French For Mathematicians:
A linguistic approach

Extended notes form a one-lecture course given at Brandeis university, in the second-year seminar

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Why should mathematicians be able to read another language?

Let us begin this course with some motivation (or propaganda, call it how you wish).

It is an advantage for a mathematician to be able to read old papers and books in their original language, especially classical papers of historical importance. Even though most of the contents of those old work have been digested and assimilated into well-crafted modern theories, that are exposed in efficient text books, there is always an element of surprise when one reads one of the papers that started it all. One discovers that many lateral ideas have been left out in the modern theories, because they did not fit so well, that methods of proofs are different from what we have learnt and can suggest new ideas of generalizations, that the motivations were unknown to us but help us make connections with other part of mathematics. Many of those books or papers have not been translated, and even when they had, often much is lost in translation.

Here are the main languages used in classical math papers and books, by periods:
1900-1990: English, German, French, Russian, Italian
1800-1900: German, French, English
1000-1800: Mostly Latin, some French, German, English, Arabic.
Before: Mostly Greek though some works are lost and exist only in Arabic or Latin translations.

Why should mathematicians be able to read French?

There are two particular reasons why learning to read mathematical works in French will be a better investment than in the other languages (assuming you already know English, of course):

1.– Nowadays, there is still a significant, though diminishing, proportion of papers, textbooks, and research books which are written in French. The proportion has gone from nearly 10% in the 1960's to 1-2% now, according to Mathscinet, but it is still much more important in the top journals. Regrettably, the production of research mathematics in any other language than English and French has essentially come to a halt (except perhaps in Chinese, but the most important papers are almost always written or immediately translated in English).

Amongst the 56 Fields medalists so far, 13 are French, 2 are French-speaking Belgians, and one more is a French-writing stateless person. Many have written or continue to write most or all their papers in French.

In certain fields, the foundational papers and books are written in French and have not been translated. Being able to read the *Éléments de Géométrie Algébrique* in French is for instance almost necessary if you want to study deeply Grothendieck-style algebraic geometry. In general, the chances that at some point of your PhD thesis, you will have to read a paper or book in French is quite high.

2.– Learning to read mathematical works in French is extremely simple, for someone who knows English. It is not really necessary to know any French grammar, though it may help. You basically need to understand vocabulary, but that is easy: most mathematical words in French are immediately recognizable by an English-speaking mathematician. More of them can also be recognized with some trainings. Only a few words (two or three dozens, mainly logical words) need to be *learnt*. This can be done very quickly.

The aim of this course

A mathematician with a good knowledge of English already knows 90% of what is needed to read a mathematical text in French. The aim of this course is to give you in a short amount of time the 10% remaining:

– some important vocabulary;

– some rules to recognize French words as cognates of English words you know, and to use it to guess their meaning. This will require an introduction to historical linguistics;

– some elements of French grammar and syntax needed to analyze complex sentences;

– the confidence that you are can read French mathematics with little effort.

Part I.

Linguistics

Conventions

In this text, I use words and sentences to talk about words and sentences. To distinguish those, instead of quotation marks, I shall adopt a color code: my discourse is written in black, while the letters, words, and sentences (in French, English and occasionally other languages) I am discoursing about are colored. I use different colors for different parts of speech: red for verbs (or verbal phrases), blue for nouns (or noun phrases), green for adjectives, and purple for all the other parts of speech. In addition, I use orange for part of words or individual letters, and brown for full sentences.

An equation of the type mot = word almost always means that the right-hand-side is an English translation of the left hand side, which is in French.

As you certainly know, languages evolve with time. They do so in many ways. The way sounds are pronounced slowly change over time until words and grammatical forms are not recognizable. Some new words are coined, or borrowed to other languages, some come out of use and eventually disappear, some see their meaning progressively or abruptly change. Grammar, morphology, and syntax are also subject to evolution.

If a language is spoken by a relatively homogeneous group with strong internal exchanges and communications, it tends to keep its unity: innovations that are adopted in some part tends to either extend to the whole domain of the language or to die. But often historical accidents cut a linguistic group in several parts with diminished or completely cut-off communications. This happens after conquests, voluntary or forced migrations, and political changes that hinder internal communications (example: the Latin-speaking world after the fall of the Western Roman Empire in the fifth century AD). In such a situation, the language will evolve into several different languages (which are said to *descend* from the original language) which over time will be so different that a speaker of one of those languages shall be unable to understand, without a special formation, the speaker of another.

Indo-European languages

Both English and French are known to descend from a language which was never written, and had been forgotten, but that has to a large extent been reconstructed by linguists. This language is the ancestor of many languages that have been or are currently spoken, including most of the languages of modern Europe (and thus also of the main languages spoken in America: Spanish, English, Portuguese, French) as well as many languages spoken in India (especially in the northern part), Pakistan, Afghanistan, and Iran. For this reason this group of languages is called the *Indo-European* group, and their common ancestor is called the *Proto-Indo-European*, abbreviated *p.i.e.*.

Today, more than 2.5 billion people on earth have as mother tongue an Indo-European language. Many more knows one as a second language. This makes the Indo-European group by far the largest family of languages.

