisions' and select New Rule from the Conditional Formatting drop down menu in the Styles area on the Home tab. Select “Use a formula to determine which cells to format” from the “Select a Rule Type” area of the New Formatting Rule dialog. Type the formula =G44< >$F44 (this entry is for psr application below) in the “Format values where this formula is true” box. Click Format and choose the format (for example, we use the fill color as yellow). Finally, each cell in the range (with 'decisions') is compared to the cell in the corresponding row of initial optimal decisions and if the values differ the chosen format is applied.
(To make visualization better, after applying New Rule, since we know the number of breakpoints of changes to optimal decisions, we just highlight the breakpoints for image below, not fully use New Rule.)

As for psr, we enter psr as the "Row input cell" and click "OK", and from New Rule, we get the image as follows. According to the optimal decisions, the yellow bars show the differences, and the breakpoint will belong to (0.55 , 0.6) because d4 changes to 'pm' when 'psr=0.55', which means we determine to use preventive maintenance when Turnco is found in condition 4. For values of profits, it is obvious that they will go down as psr decreases, and an interesting result is that the biggest drop may be at the breakpoint. Overall, they are relatively stable. Additionally, errors are also stable.

psr	0.7	0.95	0.9	0.85	0.8	0.75	0.7	0.65	0.6	0.55	0.5
v1	90892.549	91473.32	91381.14	91278.93	91164.96	91037.07	90892.55	90727.93	90538.7	90332.88	90184.99
v2	85539.369	86088.05	86000.97	85904.4	85796.73	85675.9	85539.37	85383.84	85205.07	85010.62	84870.9
v3	80481.944	81000.31	80918.04	80826.81	80725.08	80610.94	80481.94	80335.01	80166.12	79982.41	79850.41
v4	77043.658	78971.54	78665.55	78326.26	77947.92	77523.39	77043.66	76497.2	75869.06	75232	75107.29
v5	77097.594	79226.47	78888.57	78513.91	78096.13	77627.34	77097.59	76494.16	75800.54	75005.94	74145.49
v0	83277.684	85726.69	85337.98	84906.98	84426.38	83887.1	83277.68	82583.51	81785.58	80871.49	79881.66
d1	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp
d2	pm	pm	pm	pm	pm	pm	pm	pm	pm	pm	pm
d3	pm	pm	pm	pm	pm	pm	pm	pm	pm	pm	pm
d4	kp	kp	kp	kp	kp	kp	kp	kp	kp	pm	pm
d5	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp
d0	mr	mr	mr	mr	mr	mr	mr	mr	mr	mr	mr
Error	0.0104527	0.010526	0.010515	0.010503	0.010489	0.010472	0.010453	0.01043	0.010402	0.010373	0.010356

As for psm, we enter psr as the "Row input cell" and click "OK", and from New Rule, we get the image as follows. According to the optimal decisions, the yellow bars show the differences, and the breakpoint will belong to (0.74 , 0.75) because d3 changes to 'kp' when 'psm=0.74', which means we determine to keep operations when Turnco is found in condition 3. For values of profits, it is obvious that they will go down as psm decreases, and an interesting result is that the biggest drop may be at the breakpoint, too. Overall, they are relatively stable. Additionally, errors are also stable.

