Sample Solutions on HW9 (*6 exercises in total*)

Sec. 8.5 7, 10, 18

7 We need to use the formula:

$$|P \cup F \cup C| = |P| + |F| + |C| - |P \cap F| - |P \cap C| - |F \cap C| + |P \cap F \cap C|$$

where, for example, P is the set of students who have taken a course in Pascal. Thus we have $|P \cup F \cup C|$ = 1876 + 999 + 345 – 876 – 290 – 231 + 189 = 2012. Therefore, since there are 2504 students altogether, we know that 2504 – 2012 = **492** have taken none of these courses.

10 $100 - \lfloor 100/5 \rfloor - \lfloor 100/7 \rfloor + \lfloor 100/(5 \times 7) \rfloor = 100 - 20 - 14 + 2 = 68$

18 There are C(10, 1) + C(10, 2) + · · · + C(10, 10) = 2^{10} − C(10, 0) = 1023 terms on the right-hand side of the equation.

Sec. 8.6 6, 11, 16

6 Square-free numbers are those not divisible by the square of a prime. We count them as follows:

$$99 - \lfloor 99/2^2 \rfloor - \lfloor 99/3^2 \rfloor - \lfloor 99/5^2 \rfloor - \lfloor 99/7^2 \rfloor + \lfloor 99/(2^2 3^2) \rfloor = 61$$

11 Here is one approach. Let us ignore temporarily the stipulation about the most difficult job being assigned to be the best employee (We assume that this language uniquely specifies a job and an employee). Then we are looking for the number of onto functions from the set of 7 jobs to the set of 4 employees. By Theorem 1 there are 4^7- $C(4,1)3^7 + C(4,2)2^7$ - $C(4,1)1^7$ = 8400 such functions. Now by symmetry, in exactly one fourth of those assignments should the most difficult job be given to the best employee, as opposed to one of the other three employees. Therefore the answer is 8400 / 4 = **2100**.

16 There are n! ways to make the first assignment. We can think of this first seating as assigning student n to a chair we will label n. Then the next seating must be a derangement with respect to this numbering, so there are D_n second seating possible. Therefore the answer is $n!D_n$.