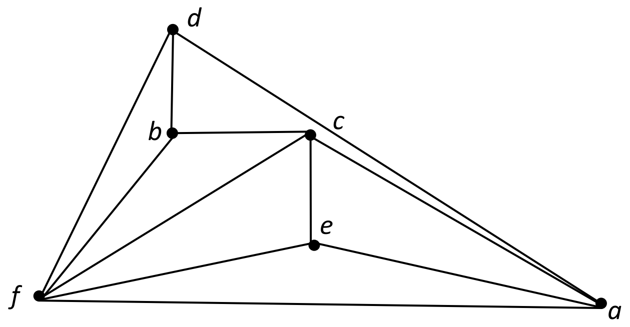
Sample Solutions on HW14 (*10 exercises in total*)

Sec. 10.7 7, 20, 22, 23, 25

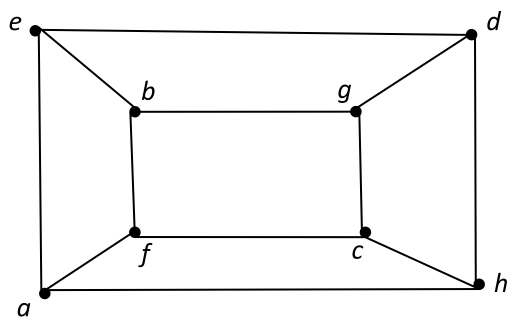
7 This graph can be untangled if we play with it long enough. The following picture gives a planar representation of it.



20 This graph is not homeomorphic to $K_{3,3}$, since by rerouting the edge between *a* and *h* we see that it is planar.

22 Replace each vertex of degree two and its incident edges by a single edge. Then the result is $K_{3,3}$: the parts are $\{a,e,i\}$ and $\{c,g,k\}$. Therefore this graph is homeomorphic to $K_{3,3}$.

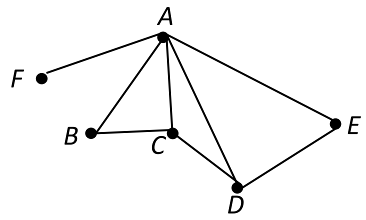
23 The instructions are really not fair. It is hopeless to try to use Kuratowski's Theorem to prove that a graph is planar. Since we would have to check hundreds of cases to argue that there is no subgraph homeomorphic to K_5 or $K_{3,3}$. Thus we will show that this graph is planar simply by giving a planar representation. Note that it is Q_3.



25 This graph is nonplanar, since it contains $K_{3,3}$ as a subgraph: the parts are $\{a,g,d\}$ and $\{b,c,e\}$. (Actually it contains $K_{3,4}$, and it even contains a subgraph homeomorphic to K_5.)

Sec. 10.8 3, 8, 9, 10, 17

3 We construct the dual graph by putting a vertex inside each region (but not in the unbounded region), and drawing an edge between two vertices if the regions share a common border. The number of colors needed to color this map is the same as the number of colors needed to color the dual graph. Three colors are clearly necessary, because of the triangle *ABC*, for instance. Furthermore three colors suffice, since we can color vertex *A* red, vertices *B*, *D*, and *F* blue, and vertices *C* and *E* green.



8 Since there is a triangle, at least 3 colors are needed. The coloring in which b and c are blue, a and f are red, and d and e are green shows that 3 colors suffice.

9 Since there is an edge, at least 2 colors are needed. The coloring in which b, d, and e are red and a and c blue shows that 2 colors suffice.

10 Since vertices b, c, h, and i form a K_4, at least 4 colors are required. A coloring using only 4 colors (and we can get this by trial and error, without much difficulty) is to let a and c be red; b, d, and f, blue; g and i, green; and e and h, yellow.

17 Consider the graph representing this problem. The vertices are the 8 courses, and two courses are joined by an edge if there are students taking both of them. Thus there are edges between every pair of vertices except the 7 pairs listed. It is much easier to draw the complement than to draw this graph itself; it is shown below. We want to find the chromatic number of the graph whose complement we have drawn; the colors will be the time periods for exams. First note that since Math 185 and the four CS courses form a K5 (in other words, there are no edges between any two of these in our picture), the chromatic number is at least 5. To show that it equals 5, we just need to color the other three vertices. A little trial and error shows that we can make Math 195 the same color as (i.e., have its final exam at the same time as) CS 101; and we can make Math 115 and 116 the same color as CS473. Therefore five time slots (colors) are sufficient.