psm	0.9	0.99	0.98	0.97	0.96	0.95	0.94	0.93	0.92	0.91	0.9	0.89	0.88	0.87	0.86	0.85
v1	90892.549	91279.1	91239.3	91198.8	91157.5	91115.4	91072.5	91028.8	90984.3	90938.9	90892.5	90845.3	90797.1	90747.9	90697.7	90646.5
v2	85539.369	86368.5	86283.2	86196.2	86107.7	86017.5	85925.5	85831.8	85736.2	85638.8	85539.4	85438	85334.5	85228.9	85121.2	85011.1
v3	80481.944	81705.8	81579.6	81451.2	81320.4	81187.2	81051.4	80913.1	80772.1	80628.4	80481.9	80332.6	80180.2	80024.8	79866.2	79704.4
v4	77043.658	77326.7	77297.5	77267.9	77237.6	77206.8	77175.4	77143.4	77110.8	77077.6	77043.7	77009.1	76973.8	76937.8	76901	76863.5
v5	77097.594	77410.1	77377.9	77345.2	77311.8	77277.8	77243.1	77207.8	77171.8	77135	77097.6	77059.4	77020.4	76980.7	76940.1	76898.7
v0	83277.684	83637.2	83600.2	83562.5	83524.1	83484.9	83445.1	83404.4	83363	83320.8	83277.7	83233.7	83188.9	83143.2	83096.5	83048.8
d1	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp
d2	pm	pm	pm	pm	pm	pm	pm	pm	pm	pm	pm	pm	pm	pm	pm	pm
d3	pm	pm	pm	pm	pm	pm	pm	pm	pm	pm	pm	pm	pm	pm	pm	pm
d4	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp
d5	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp
d0	mr	mr	mr	mr	mr	mr	mr	mr	mr	mr	mr	mr	mr	mr	mr	mr
Error	0.010	0.01053	0.01052	0.01051	0.0105	0.0105	0.01049	0.01048	0.01047	0.01046	0.01045	0.01044	0.01043	0.01042	0.01041	0.0104

psm	0.84	0.83	0.82	0.81	0.8	0.79	0.78	0.77	0.76	0.75	0.74	0.73	0.72	0.71	0.7
v1	90594.2	90540.8	90486.2	90430.4	90373.4	90315.2	90255.6	90194.7	90132.4	90068.6	90020.5	89981.1	89940.7	89899.3	89856.8
v2	84898.8	84784	84666.7	84546.9	84424.5	84299.2	84171.2	84040.2	83906.2	83769.1	83644.6	83526.4	83405.2	83281	83153.5
v3	79539.2	79370.5	79198.3	79022.4	78842.7	78659.1	78471.4	78279.5	78083.3	77882.6	77793.8	77770	77745.7	77720.7	77695.2
v4	76825.2	76786.1	76746.1	76705.3	76663.6	76620.9	76577.3	76532.7	76487.1	76440.4	76405.2	76376.3	76346.7	76316.4	76285.3
v5	76856.4	76813.2	76769	76724	76677.9	76630.8	76582.6	76533.4	76483	76431.5	76392.6	76360.7	76328	76294.5	76260.2
v0	83000.2	82950.5	82899.7	82847.9	82794.9	82740.7	82685.3	82628.6	82570.7	82511.4	82466.6	82430	82392.4	82353.9	82314.3
d1	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp
d2	pm	pm	pm	pm	pm	pm	pm	pm	pm	pm	pm	pm	pm	pm	pm
d3	pm	pm	pm	pm	pm	pm	pm	pm	pm	pm	kp	kp	kp	kp	kp
d4	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp
d5	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp
d0	mr	mr	mr	mr	mr	mr	mr	mr	mr	mr	mr	mr	mr	mr	mr
Error	0.01039	0.01038	0.01037	0.01036	0.01034	0.01033	0.01031	0.0103	0.01028	0.01026	0.01025	0.01024	0.01023	0.01022	0.01021

As for dff, we enter psr as the "Row input cell" and click "OK", and from New Rule we get the image as follows. According to the optimal decisions, the yellow bars show the differences. Compared with the sensitivity analysis above for probabilities, that for discount factor is quite special. Firstly, it has two breakpoints. The 1st one belongs to (0.91 , 0.92), and the 2nd one belongs to (0.9 , 0.91). The optimal actions of d3 changes to 'kp' when 'psm=0.91', and that of d2 also changes to 'kp' when 'psm=0.9)'. That means we determine to keep operations when Turnco is found in condition 2 & 3. For values of profits and error, it is surprising that they all go down dramatically as psr decreases gradually. Therefore, discount factor is not robust at all.