On the next page stands a simplified family tree of the Indo-European group, found on the internet. As the author points out, dozens of languages, some extant, some not, are missing.
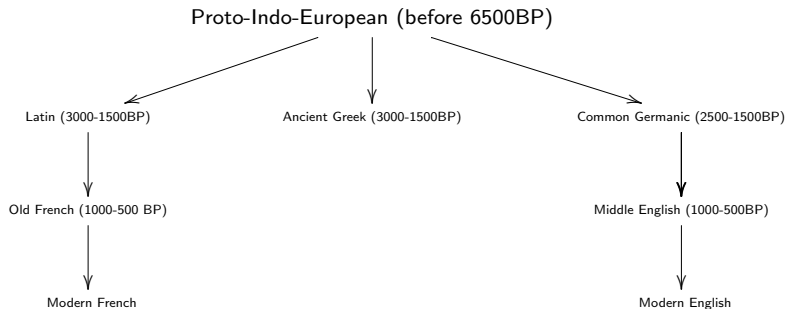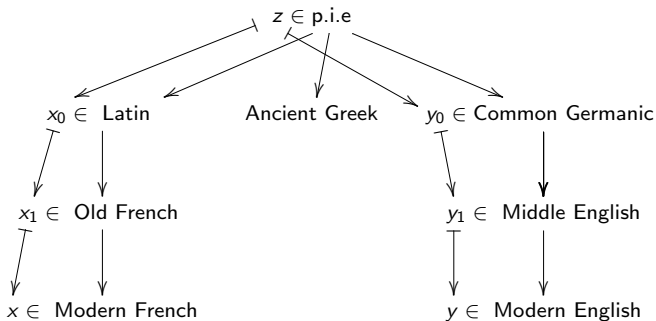
Proto-Indo-European
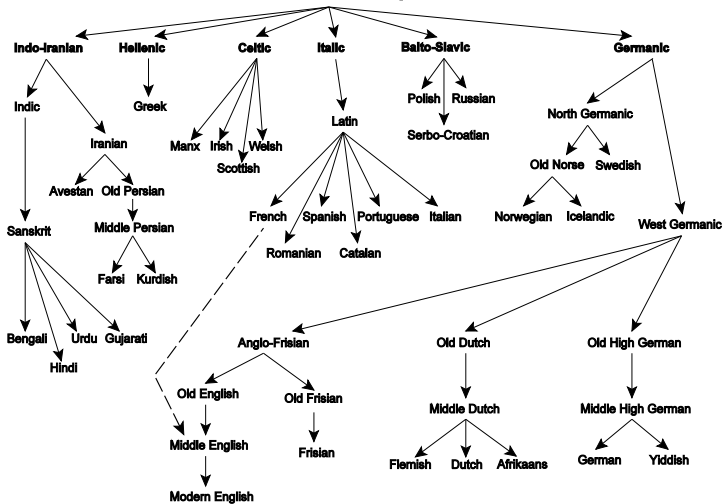
A brief history of English and French

Let us simplify it even more to restrain our attention to the few languages that are of concern for an English speaker willing to learn mathematical French.

Proto-Indo-European (before 6500BP)

Latin (3000-1500BP)   Ancient Greek (3000-1500BP)   Common Germanic (2500-1500BP)

Old French (1000-500 BP)   Middle English (1000-500BP)

Modern French   Modern English

BP means *Before Present*. The dates given must be taken with two important *caveat*. First, as languages evolve continuously, there is a lot of arbitrariness about when a language becomes another one (when exactly did ancient Greek change into modern Greek? The question, clearly, does not admit a precise answer). Moreover the dates for Proto-Indo-European are controversial. The date given is the opinion of the majority of historical linguists, and also my opinion, but some scientists defend a much higher date (before 10000BP).

When a modern English word y descends by the arrows of the preceding diagram (that is, via Common Germanic and middle English) from a word z in Proto-Indo-European, it is said to be of the *Indo-European stock*. It is estimated that this stock represents less than half of the modern English vocabulary, but many of the most basic words of the everyday language are of this stock (e.g. small numbers, parts of the body, family relation, natural phenomena, etc.). When the word z has also a descendent x in modern French by the arrows of the preceding diagram (i.e. via Latin and Old French), we say that the pair x/y is a pair of *i.e. cognates*.

Since words have experienced dramatic changes in the 6500+ years before the Proto-Indo-European times, it is often difficult to recognize cognates and certainty in these matters can only be attained by a careful study and the comparison of the many other known Indo-European languages, which is beyond the scope of this course. Moreover the meanings of the words have also changed, so the meaning of the two terms in a pair of cognates are sometimes quite different.

French/English Indo-European Cognates

Here is small list of true i.e. cognates, given without even a beginning of proof (but see the appendix below). If you have doubt regarding some of them, this is a perfectly healthy skepticism. Remember that they are separated by at least 13000 years of independent evolution.

All the numbers up to ten, e.g: un/one; deux/two; trois/three; six/six; sept/seven;....

Family: père/father; mère/mother; frère/brother; soeur/sister; ...

Parts of the body: chef/head (chef means chief or master in modern French, but the ancient meaning was head; couvre-chef means hat, for instance); arme/arm (arme only means weapon in modern French); genou/knee; pied/foot; œil/eye; nez/nose; sueur/sweat; corne/horn (in algebraic topology, an increasing sequence of simplices)...

Animals or plants (or their productions): bœuf/cow; chien/hound (old word for dog, as in Greyhound); loup/wolf; œuf/egg; nid/nest; grain/corn; Lait/Milk; ...

Various: cru/rare (for a hamburger, say); sel/salt; peuple/folk; joug/yoke (cf. Sanskrit yoga); meurtre/murder; soleil/sun; mois/moon (mois means a month in French); étoiles/stars; nuit/night; onde/water (again, the meanings are off; onde means wave in French, cf. in mathematics wavelets = ondelettes); vent/wind; cercle/wheel (cercle means circle), etc.

English words not of Indo-European stock

If words of the old Indo-European stock make only for less than half of modern English words, where do the other come from? Well, there are words of basically any origin in English, but by far the largest part of the remaining words result from two important historical processes:

First, massive borrowing from Old French during the eleventh to fifteenth century.

Second, the building of a technical, medical, scientific, and philosophical vocabulary, common to English and French (and to a lesser extent to some other modern European languages), based on Ancient Greek and Latin written words, from the sixteenth century to our days.

We shall now discuss these processes in some details.

First process: Old French to Middle English borrowings, I

After the Norman invasion of England (1066) and up to the fifteenth century, the English kings and the nobility were French-speaking. Many words were then borrowed from Old French to Middle English. We will call a descendent in Modern English of such a word *of Old French stock*, and the pair consisting of the descendent in modern French and the descendent in modern English of such a word a pair of *Old French cognates*.

