Sample Solutions on HW1 (32 questions in total)

Sec. 1.1 2(a, b, d, e ,f), 8(e,h), 14(d,e), 18(c), 22, 28(b), 32(f), 46, 48

2.

a) This is not a proposition; it's a command.

b) This is not a proposition; it's a question.

d) This is not a proposition; its truth value depends on the value of x.

e) This is a proposition that is false.

f) This is not a proposition; its truth value depends on the value of n.

8.

e) I bought a lottery ticket this week if and only if I won the million dollar jackpot on Friday.

h) Either I did not buy a lottery ticket this week, or else I did buy one and won the million dollar jackpot on Friday.

14

(d) $p \wedge \neg q \wedge r$

(e) $(p \wedge q) \to r$

18(c) This is $T \to F$, which is false.

22

a) If you get promoted, then you wash the boss's car.

b) If the winds are from the south, then there will be a spring thaw.

c) If you bought the computer less than a year ago, then the warranty is good.

d) If Willy cheats, then he gets caught.

e) If you can access the website, then you must pay a subscription fee.

f) If you know the right people, then you will be elected.

g) If Carol is on a boat, then she gets seasick.

28(b)

Converse: Whenever I go to the beach, it is a sunny summer day. **Contrapositive**: Whenever I do not go to the beach, it is not a sunny summer day.

Inverse: Whenever it is not a sunny day, I do not go to the beach.

32(f)

p	q	$\neg q$	$p \leftrightarrow q$	$p \leftrightarrow \neg q$	$(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$
T	T	F	T	F	T
T	F	T	F	T	T
F	T	F	F	T	T
F	F	T	T	F	T

46 The truth value of "Fred and John are happy" is min(0.8, 0.4) = 0.4. The truth value of "Neither Fred nor John is happy" is min(0.2, 0.6) = 0.2, since this statement means "Fred is not happy, and John is not happy."

48 This cannot be a proposition, because it cannot have a truth value. Indeed, if it were true, then it would be truly asserting that it is false, a contradiction; on the other hand if it were false, then its assertion that it is false must be false, so that it would be true – again a contradiction. Thus this string of letters, while appearing to be a proposition, is in fact meaningless.

Sec. 1.2 4, 10, 18

4 The condition stated here is that if you use the network, then either you pay the fee or you are a subscriber. Therefore the proposition in symbols is $w \to d \vee \boldsymbol{s}$.

10 We write these symbolically: $u \to \neg\, a$, $a \to s$, $\neg\, s \to \neg u$. Note that we can make all the conclusion true by making ***a*** false, ***s*** true, and ***u*** false. Therefore if the users cannot access the file system, they can save new files, and the system is not being upgraded, then all the conditional statements are true. Thus the system is consistent.

18 We will translate these conditions into statements in symbolic logic, using ***j***, ***s***, and ***k*** for the propositions that Jasmine, Samir, and Kanti attend, respectively. The first statement is $j \to \neg\, s$. The second statement is $s \to k$, . The last statement is $\neg\, k \vee j$, , because “unless” means “or.” (We could also translate this as $k \to j$. From the comments following Definition 5 in the text, we know that $jp \to q$ is equivalent to “q unless $\neg p$. In this case p is $\neg j$ and q is $\neg k$.) First, suppose that ***s*** is true. Then the second statement tells us that ***k*** is also true, and then the last statement forces ***j*** to be true. But now the first statement forces ***s*** to be false.

So we conclude that ***s*** must be false; Samir cannot attend. On the other hand, if ***s*** is false, then the first two statements are automatically true, not matter what the truth values of ***k*** and j are. If we look at the last statement, we see that it will be true as long as it is not the case that ***k*** is true and ***j*** is false. So the only combinations of friends that make everybody happy are Jasmine and Kanti, or Jasmine alone (or no one!).

Sec. 1.3 6, 8(a,b), 12(b), 32, 40, 51,58

6 We see that the fourth and seventh columns are identical.

p	q	$p \wedge q$	$\neg (p \wedge q)$	$\neg p$	$\neg q$	$\neg p \vee \neg q$
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T

8

a) Kwame will not take a job in industry and will not go to graduate school.

b) Yoshiko does not know Java or does not know calculus.

12(b)

$$[(p \to q) \wedge (q \to r)] \to (p \to r)$$

$$\equiv \neg[(\neg p \vee q) \wedge (\neg q \vee r)] \vee (\neg p \vee r)$$

$$\equiv [\neg(\neg p \vee q) \vee \neg(\neg q \vee r)] \vee (\neg p \vee r)$$

$$\equiv [(p \wedge \neg q) \vee (q \wedge \neg r)] \vee \neg p \vee r$$

$\equiv [(p \wedge \neg q) \vee \neg p] \vee [(q \wedge \neg r) \vee r]$

$\equiv [(p \vee \neg p) \wedge (\neg q \vee \neg p)] \vee [(q \vee r) \wedge (\neg r \vee r)]$

$\equiv (\neg q \vee \neg p) \vee (q \vee r)$

$\equiv (\neg q \vee q) \vee \neg p \vee r$

$\equiv T \vee \neg p \vee r \equiv T$

32 We just need to find an assignment of truth values that makes one of these propositions true and the other false. We can let p be true and the other two variables be false. Then the first statement will be F $\rightarrow$ F, which is true, but the second will be F$\wedge$T, which is false.

40 Following the hint, we see that the answer is $p \wedge q \wedge \neg r$

51 Using the results of Exercise 50, parts (a) and (b), we can derive

$$p \rightarrow q \equiv (((p \downarrow p) \downarrow q) \downarrow ((p \downarrow p) \downarrow q))$$

58 If we want the first two of these to be true, then p and q must have the same truth value. If q is true, then the third and fourth expressions will be true, and if r is false, the last expression will be true. So all five of these disjunctions will be true if we set p and q to be true, and r to be false.