dff	0.95	0.99	0.98	0.97	0.96	0.95	0.94	0.93	0.92	0.91	0.9
d1	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp
d2	pm	pm	pm	pm	pm	pm	pm	pm	pm	pm	kp
d3	pm	pm	pm	pm	pm	pm	pm	pm	pm	kp	kp
d4	kp	pm	pm	kp	kp	kp	kp	kp	kp	kp	kp
d5	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp
d0	mr	mr	mr	mr	mr	mr	mr	mr	mr	mr	mr
Error	0.010	19684.26	417.3409	11.42734	0.340433	0.010453	0.000323	9.88E-06	2.98E-07	8.83E-09	2.62E-10
v1	90892.549	430711.4	225208	150776.5	113358	90892.55	75909.18	65201.85	57167.36	50944.05	46022.64
v2	85539.369	425379.6	219872.6	145434.7	108010.3	85539.37	70550.96	59838.97	51800.19	45570.05	40840.56
v3	80481.944	420107	214655.5	140270.4	102899.2	80481.94	65547.58	54889.99	46905.9	41045.23	36599.41
v4	77043.658	414892.9	209688.7	135848.2	98985.53	77043.66	62554.11	52313.1	44719.5	38901.24	34336.48
v5	77097.594	415655.3	210491.5	136376.7	99265.31	77097.59	62402.96	51975.69	44212.92	38244.08	33549.07
v0	83277.684	423024.4	217541.5	143126.8	105725.5	83277.68	68312.31	57623.31	49607.43	43399.4	38485.29

dff	0.89	0.88	0.87	0.86	0.85	0.84	0.83	0.82	0.81	0.8
d1	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp
d2	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp
d3	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp
d4	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp
d5	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp
d0	mr	mr	mr	mr	mr	mr	mr	mr	mr	mr
Error	0	0	0	0	0	0	0	0	0	0
v1	42017.02	38666.89	35822.12	33375.37	31247.8	29380.19	27727.18	26253.45	24931.06	23737.62
v2	37153.19	34091.25	31508.29	29300.08	27390.46	25722.53	24252.92	22948.05	21781.47	20732.13
v3	33014.34	30057.3	27580.32	25477.99	23673.29	22108.66	20740.26	19534.17	18463.72	17507.68
v4	30658.22	27624.52	25085.35	22933.49	21090.34	19496.93	18108.22	16889.2	15812.25	14855.33
v5	29754.91	26614.53	23977.36	21735.65	19810.28	18141.8	16684.69	15403.48	14270.14	13262.22
v0	34483.77	31139.68	28302.67	25865.19	23748.24	21892.45	20252.32	18792.42	17484.69	16306.66

Scenario summary for profits

Then we consider the changes to profits. We use Excel's Scenario Manager from the What-If Analysis drop down menu in the Data Tools area on the Data tab to improve the accuracy of sensitivity analysis. We create 21 different scenarios based on 21 different (a , 1-a) coordinates. The expression is similar to previous tables but here we find the changing cells take up three whole rows rather than only one.

According to the optimal decisions, the yellow bars show the differences, and the breakpoint will belong to ((0.05 , 0.95) , (0 , 1)) because both d2 and d3 change to 'kp' when fully apply Scenario B, which means we determine to keep operations when Turnco is found in any other condition until it is found below the required standard. For values of profits, it is obvious that they will go up as tends to Scenario B, and an interesting result is that the biggest drop may be at the breakpoint, too. Overall, they are relatively stable. Additionally, errors are also stable. To some extent, we can claim the profits of Scenario are robust because the results only differ at the last time.