Examples of such cognates abound, as the majority of words in modern English belongs to that stock. Here are some chosen among words that are used in mathematical texts: appliquer/to apply; associé/associate; choix/choice; continuer/to continue; dériver/to derivate; fini/finite; fraction/fraction; neutre/neutral; chercher/search.

It is much easier to recognize a pair of French/English cognates of Old French origin than a pair of Indo-European origin. After all, the two words are separated by less that 2000 years of evolution, rather than more than 13000 years. Moreover, while the pronunciation of the words is often quite different, the spellings tend to be much closer, because both in French and English, the spelling is conservative and tends to represent the pronunciation used centuries ago. Thus table/table are pronounced quite differently, but they are spelled exactly the same.

To recognize Old French cognates in some dubious cases, it may help to know two systematic evolutions that French experienced after the period of massive borrowings to English. Apply the inverse of these changes to a French word and you may obtain that you can recognize in English:

– in French, a group s + stop at the beginning of a word got transformed into é + stop (sometimes es + stop). Recall that a *stop* (or *occlusive*) is a consonant that you cannot pronounce for an arbitrary long time (e.g. b, p, d, k, t are stops while l, s, r, n are not). Examples: école/school; épars/sparse; étude/study; étrange/strange; espèce/specy; espace/space; écran/screen.

– In modern French a word ending in au and eau most of the time comes from a word ending in l in old French and in Latin. Often, the l is still present in other forms of the same word or in derived words. Example: anneau comes from Latin annulus (root Annul-) and a ringed spaced in French is called an espace annelé; niveau means level and those two nouns are Old French cognates. The n in niveau is a linguistic accident, a mutation (this particular type of mutation is called *dissimilation* by linguists): modern Italian still has livello.

Second process: Ancient Greek and Latin to French/English borrowings

The Renaissance (16th century in France and England) involved an acute interest in the culture of the Greek and Roman antiquity that continues until our days (or so I hope). During the same period of time, the progress of science and technics created a lot of new artifacts and concepts that needed to be named. To this aim, the modern languages of Europe borrowed words from Greek and Latin. Of course, at that time, Greek and Latin were long dead (except that the Catholic church spoke a form of Latin), but they were known from an uninterrupted tradition and from the many texts which had survived from the antiquity. We call modern English words or modern French words borrowed from ancient Greek *of Greek stock* and from Latin *of Latin stock*. We define in the obvious ways a *pair of Latin cognates* and *pair of Greek cognates*

An important fact is that during that period, the French and English scientific and cultural elites were so intertwined that among the many ways to choose a Greek or Latin world to denote a modern object or idea, French and English **always** use the same, up to trivial spelling changes.

Compare the pairs of Greek cognates: téléphone/telephone; télégraphe/telegraph; endoscopie/endoscopy; homomorphisme/homomorphism; caractère/character; and thousand of examples.

There also many pairs of Latin cognates: transformation/transformation; civilisation/civilization; foncteur/functor; adjoint/adjoint etc.

There are even a few pairs of mixed Latin-Greek cognates (that purists do not like): télévision/television; endofoncteur/endofunctor; pseudoreprésentation/pseudorepresentation.
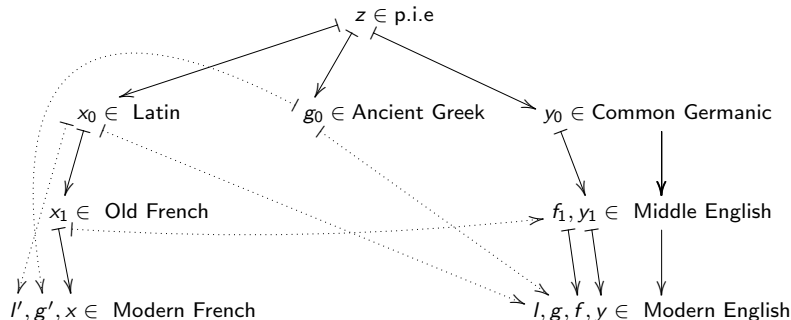
Summary Diagram

Here is a diagram summarizing what we have said above:
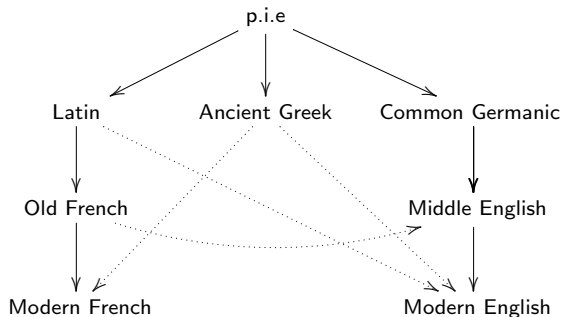
The plain arrows represent natural evolution, while the dotted arrows represent borrowing. Mathematically, an arrow from L_1 to L_2 can be thought of as a function from the set of words (or *lexis*) of L_1 to that of L_2, which *is not defined everywhere*: indeed, for a plain arrow, words sometimes disappear, and for a dotted arrow, certainly not every words of L_1 is borrowed into L_2. These functions tend to be injective on their domain though: the *merging* of two words is extremely rare.

Summary Diagram, II

Note that the diagram above is absolutely **not** commutative. In other words, a word z in Proto-Indo-European may have several descendants in modern English, one, say y, following the natural road through Germanic, Old and Middle English; one borrowed from Old French, say f; and one recently borrowed from ancient Greek, say g or from Latin, say l. This is summarized in this diagram.

$$z \in \text{p.i.e}$$

$x_0 \in$ Latin $\qquad g_0 \in$ Ancient Greek $\qquad y_0 \in$ Common Germanic

$x_1 \in$ Old French $\qquad\qquad f_1, y_1 \in$ Middle English

$l', g', x \in$ Modern French $\qquad\qquad l, g, f, y \in$ Modern English

In this diagram, with our terminology, the pairs l'/l, g'/g, x/f, x/y are respectively Latin, Greek, Old French, and Indo-European cognates.

