In class, we wanted to show

$$\times : C_*(*) \otimes C_*(*) \longrightarrow C_*(* \times *)$$

induces an isomorphism on homology by a direct calculation of $H_*(C_*(*) \otimes C_*(*))$:

$\mathbb{Z}, 0, 0, \ldots$

From $C_*(*) = \mathbb{Z} \xleftarrow{0} \mathbb{Z} \xleftarrow{1} \mathbb{Z} \xleftarrow{0} \mathbb{Z} \xleftarrow{1} \cdots$

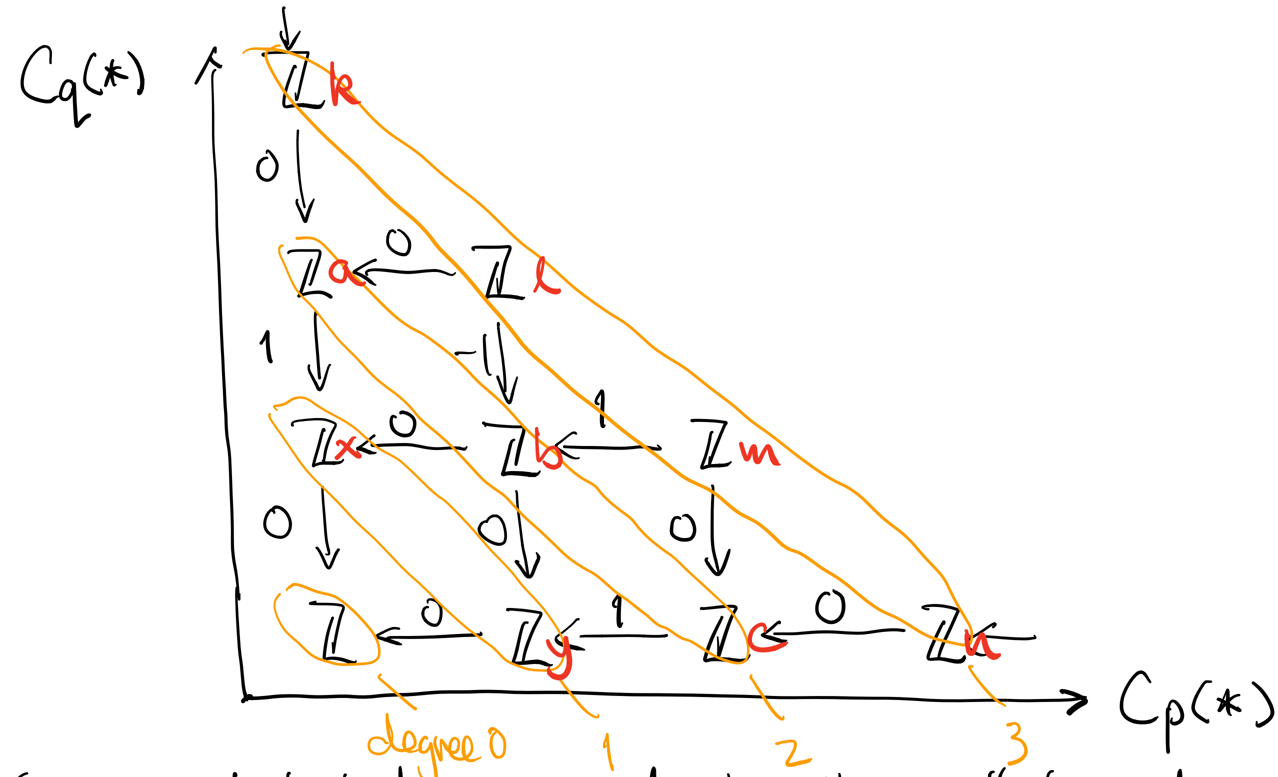
we drew

$$\begin{tikzcd}
C_q(*) & \arrow[d] \\
& \mathbb{Z} \arrow[d, "0"'] \\
& \mathbb{Z} \arrow[d, "1"'] \\
& \mathbb{Z} \arrow[d, "0"'] \\
& \mathbb{Z} & \mathbb{Z} \arrow[l, "0"'] & \mathbb{Z} \arrow[l, "1"'] & \mathbb{Z} \arrow[l, "0"'] & \arrow[l] \\
& & & & & C_p(*)
\end{tikzcd}$$

According to the boundary formula for $C_*(*) \otimes C_*(*)$

$$\partial_n = \sum_{p+q=n} \partial_p \otimes id + (-1)^p \, id \otimes \partial_q$$

we then have



(we mislabeled some of the "inner" boundary maps).

Now, as examples, we calculate:

- in degree 1,

  $\ker \partial_1 = \langle x, y \rangle$
  $\operatorname{im} \partial_2 = \langle x, y \rangle$, with $a \mapsto x$, $c \mapsto y$
  $\Rightarrow H_1 = 0$

- in degree 2,

  $\ker \partial_2 = \langle b \rangle$
  $\ker \partial_3 = \langle b \rangle$, with $-\ell \mapsto b$, $m \mapsto b$
  $\Rightarrow H_2 = 0$