Scenario Summary	A	0.95A+0.05B	0.90A+0.10B	0.85A+0.15B	0.80A+0.20B	0.75A+0.25B	0.70A+0.30B	0.65A+0.35B	0.60A+0.40B	0.55A+0.45B	0.50A+0.50B
Changing Cells:											
reward condition 2	4200	4230	4260	4290	4320	4350	4380	4410	4440	4470	4500
reward condition 3	3400	3440	3480	3520	3560	3600	3640	3680	3720	3760	3800
reward condition 4	2600	2630	2660	2690	2720	2750	2780	2810	2840	2870	2900
Result Cells:											
Value when in condition 1 initially	90714.909	90732.673	90750.437	90768.201	90785.965	90803.729	90821.493	90839.257	90857.021	90874.785	90892.549
Value when in condition 2 initially	85371.543	85388.326	85405.108	85421.891	85438.673	85455.456	85472.238	85489.021	85505.803	85522.586	85539.369
Value when in condition 3 initially	80323.391	80339.246	80355.102	80370.957	80386.812	80402.668	80418.523	80434.378	80450.234	80466.089	80481.944
Value when in condition 4 initially	75870.119	75987.473	76104.827	76222.181	76339.535	76456.888	76574.242	76691.596	76808.95	76926.304	77043.658
Value when in condition 5 initially	76953.973	76968.335	76982.697	76997.059	77011.421	77025.783	77040.145	77054.507	77068.869	77083.232	77097.594
Value when below standard initially	83112.467	83128.989	83145.51	83162.032	83178.554	83195.076	83211.597	83228.119	83244.641	83261.163	83277.684
Decision when in condition1	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp
Decision when in condition1	pm	pm	pm	pm	pm	pm	pm	pm	pm	pm	pm
Decision when in condition1	pm	pm	pm	pm	pm	pm	pm	pm	pm	pm	pm
Decision when in condition1	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp
Decision when in condition1	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp
Decision when below standard	mr	mr	mr	mr	mr	mr	mr	mr	mr	mr	mr
Error	0.0104813	0.0104798	0.0104781	0.0104764	0.0104746	0.0104719	0.0104692	0.010466	0.0104622	0.010458	0.0104527

Scenario Summary	0.45A+0.55B	0.40A+0.60B	0.35A+0.65B	0.30A+0.70B	0.25A+0.75B	0.20A+0.80B	0.15A+0.85B	0.10A+0.90B	0.05A+0.95B	B
Changing Cells:										
reward condition 2	4530	4560	4590	4620	4650	4680	4710	4740	4770	4800
reward condition 3	3840	3880	3920	3960	4000	4040	4080	4120	4160	4200
reward condition 4	2930	2960	2990	3020	3050	3080	3110	3140	3170	3200
Result Cells:										
Value when in condition 1 initially	90910.313	90928.077	90945.841	90963.605	90981.369	90999.133	91016.897	91034.661	91052.425	91150.962
Value when in condition 2 initially	85556.151	85572.934	85589.716	85606.499	85623.282	85640.064	85656.847	85673.629	85690.412	85874.919
Value when in condition 3 initially	80497.8	80513.655	80529.51	80545.366	80561.221	80577.077	80592.932	80608.787	80624.643	80912.821
Value when in condition 4 initially	77161.012	77278.366	77395.72	77513.074	77630.428	77747.782	77865.136	77982.49	78099.844	78276.337
Value when in condition 5 initially	77111.956	77126.318	77140.68	77155.042	77169.404	77183.766	77198.128	77212.49	77226.852	77306.519
Value when below standard initially	83294.206	83310.728	83327.25	83343.772	83360.293	83376.815	83393.337	83409.859	83426.38	83518.027
Decision when in condition1	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp
Decision when in condition1	pm	pm	pm	pm	pm	pm	pm	pm	pm	kp
Decision when in condition1	pm	pm	pm	pm	pm	pm	pm	pm	pm	kp
Decision when in condition1	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp
Decision when in condition1	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp
Decision when below standard	mr	mr	mr	mr	mr	mr	mr	mr	mr	mr
Error	0.0104469	0.0104402	0.0104318	0.0104222	0.0104108	0.0103967	0.0103793	0.0103563	0.0103216	0.0103061

The steps for achieving Scenario Summary above:

1.Select Data, What-If Analysis, Scenario Manager.
2.Click the Add... button in the Scenario Manager dialog.
3.Type a name for the scenario using the current values.
4.Specify the input cells (reward in condition 2, 3, and 4) by clicking the first cell and Ctrl+clicking the other input cells. Click OK.
5.Check and specify the input cells (reward in condition 2, 3, and 4). Excel will show you the current input values. (maybe not for desired scenario)
6.Verify the values for the original scenario. Click Add. You will go back to the Add Scenario dialog.