[In real life, the distinctions between various type of cognates are sometimes more blurred than this mathematical presentation may suggest. For instance, some Old French words have sometimes been re-latinized in either modern French, modern English, or both, that is their spelling has been changed to be closer from the Latin word where they originate. When this happens, one may hesitate to classify the pair of cognates as from Old French or from Latin. For the sake of consistency below, I have chosen to consider such a situation as Old French cognates.]

Doublets in English

Let z be a word in p.i.e. With the notations of the above diagram, a subset with 2 elements of the set $\{l, g, f, y\}$ is called in Linguistics a *doublet*, and its elements are said to be *etymological twins*. Similarly one defines a *triplet*, and a *quadruplet*. Also, if x_0 is a word in Latin not of known Indo-European origin, it may have two descendants l and f in Modern English, and $\{l, f\}$ is still called a *doublet*.

The English language is full of doublets, triplets, and sometimes, quadruplets. To a speaker without knowledge of historical linguistics, they appear as groups of words that look vaguely, but inexplicably, similar in forms and meanings.

Morality: to understand fully the internal structure of his/her language, an English speaker must learn Proto-Indo-European. (This is also true for a French speaker, though if one puts aside the words of Greek origin, learning Latin is enough. Morality of the morality: speak French, it's simpler).

Here are examples:
$\{l, y\} = \{\text{verb}, \text{word}\}$.
$\{l, y\} = \{\text{young}, \text{juvenile}\}$.
$\{l, f\} = \{\text{secure}, \text{sure}\}$.
$\{g, f, y\} = \{\text{neo-}, \text{novel}, \text{new}\}$.
$\{l, g, f, y\} = \{\text{video}, \text{idea}, \text{vision}, \text{wit}\}$.

Practical conclusion

We have seen that there were three basic categories of French words (the Indo-European stock, the Greek stock and the Latin stock) and four in English (the same plus the one of Old French stocks).

Suppose you read a French word in a mathematical text that you have never seen. What do you do? Look for a cognate in English.

If your word is of Greek or Latin stock, chances are that you will immediately find an English word which is a Greek or Latin cognate of your French word, with exactly the same meaning. That's the easy case, but it is quite frequent in Mathematics.

If your French word is not of the Latin or Greek stock it is very likely of the old French stock, and then it might be a little more difficult to find a cognate in English, but fortunately you have two tries: you can try first to find an Old french cognates in English.

if this fails, or even if it succeeds, you can also try to find an Indo-European cognate in English.

When you have found one cognate, or if you're lucky, several, you have a good approximation for the meaning of the French word. In general the context will help you confirm your guess, and fix more precisely the meaning. Even better, you will be much more likely to remember the French word you had learnt this way than by looking up a dictionary or the internet.

In spoken languages, a word is a sequence of *sounds*. Most of the evolution of words results of the change of the individual sounds that compose them. Quoting wikipedia, "*sound change* can consist of the replacement of one speech sound by another, the complete loss of the affected sound, or even the introduction of a new sound in a place where there previously was none. Sound changes can be environmentally conditioned, meaning that the change only occurs in a defined sound environment, whereas in other environments the same speech sound is not affected by the change."

A crucial tenet of historical linguistic, well-confirmed directly by observation over languages we know over a long period of time (e.g. Latin and its descendants French, Italian, Spanish, Portuguese, Romanian, etc), and indirectly in many ways, is that sound change is *regular*. That is, it applies in the same way to a given sound in some given immediate sound context, deterministically, and independently of the word to which the sound belongs.

To make this *law of regularity of sound changes* more concrete, let us give one example of *sound with immediate context*, and of some regular changes that have affected it.

An example of a *sound with context* in Latin is, for instance *an s between two vowels* (that is, both immediately preceded and followed by a vowel). Then an example of change rule is: *in Latin, around the fifth century BC, an s between vowels becomes an r*. This rules is called Latin's *rhotacism*. Thus, the old Roman name Valesius (or Valesios) attested on old inscriptions became in classical Latin time Valerius, still used as Valery, Valerian, etc. Similarly, the old genitive genesis of the word whose nominative is genus has become generis (nominative plural genera), while the nominative has stayed *genus* with an s, since in that form s is not followed by a vowel. Similarly, from the old from jus (= law), where the s is still present in justice, just, have arose forms with an r, like in jury.

This example illustrates the blind and ruthless nature of sound changes: it applies each time the sound appear, not sparring proper names, tearing family of words apart, etc.

If we call L_1 the set of words and word forms of a given language at a time t_1, and L_2 the set of words of the "same" language at a later time t_2, we considered above the natural evolution of language as maps $L_1 \rightarrow L_2$ (represented by arrows in the diagrams of the preceding pages); those maps, as we noticed, were only defined on a subdomain of their source (some words disappear), and in general tend to be injective wherever defined (fusion of words are rare).

We now see that one can instead describe the natural changes as maps $S_1 \rightarrow S_2$ where S_i are the sets of sounds (with immediate contexts) of our language at time t_1 and t_2, our old map $L_1 \rightarrow L_2$ beine the one induced by $S_1 \rightarrow S_2$ in an obvious sense. What we gain from our new point f view should be obvious from the fact that typically sets like L_1 have thousands of elements, while sets like S_1 only a few dozens.

It may be interesting to note that the maps $S_1 \rightarrow S_2$ describing sound change are defined everywhere (it happens something to any sound in a language) but tend not to be injective; for instance, both s between vowels and r in ancient Latin ends up being the same r in classical Latin. However, the sound changez $S_1 \rightarrow S_2$ seem to happen in such a way that they induce injective maps $L_1 \rightarrow L_2$. See André Martinet's books for a discussion of how this may happen).

