Sample Solutions on HW13 (*9 exercises in total*)

Sec. 10.5 4, 6, 31, 34, 38, 41

4 The graph has no Euler circuit, since the degree of vertex *c* (for one) is odd. There is an Euler path between the two vertices of odd degree. One such path is *f, a, b, c, d, e, f, b, d, a, e, c*.

6 This graph has no Euler circuit, since the degree of vertex *b* (for one) is odd. There is an Euler path between the two vertices of odd degree. One such path is *b, c, d, e, f, d, g, i, d, a, h, i, a, b, i, c*.

31 It is clear that *a, b, c, d, e, a* is a Hamilton circuit.

34 This graph has no Hamilton circuit. If it did, then certainly the circuit would have to contain edges {*d*,*a*} and {*a*,*b*}, since these are the only edges incident to vertex *a*. By the same reasoning, the circuit would have to contain the other six edges around the outside of the figure. These eight edges already complete a circuit, and this circuit omits the nine vertices on the inside. Therefore there is no Hamilton circuit.

38 This graph has the Hamilton path *a, b, c, d, e*.

41 There are eight vertices of degree 2 in this graph. Only two of them can be the end vertices of a Hamilton path, so for each of the other six their two incident edges must be present in the path. Now if either all four of the "outside" vertices of degree 2 (*a*, *c*, *g*, and *e*) or all four of the "inside" vertices of degree 2 (*i*, *k*, *l*, and *n*) are not end vertices, then a circuit will be completed that does not include all the vertices – either the outside square or the middle square. Therefore if there is to be a Hamilton path then exactly one of the inside corner vertices must be an end vertex, and each of the other inside corner vertices must have its two incident edges in the path. Without loss of generality we can assume that vertex *i* is an end, and that the path begins *i, o, n, m, l, q, k, j*. At this point, either the path must visit vertex *p*, in which case it gets stuck, or else it must visit *b*, in which case it will never be able to reach *p*. Either case gives a contradiction, so there is no Hamilton path.

Sec. 10.6 3, 17a), 26

3 By applying Dijkstra's algorithm, we can see a shortest path between *a* and *z* is *a, c, d, e, g, z* with length 16.

17(a) The shortest routes are Newark to Woodbridge to Camden, and Newark to Woodbridge to Camden to Cape May. (The map is obviously not drawn to scale.)

26 The following table shows the twelve different Hamilton circuits and their weights.

Circuit	Weight
$a-b-c-d-e-a$	3+10+6+1+7=27
$a-b-c-e-d-a$	3+10+5+1+4=23
$a-b-d-c-e-a$	3+9+6+5+7=30
$a-b-d-e-c-a$	3+9+1+5+8=26
$a-b-e-c-d-a$	3+2+5+6+4=20
$a-b-e-d-c-a$	3+10+1+6+8=20
$a-c-b-d-e-a$	8+10+9+1+7=35
$a-c-b-e-d-a$	8+10+2+1+4=25
$a-c-d-d-e-a$	8+6+9+2+7=32
$a-c-e-b-d-a$	8+5+2+9+4=28
$a-d-b-c-e-a$	4+9+10+5+7=35
$a-d-c-b-e-a$	4+6+10+2+7=29

Thus we see that the circuits $a - b - e - c - d - a$ and $a - b - e - d - c - a$ (or the same circuits starting at some other point but traversing the vertices in the same or exactly opposite order) are the ones with minimum total weight.