7.Enter a new scenario name and description. Click OK. You will go to the Scenario Values dialog.
8.Repeat steps 4 to 6 for each additional scenario.
9.Create 20 new scenarios using steps above.
10.When we are done entering scenarios, click OK instead of Add in the Scenario Values dialog.
11.In the Scenario Manager dialog, choose any scenario and click Show to show that scenario in the worksheet.
12.To see a comparison of all 21 different scenarios, click Summary.
13.In the Scenario Summary dialog, specify the output cells to include in the report.
14.Specify output cells. A new worksheet is inserted as the image above. Such summary report compares the scenarios.

Changes to maintenance and repair cost

After considering profits, it is quite natural to test how robust changes to costs are. To say in detail, we set the range of pmc as (150 , 1300) with 'step=50', and the range of mrc (450 , 1800) with 'step=50'. Initially, we enter the first column with the value '300' and '900' respectively since they are the initial values used for our model above. Secondly, enter the optimal results for each label on the left hand side. Finally, get Excel filled the table. We choose all the cells and select Data Table from the What-If Analysis drop down menu in the Data Tools area on the Data tab & select New Rule from the Conditional Formatting drop down menu in the Styles area on the Home tab to accomplish Sensitivity Analysis.

As for pmc, we enter pmc as the "Row input cell" and click "OK", and from New Rule we get the image as follows. According to the optimal decisions, the yellow bars show the differences, and the breakpoint will belong to (1150 , 1200) because d3 changes to 'kp' when 'pmc=1200', which means we determine to keep operations when Turnco is found in condition 3. For values of profits, it is obvious that they will go down as pmc increases. Overall, they are relatively stable. Additionally, errors are also stable.

pmc	300	150	200	250	300	350	400	450	500	550	600	650
v1	90892.549	91032.1	90985.6	90939.1	90892.5	90846	90799.5	90753	90706.5	90660	90613.5	90567
v2	85539.369	85836.9	85737.7	85638.6	85539.4	85440.2	85341	85241.8	85142.6	85043.4	84944.2	84845.1
v3	80481.944	80928.8	80779.9	80630.9	80481.9	80333	80184	80035.1	79886.1	79737.2	79588.2	79439.3
v4	77043.658	77145.8	77111.8	77077.7	77043.7	77009.6	76975.6	76941.5	76907.5	76873.4	76839.4	76805.3
v5	77097.594	77210.4	77172.8	77135.2	77097.6	77060	77022.4	76984.8	76947.2	76909.6	76872	76834.4
v0	83277.684	83407.5	83364.2	83320.9	83277.7	83234.4	83191.2	83147.9	83104.7	83061.4	83018.2	82974.9
d1	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp
d2	pm	pm	pm	pm	pm	pm	pm	pm	pm	pm	pm	pm
d3	pm	pm	pm	pm	pm	pm	pm	pm	pm	pm	pm	pm
d4	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp
d5	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp
d0	mr	mr	mr	mr	mr	mr	mr	mr	mr	mr	mr	mr
Error	0.010	0.01048	0.01047	0.01046	0.01045	0.01044	0.01043	0.01042	0.01041	0.01041	0.0104	0.01039

pmc	700	750	800	850	900	950	1000	1050	1100	1150	1200	1250	1300
v1	90520.5	90474	90427.5	90381	90334.5	90288	90241.4	90194.9	90148.4	90101.9	90057.7	90030.8	90003.9
v2	84745.9	84646.7	84547.5	84448.3	84349.1	84249.9	84150.8	84051.6	83952.4	83853.2	83756.2	83675.5	83594.9
v3	79290.3	79141.3	78992.4	78843.4	78694.5	78545.5	78396.6	78247.6	78098.7	77949.7	77816.1	77799.9	77783.8
v4	76771.2	76737.2	76703.1	76669.1	76635	76601	76566.9	76532.9	76498.8	76464.8	76432.4	76412.7	76393
v5	76796.8	76759.2	76721.6	76684	76646.4	76608.8	76571.2	76533.6	76496	76458.4	76422.6	76400.9	76379.1
v0	82931.6	82888.4	82845.1	82801.9	82758.6	82715.4	82672.1	82628.9	82585.6	82542.3	82501.2	82476.2	82451.2
d1	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp
d2	pm	pm	pm	pm	pm	pm	pm	pm	pm	pm	pm	pm	pm
d3	pm	pm	pm	pm	pm	pm	pm	pm	pm	pm	kp	kp	kp
d4	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp
d5	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp
d0	mr	mr	mr	mr	mr	mr	mr	mr	mr	mr	mr	mr	mr
Error	0.01038	0.01037	0.01035	0.01034	0.01033	0.01032	0.01031	0.0103	0.01029	0.01027	0.01026	0.01025	0.01025