Unfortunately, things are made more complicated by other sources of changes in words. Beside borrowings that have been abundantly discussed above, other words change includes regularization (when a form of a word is changed to make it more similar to another grammatical form of the same word), false etymology, and various accidents (as in the example of French niveau discussed above).

Nevertheless, pre-composing the map $S_{\text{p.i.e.}} \to S_{\text{Modern English}}$ with the *inverse* of the map $S_{\text{Modern French}} \to S_{\text{p.i.e.}}$, one obtains a multivalued map (when one inverses a non-injective map, one gets something multi-valued) from the set of French sounds (with immediate context) to the set of English sounds, which should send a French word of i.e. stock to an English word which is its i.e. cognate.

In what follows I will describe very roughly and very imprecisely that map. Note that this maps applies to "sounds", while for the sake of practicality I will describe it on "letters", a very dangerous simplification as the correspondence between sound and letters is quite irregular both in French and English.

When seeing a written word in French, proceed as follows:

1.– Focus on the beginning of the words, essentially the first syllabi and first consonant of the second one.

2.– Apply the following changes:
ch, c > h
t > th (or d, or t, when not in initial position)
p > f (or v, when not in initial position)
qu > h or wh
g > k; d > t; b > p (or c sometimes in initial position)
f > b or d or g (especially in initial position)
s>s; l>l; m>m; n>n
r>r (or s, when between vowels)
i or j > i,y
u or v > u or w

3.– Ignore completely the vowels e, a, o, and the composed vowels (eau, au, ai, etc.) There are rules for them but they are far more complicated (beginning to understand them is one important contribution of the famous Linguist F. de Saussure). Change them into other vowels, remove them, introduce new vowels as you like.

If by doing this you find an English word you recognize, then there is chance you found an English Cognate. Use the contest to see if a meaning at least vaguely related for the French word to the meaning of its cognate makes sense.

Here is a trick to improve your chances in this game: Instead of starting with the French word, try to find the Ancient French or Latin word it comes from, and apply the same rules to it. You don't know Ancient French and Latin? Sure, you do, and better than most French people! There are so many words from these languages in English, that you basically know them. And since the sound transform rules from Latin to French are highly non-injective (basically, everything after the word accent is destroyed), it is especially helpful if you can go back all the way to Latin.

Example: say you want to understand the pairs of cognates père/father. Well, the initial p gives an f in English, the r is there, and if you neglect the vowels, everything is accounted for excepted the th in the middle of Father. But even if you not really big on Latin, you probably know that the word for father in Latin is pater, or even if you don't, you can guess it from you knowledge of English words such as paternalism. Now the t is here and "explains" the th in English.

Exercises: 1.– Consider the French adjective ténu. Find two cognates of this word in English, one form Old French and one Indo-European. Use it to guess the meaning of ténu.

2.– Are the verbs avoir and to Have cognates?

Part II.

Vocabulary

Vocabulary I: statements and proofs

Greek cognates: théorème = theorem; lemme = lemma; axiome = axiom.

Latin cognates: proposition = proposition; démonstration = demonstration; définition = definition ; scholie = scholium; postulat = postulate; conjecture = conjecture.

Old French cognates: remarque = remark; preuve = proof; esquisse = sketch; exemple = example; contre-exemple = counter-example.

How easy!

Vocabulary II: Names of mathematical fields

Greek cognates: logique = logic; mathématiques = mathematics; physique = physics; analyse = analysis; géométrie = geometry; topologie = topology; arithmétique = arithmetics.

Latin cognates: combinatoire = combinatorics; calcul différentiel = (differential) calculus; probabilités = probabilities.

Old French cognates: math. appliquées = applied math.; math. pures = pure math.

Other: algèbre = algebra (from Arabic).

Exercise. Translate: Théorie géométrique des groupes, géométrie différentielle, combinatoire énumérative.

Vocabulary III: Elementary mathematics

Greek cognates: logarithme = logarithm; polygone = polygon; pentagone = pentagon; hexagone=hexagon; heptagone=heptagon; but octogone = octagon.

Latin cognates: addition = addition; multiplication = multiplication; division = division; équation = equation; factoriser = to factor; intersection = intersection; union, réunion = union; complementaire = complement.
soustraction = subtraction (note the slight difference: the English word is borrowed directly from Latin, the French word has been modernized).
existence = existence; cercle = circle (a Greek word of the same family has also been borrowed in French and English, and is used in math: cycle = cycle; An Indo-European cognate of all these words is wheel).

Old French cognates: angle = angle ; aire = area; boule = ball; carré = square; nombre = number; somme = sum; produit = product; puissance = power; unique = unique; unicité = uniqueness ; acute = aigu; obtu = obtuse; nombre premier = prime number, prime; p.g.c.d (plus grand commun diviseur) = g.c.d; p.p.c.m (plus petit commun multiple) = l.c.m.; long = long.

Indo-European cognates: racine = root; longueur = length; court = short; droit = right (for an angle; precisely the i.e. cognate of right is Latin rectus; droit comes from di-rectus, which is the source of the pair of cognates direction/direction).

Non related: chiffre = digit (but doigt/digit are Old French cognates; doigt means finger). développer = to expand . application = function (French uses also fonction for application, especially when the target is a vector space over $\mathbb{R}$ or $\mathbb{C}$.)

Exercise. Translate: Le volume d'une boule est proportionnel au cube de son diamètre.

Vocabulary IV: Algebra

Greek cognates: caractère = character; caractéristique = characteristic (for a subgroup)

Latin Cognates: centre = center, centre (from Latin centrum, from Greek κεντρον, centron).
commutatif = commutative; associatif = associative; module = module.
représentation = representation; conjugués = conjugate

Old French Cognates: groupe = group; sous-groupe = subgroup; action = action; normal, distingué = normal (for a subgroup).

Indo-European cognates: fidèle = faithful;

Unrelated: anneau = ring. (An example of *sense borrowing*. French anneau and English, German ring are unrelated but have the same concrete sense – something you put around your finger. When, in German, Hilbert gave its mathematical meaning to ring, the sense was also imported in French and English.)
corps = field; corps gauche or algèbre a division = skew field or division algebra; anneau intègre = (integral) domain; noyau = kernel.

Exercise. Translate: "corps algébriquement clos", corps des fractions, anneau quotient, anneau gradué, groupe dérivé, abélianisé d'un groupe, classe de conjugaison.

Vocabulary V: Topology

Latin Cognates: interieur = interior; dense = dense; compact(e) = compact; connexe = connected; continu(e) = continuous ; discret, discrète = discrete; tubulaire = tubular (for a neighborhood); genre = genus; immersion = immersion.

Indo-European Cognates: plat = flat (for a connection; in Algebraic Geometry as well, for a morphism.)

Unrelated: ouvert = open; fermé = closed; voisinage = neighborhood (three examples of sense borrowing).
adhérence = closure; borné = bounded.
fort, forte = strong; faible = weak, grossière = coarse (for a topology. All these adjectives have an Old French cognate in English: force, feeble, gross).

Exercise. Translate: Partition de l'unité, complet, application propre, espace métrique, Homéomorphisme, point isolé.

Vocabulary VI: Algebraic Topology, Homological algebra

Greek Cognates: squelette = skeletton; homologie = homology; homotopie = homotopy; cycle = cycle.

Latin Cognates: suite exacte = exact sequence; complexe = complex; simplement connexe = simply connected; cobordisme = cobordism.
nerf = nerve (from Latin nervus, an Indo-European cognate of Greek νευρον, neuron)

Old French Cognates: chaîne = chain (chaîne is the French natural evolution of latin catena, also borrowed in French and English in caténaire = catenary, used in ring theory).

Indo-European cognates: corne = horn.

Unrelated: bord = boundary; cobord = coboundary.

Latin Cognates: transformation de Fourier = Fourier transform; distribution = distribution.

Old French Cognates: mesure = measure.

Unrelated: noyau = kernel (as in Poisson kernel, etc.; kernel is of the family of corn, i.e. cognate of French grain.)

Vocabulary VIII : Geometry

Greek cognates: géodésique = geodesic.

Latin Cognates: variété = varieté (in algebraic and analytic geometry)
connexion = connection; forme différentielle = differential form

Old French Cognates: courbure = curvature; courbe = curve; feuilletage = foliation;
torsion = torsion;

Indo-European cognates: noeud = knot.

Unrelated: variété = manifold (in differential geometry) (in old texts, 19th and first half
of 20th centuries, French uses multiplicité instead of variété. multiplicité is a partial Indo-European
cognates to manifold, as plier/to fold are i.e. cognates.)
champ de vecteurs = vector field (champ = field in the current language)
crochet de Lie = Lie bracket (unrelated: French crochet comes by borrowing from Old Norse
croc; bracket comes from Old French braguette = fly, zipper.)
tresse = braid (tresse comes from trois).

Vocabulary X: Probabilities

Greek Cognates: stochastique = stochastic.

Old French Cognates: martingale = martingale; biais = bias; mouvement brownien = brownian motion;

Unrelated: espérance = expectation; moyenne = average, mean; aléatoire = random;

Note the cognates: chance/chance, risque/risk (from Old French); hasard/hazard (from Arabic), but be careful that they do not have exactly the same meaning and usage in French and English: par chance = luckily; par hasard = by chance; risque = hazard; Quelles sont les chances de ... ? = What are the odds of ... ?

Vocabulary XI : Various mathematical terms

Greek Cognates graphe = graph.

Old French Cognates: ordre = order; forêt = forest; face = face
attaché(e) à = attached to; associé(e) à = associated to;

Unrelated: réseau = lattice (in group theory)
treillis = lattice (as an ordered set)
sommet = vertex; arête = edge; muni(e) de = provided with.

Vocabulary XII : conjonctions and logical words

Unfortunately, those words are very often unrelated in French and English. It is necessary to learn them.

et = and ; ou = or; mais = but; donc = therefore;
car, parce que, puisque = because; à cause de = because of; cependant, néanmoins = however;
si = if ; alors = then; ainsi = thus, this way;
ainsi que = as well as; aussi = too, as well, also;
par ailleurs = in addition, also; d'ailleurs = besides, moreover, what's more;
de plus, qui plus est = moreover.
il faut = it is necessary ; il suffit = it is sufficient.
French or is hard to translate. It is used to introduce a new fact. Can correspond to and, but, also, in addition.

Translate: On suppose de plus que la somme et le produit de deux objets quelconques A et B de **C** existent. Il suffit d'ailleurs de postuler l'existence de la somme *ou* du produit. (Grothendieck, Tohoku).

Si G est un groupe, le sous-ensemble des éléments de G qui commutent avec tout G est un sous-groupe de G, qui plus est caractéristique. On note $Z(G)$ le sous-groupe de G ainsi défini, et on l'appelle le centre de G.

Part III.

Grammar

While you do not need much French grammar to understand French mathematical writings, a few basics may sometimes be useful to decipher a complex French sentence. I will do a very brief and incomplete review of French grammar, focusing on things that might be useful to know,

The *parts of speech* in French are roughly the same as in English: Nouns, Determiners, Pronouns, Adjectives, Verbs, Adverbs, Prepositions, Conjunctions. We shall cover them one by one.

Nouns

In French as in English, a *noun* (French *nom*) is "a part of speech that denotes a person, animal, place, thing, or idea" (wikipedia). A noun in French or English has a *gender*, which is fixed, and a *number*, which may vary. As in English, there are two numbers in French, *singular* and *plural*. However, there are only two genders in French, *masculine* and *feminine*. Except for nouns that are obviously sexualized, which have the gender corresponding to their sex (e.g. homme = man is masculine, femme = women is feminine), the gender of a noun in French is essentially arbitrary. Groupe, anneau, corps, vecteur, foncteur, espace are masculine, but variété, algèbre, action, probabilité, fonction, unicité are feminine.