As for mrc, we enter mrc as the "Row input cell" and click "OK", and from New Rule we get the image as follows. According to the optimal decisions, the yellow bars show the differences, and the breakpoint will belong to (1550 , 1600) because d4 changes to 'pm' when 'mrc=1600', which means we determine to use preventive maintenance when Turnco is found in condition 4. For values of profits, it is obvious that they will go down as mrc increases. Overall, they are relatively stable. Additionally, errors are also stable.

mrc	900	450	500	550	600	650	700	750	800	850	900	950	1000	1050	1100	1150
v1	90892.549	91084.04	91158.5	91174.46	91161.16	91133.24	91098	91059.1	91018.38	90976.74	90934.64	90892.32	90849.88	90807.39	90764.86	90722.32
v2	85539.369	85720.28	85790.63	85805.71	85793.14	85766.76	85733.47	85696.72	85658.25	85618.91	85579.14	85539.15	85499.06	85458.91	85418.74	85378.55
v3	80481.944	80652.86	80719.32	80733.57	80721.7	80696.77	80665.32	80630.6	80594.25	80557.09	80519.52	80481.74	80443.86	80405.93	80367.98	80330.01
v4	77043.658	77679.31	77926.5	77979.47	77935.33	77842.63	77725.66	77596.54	77461.35	77323.13	77183.4	77042.9	76902.02	76760.96	76619.8	76478.59
v5	77097.594	77799.51	78072.48	78130.97	78082.23	77979.86	77850.69	77708.11	77558.84	77406.21	77251.9	77096.76	76941.19	76785.42	76629.54	76473.62
v0	83277.684	84085.15	84399.17	84466.46	84410.38	84292.63	84144.03	83980.01	83808.29	83632.7	83455.19	83276.72	83097.77	82918.57	82739.25	82559.88
d1	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp
d2	pm	pm	pm	pm	pm	pm	pm	pm	pm	pm	pm	pm	pm	pm	pm	pm
d3	pm	pm	pm	pm	pm	pm	pm	pm	pm	pm	pm	pm	pm	pm	pm	pm
d4	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp
d5	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp
d0	mr	mr	mr	mr	mr	mr	mr	mr	mr	mr	mr	mr	mr	mr	mr	mr
Error	0.010	0.010476	0.010485	0.010487	0.010485	0.010482	0.010478	0.010473	0.010468	0.010463	0.010458	0.010453	0.010447	0.010442	0.010436	0.010431

1150	1200	1250	1300	1350	1400	1450	1500	1550	1600	1650	1700	1750	1800
90722.32	90679.78	90637.23	90594.68	90552.13	90509.57	90467.02	90424.47	90381.92	90344.59	90320.32	90296.05	90271.78	90247.52
85378.55	85338.35	85298.15	85257.95	85217.75	85177.55	85137.35	85097.15	85056.95	85021.68	84998.76	84975.83	84952.9	84929.97
80330.01	80292.03	80254.06	80216.08	80178.1	80140.12	80102.14	80064.16	80026.18	79992.86	79971.2	79949.54	79927.88	79906.22
76478.59	76337.36	76196.12	76054.87	75913.61	75772.36	75631.11	75489.85	75348.6	75241.87	75221.41	75200.95	75180.48	75160.02
76473.62	76317.66	76161.69	76005.72	75849.74	75693.76	75537.78	75381.8	75225.82	75074.06	74932.86	74791.67	74650.47	74509.27
82559.88	82380.47	82201.05	82021.62	81842.18	81662.75	81483.31	81303.88	81124.44	80949.86	80787.43	80625	80462.57	80300.14
kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp
pm	pm	pm	pm	pm	pm	pm	pm	pm	pm	pm	pm	pm	pm
pm	pm	pm	pm	pm	pm	pm	pm	pm	pm	pm	pm	pm	pm
kp	kp	kp	kp	kp	kp	kp	kp	kp	pm	pm	pm	pm	pm
kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp	kp
mr	mr	mr	mr	mr	mr	mr	mr	mr	mr	mr	mr	mr	mr
0.010431	0.010425	0.010419	0.010414	0.010408	0.010402	0.010396	0.010391	0.010385	0.01038	0.010379	0.010377	0.010375	0.010373

Hence, we have completed Sensitivity Analysis based on the meanings of parameters in order.

Proportions of time Calculation

Firstly, we are interested in the probability that the process is in a particular state after some time (after some steps). We define π_i^n to be the probability that the process is in state **i** after **n** transitions. Conditioning on the previous state, we can get,

$$\pi_i^n = \pi_1^{n-1} p_{1,i} + \pi_2^{n-1} p_{2,i} + \ldots + \pi_f^{n-1} p_{f,i} \; and \; get \; \pi^n = \pi^{n-1} P$$

Here $\pi^n = (\pi_1^n, \pi_2^n, \pi_3^n, \pi_4^n, \pi_5^n, \pi_f^n)$ is the probability distribution of the state of the process after **n** steps. P is the transition matrix.

In this MDP problem, Turnco operates in a long-run process. We can calculate the proportion of time with respect to each state by equilibrium distribution π. In the long-run distribution, since an aperiodic and ergodic Markov chain has a unique equilibrium distribution, π, such that, for any probability distribution of initial state, π^n tends to π as **n** tends to infinity.

As we know, the above π fits the equation $\pi = \pi P$. Additionally, $\pi = (\pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_f)$, π_i is the long-run proportion of time the process spends in state i. For such an aperiodic, ergodic Markov chain, the equilibrium probabilities are the unique solutions to the following linear equations:

$$\begin{aligned}
\pi_1 &= \pi_1 P_{1,1} + \pi_2 P_{2,1} + \pi_3 P_{3,1} + \pi_4 P_{4,1} + \pi_5 P_{5,1} + \pi_f P_{f,1} \\
\pi_2 &= \pi_1 P_{1,2} + \pi_2 P_{2,2} + \pi_3 P_{3,2} + \pi_4 P_{4,2} + \pi_5 P_{5,2} + \pi_f P_{f,2} \\
\pi_3 &= \pi_1 P_{1,3} + \pi_2 P_{2,3} + \pi_3 P_{3,3} + \pi_4 P_{4,3} + \pi_5 P_{5,3} + \pi_f P_{f,3} \\
\pi_4 &= \pi_1 P_{1,4} + \pi_2 P_{2,4} + \pi_3 P_{3,4} + \pi_4 P_{4,4} + \pi_5 P_{5,4} + \pi_f P_{f,4} \\
\pi_5 &= \pi_1 P_{1,5} + \pi_2 P_{2,5} + \pi_3 P_{3,5} + \pi_4 P_{4,5} + \pi_5 P_{5,5} + \pi_f P_{f,5} \\
\pi_f &= \pi_1 P_{1,f} + \pi_2 P_{2,f} + \pi_3 P_{3,f} + \pi_4 P_{4,f} + \pi_5 P_{5,f} + \pi_f P_{f,f} \\
1 &= \pi_1 + \pi_2 + \pi_3 + \pi_4 + \pi_5 + \pi_f
\end{aligned}$$

Therefore, we just need the transition probabilities $p_{i,j}$, which is also the entry $P_{i,j}$ of the transition matrix P, to achieve the proportions of time. We can find these $p_{i,j}$ are not the same as 'Probabilities of transitions' at the very first of the Appendix, since it also depends on the actions we take at each state. To explain in detail, under our determining optimal policy, Turnco may not naturally transit from state 2 to state 3 with the probability 0.05 by normal operations because we decide to apply preventive maintenance for Turnco in condition 2, so the $p_{2,3}$ here is