There is no need to learn which French words are masculine and which are feminine (except if you want to learn to speak French of course), but it may be useful to be able to recognize them on the spot, because it may be necessary to determine which pronoun refers to which noun (see below). The simplest way to do that is to look at the *determiner* that almost always precedes the noun.

Determiners, I

In French as in English, "a *determiner* is a word that occurs together with a noun (always before it) and serves to express the reference of that noun or noun phrase in the context." (Wikipedia).

Determiners in French have a variable gender and number: they take the gender and the number of the noun that they occur with.

Determiners, II

Articles: The most common and basic type of determiners are the *articles*. In French as in English, there are two kind of articles, the *definite* article and the *indefinite* article.

The forms of the definite article are: le (masculine singular), la (feminine singular), les (plural, both genders). When the following word begins by a vowel, le and la are replaced by l'. All those forms corresponds to English the.

The forms of the indefinite article are: un (masculine singular), une (feminine singular), des (plural, both genders). In English, un and une are translated by a, des is replaced by an absence of determiner.

The semantic difference between the two kind of articles is rather subtle, but easy to understand and important for a mathematician: the definite article (le, la, les) used for a noun which represents an object which is *canonically* or *uniquely* defined by the context, and the indefinite article otherwise.

Compare: Soit D le sous-groupe d'ordre 4 du groupe alterné A_4 (the word le indicates that there is only one such subgroup: this is a mathematical assertion in just one word); Soit H un sous-groupe d'ordre 4 du groupe symétrique S_4 (there are several of them); la clôture algébrique d'un corps K ou le revêtement universel d'un espace X (when you want to insist that it is unique up to isomorphism) but une clôture algébrique d'un corps K or un revêtement universel d'un espace (when it matters for you that these are defined only up to a non-canonical isomorphism).

The article gives an easy way to learn the gender of a noun that you meet in a text, at least when this noun is singular and (for the definite article) when it does not begin by a vowel.

Demonstrative Determiners: They are used for a noun which refers to an object that has already been mentioned before (or in the oral language, to an an object that you can show with your finger or otherwise indicates).

The forms are ce (masculine singular), cette (feminine singular), ces (plural, both genders).

They are translated by this or that for the singular forms, these or those for the plural forms.

Determiners, IV

Indefinite Determiners: that's a class of determinants that are used frequently in mathematics and that are very important to know.

aucun (masculine singular), aucune (feminine singular) = no. Translate: Aucun zéro non-trivial de la fonction zêta n'a jamais été trouvé, à part sur la droite critique.

un autre (masculine singular), une autre (feminine singular), d'autres, des autres (plural) = another, other.

certain (masculine singular), certaine (masculine singular), certains (masculine plural), certaines (feminine plural) = some.

chaque (singular) = any, every

tout (masculine singular), toute (masculine singular), tous les (masculine plural), toutes les (feminine plural) = every (singular), all (plural).
Toute algèbre à division finie est commutative = Toutes les algèbres à division finies sont commutatives = Every finite division algebra is commutative = All finite division algebras are commutative. Be careful: tout le, toute la (singular) means the full, the entire. Je ne donnerai pas toute la démonstration = I won't give the entire proof.

nul (masculine singular), nulle (feminine singular) = aucun, aucune.

plusieurs (plural) = several.

le même, la même, les mêmes = the same. (the word même was recently borrowed in English by the biologist Richard Dawkins as meme, defined as an element of a culture or system of behavior that may be considered to be passed from one individual to another by nongenetic means, especially imitation. cf. an internet meme)

un quelconque, une quelconque = any.

Possessive Determiner:

mon, ton, son (possessed noun masculine singular, one possessor) = my, your, his/her/its.
ma, ta, sa (possessed noun feminine singular, one possessor) = my, your, his/her/its. (Replaced by mon, ton, son if the next word begins by a vowel.)
mes, tes, ses (possessed noun plural, one possessors) = my, your, his/her/its.
notre, votre, leur (possessed noun singular, several possessors) = our, your, their
nos, vos, leurs (possessed noun plural, several possessors) = our, your, their.

The form of a possessive determiner may *a priori* depends on five things: the gender and number of the possessed object, and of the gender, the gender and number and person of the possessor (the *Persons* are three in both English and French: the speaker (first person), the person spoken to (second person), anything or anyone else (third person)). In English the form of a possessive adjective depends on the gender, the number, and the person of its possessor; the possessed object doesn't matter. In French it depends on the number and the person of its possessor, and of the gender and number of the possessed object. Weird.

Determiners, VI

Numeral Determiner:

un/une, deux, trois, quatre, etc. = one, two, three, four, etc.. Il y a trois cas à traiter = there are three cases to deal with.

premier, deuxième or second, troisième, quatrième, etc. = first, second, third, fourth, etc.; n-ième or $n^{\text{ième}}$ = n-th.
Le premier cas est le plus facile=the first case is the easiest one.

Adjectives

Adjectives are words that are used to modify, qualify, or precise a noun. In mathematics, while nouns in general represent mathematical objects, adjectives usually represent properties that those mathematical objects may or may not have.

While in English adjectives are invariable, in French their form change according to the gender and number of the noun they modify. In general (but there are many exceptions), the feminine singular form is obtained by adding e to the masculine singular form, the masculine plural form is obtained by adding s, and the feminine plural form by adding es. Compare: un espace compact, des espaces compacts, une variété compacte, des variétés compactes.

In French as in English the adjective can be used in *attributive* position (called *épitèthe* in French) as in the blue cat, or in predicative position (called *attribut* in French, confusingly), as in the cat is blue. In the sentence un sous-espace fermé et borné est compact, fermé et borné are in attributive position, while compact is in predicative position. Note that an adjective changes its form according to the noun it modifies even when it is *un attribut*. Example: Les variétés X et Y sont compactes.

As in English, but much more frequently, adjectives can be used as nouns in French. Example: Dans un espace topologique, la réunion de deux fermés est un fermé. Here the adjective fermé is used as a noun, while English will rather use closed subset.