0 indeed. Just consider the probabilities for other $p_{i,j}$ and we can get the transition matrix p has the expression below,

$$\begin{pmatrix} 0.94 & 0.03 & 0.01 & 0.01 & 0 & 0.01 \\ 0.90 & 0.10 & 0 & 0 & 0 & 0 \\ 0 & 0.90 & 0.10 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.75 & 0.09 & 0.16 \\ 0 & 0 & 0 & 0 & 0.65 & 0.35 \\ 0.70 & 0 & 0 & 0 & 0 & 0.30 \end{pmatrix}$$

By applying the matrix above for the linear equations, we can get the final results for π, that is,

$$\pi = (0.8815, 0.0392, 0.0098, 0.0353, 0.0091, 0.0252)$$

Hence, we can get π_i for each **State i**,

$$\pi_1 = 0.8815, \pi_2 = 0.0392, \pi_3 = 0.0098, \pi_4 = 0.0353, \pi_5 = 0.0091, \pi_f = 0.0252$$

Based on optimal actions we determined, we add the proportion of state 1, state 4, and state 5 for the proportion of operations time together. As both preventive maintenance and manufacturer repair need one day out of use, we add the proportion of time of the remained states together.

Hence we can get the proportion of normal operations is 0.9259 ($\pi_1 + \pi_4 + \pi_5$), and the proportion of out of use is 0.0741 (($\pi_2 + \pi_3 + \pi_f$), which means under such optimal policy, Turnco will operate for approximately 92.59% of the total time, and the remained time (7.41%) is out of use due to preventive maintenance or manufacturer repair.

Part B

The main strengths can be considered in three aspects.

The first one is setting the initial model reasonably. To start with, we set parameters rigorously. We do not just take Scenario A or Scenario B into account, but create a new Scenario C with the combination of A & B (Half and Half), and also, set the probability of successful preventive maintenance as 0.9, higher than that of the successful manufacturer repair (0.7) since preventive maintenance is much easier to be applied than manufacturer repair. What's more, we use all the figures from the problem sheet to indicate the validity of the further process.

The second is doing a useful sensitivity analysis. To say in detail, it is clear that we consider most of the changes in parameters that may influence the original results, which means we try to change values of probability or discount factor, and also several profits. We use not only data table tool, but also accurate scenario method tool to test how robust the changes in Scenario are, not just for single profit in any condition.

The third one is for decision criterion, it is based on the strong convergence of expected reward and small error. Considering our optimal policy, we control the error as less than 0.01 for stopping iterations, and also, our $|v_j^n - v_j^{n-1}|$ are all equal to 0.001, which proves the validity of decision criterion. What's more, our optimal policy seems reasonable as explained in the Appendix.

The main weaknesses can be considered in two aspects.

The first one is the basic assumptions for decisions. As you as see, we consider the two-decision model - Turnco can only be repaired by manufacturer when it is below the standard required for normal operation, but not apply a more complex three-decision model - Turnco can take the optimal action from three actions in any condition (Turnco can be repaired by the manufacturer at any state). However, in fact, we have several reasons for choosing the present model; we think the frequent repair is relatively complex that it is kind of rare in real-life (preventive maintenance is more common to be frequently applied, but not the ability to apply manufacturer repair at every state). Yet we also have applied the three-decision model and find that has approximately more 1500 pounds in each condition than the two-decision model under the same parameter settings. Therefore, we admit that our assumption is less profitable.

The second one is for sensitivity analysis. Although we have done useful analysis above, we think we can do more, because we have not considered more combinations of parameters with simultaneous changes in scenario summary. For example, we do not change both the profits and probabilities in the meantime, which indicates the incompleteness of sensitivity analysis. Nevertheless, we also have reasons for that; we can not list all combinations of parameters changing in reality, which means we have to give up for some not strongly-related combinations in scenario summary.