Verbs, I

As in English, *verbs* in French are words that are used (roughly) to denote an action or a state. As in English, a same verb can take many forms depending on the *person* (first, second or third) and the *number* (singular or plural) of the *subject* (what performs the action or is in the state expressed by the verb), as well as the *mood* (in French: infinitive, indicative, subjunctive, conditional, imperative, participle), *tense* (in French: present, future, composed past, imperfect, and four others) and *voice* (active or passive – the passive exists only for verb that admits a direct object, which are called *transitive* verbs).

Learning all the forms of a verb (in French, *conjuguer le verbe*) is a hard task for anyone learning French seriously, all the more because there are many irregular verbs. Fortunately, this is not really necessary to understand a French mathematical text. For that, the following review of the various moods should be more than sufficient.

Verbs, II

The **infinitive** is the mood of the verb considered independently of its subject. In French all (present tense, active) infinitive ends up with the two letters er, ir, or re. Example: calculer = to calculate; prendre = to take.

Besides the infinitive present active, the infinitive also exists in one other tense, past, and in the passive voice – for a total of four forms. All the other forms are composed of several words including the *past-participle* of the verb (see below). Examples: avoir calculé = to have calculated, avoir pris = to have taken; être calculé = to be calculated, être pris = to be taken; avoir été calculé = to have been calculated; avoir été pris = to have been taken;

Verbs, III

The **participle** is a mood of the verb which can be used as an adjective in a sentence. Its exists in only two forms, that are called present-participle and past-participle, but which should be more logically called *active-participle* and *passive-participle*. Present participle: calculant = calculating, prenant = taking; Past participle: calculé = calculated; pris = taken.

The present participle is very often formed by replacing the last two letters (sometimes three letters) of the (present active) infinitive form of the verb by *ant* (or, for some verbs ending with ir, by issant). When used as an adjective, the present participle is often invariable. The meaning of that adjective is to indicate that the noun modified by it is performing the action (or is in the state) expressed by the verb. Example: Soit C un cercle, D une droite coupant C en deux points A et B = Let C be a circle, D a line cutting (or intersecting) C at two points A and B. The present participle can also be used with the preposition en, to form the *gerundive*. Example: En remplaçant X par son adhérence, on se ramène au cas où X est fermé, donc complet = by replacing X by its closure, one is reduced to the case when X is closed, hence complete.

The formation of the past participle is more irregular that the present participle (as in English). For most verbs ending in er at infinitive, it is formed by replacing that er by é. For many verbs ending in ir, it is formed by replacing ir by i (example: choisir, choisi = to choose, chosen.)
Examples of irregular past participles: être = to be, été; avoir = to have, had = eu; faire = to do, done = fait; dire = to say, said = dit; voir = to see, seen = vu; savoir = to know, known = su; devoir = must, past participle dû.

Verbs, IV

The **indicative** mood is the most basic mood, used for a statement or a question. It exists in the 8 possible tenses, 3 possible persons, 2 possible numbers, 2 possible voices, that is 96 forms for each verb, fortunately not all different. In a mathematical text, you will mostly meet the *present* tense of the indicative mood. Let us indicate its *conjugaison*: je calcule = I calculate; tu calcules = you calculate; il, elle, on calcule = he (or it), she (or it), one calculates; nous calculons = we calculate; vous calculez = you (guys) calculate; ils,elles calculent = they calculate. Here je, tu, il/elle/on, nous, vous, ils/elles are called *personal pronouns*, rather improperly, as it is often more exact to consider them as part of the conjugation of the verb.

Note the personal pronoun on which is used to represent any person or group of persons. It is often used in the mathematical literature. On vérifie facilement que ... = One checks easily that .... Who is checking? The author? The reader? The referee? Nobody? On conveniently allows not to choose. Fun facts: 1.– on comes from the nominative (subject case) of the latin word homen, meaning man or human being); the accusative (object form) of the same word, hominem has given French homme. 2.– on/one are Ancient French cognates; in particular, one in English has no etymological relation with the numeral one.

Here are some other present active indicative *conjugaisons*, for the irregular verbs être = to be, avoir = to have; prendre = to take:
je suis, tu es, il est, nous sommes, vous êtes, ils sont;
j'ai, tu as, il a, nous avons, vous avez, ils ont;
je prends, tu prends, il prend, nous prenons, vous prenez, ils prennent.
Note that: there is always an nt at the end of the third person plural, almost always an ons at the end of the first person plural, always an s at the end of the second person singular, and never an s at the end of the third person singular.

Other tenses used at active indicative are the future tense and the composed past tense.

The *future tense* is formed for almost all verbs as follows: one takes the infinitive present of the verb (minus *e* if the last letter is an *e*) and one adds the last syllabi of the suitable form of the present of the verb avoir: je calculerai, tu calculeras, il calculera, nous calculerons, vous calculerez, ils calculeront; je prendrai; tu prendras, il prendra, nous prendrons, vous prendrez, ils prendront. Note some exceptions: être: je serai, tu seras, etc.; avoir: j'aurai, tu auras, etc.; voir; je verrai, tu verras, etc..

The *composed-past tense* is formed of the verb avoir (or sometimes être) in the present tense followed from the past participle: j'ai calculé, tu as calculé, il a calculé, etc.; j'ai pris, tu as pris, il a pris, etc.; je suis venu = I have come (not-so-ancient English: I am come). The formation of this tense is thus exactly the same as the *present perfect* in English (this is probably due to a borrowing of the French composed-past to ancient germanic languages)

The three other moods are:

The **conditional**, used to express a statement depending on a condition that is not known to be satisfied. It is sometimes used in short *reductio ad absurdum*.

The **subjunctive** used to express a supposition, an opinion, a condition, a will. In mathematical texts, you will mainly see soit, soient, third person singular and plural present active subjunctive of the verb être.
Examples:
Soient x et y deux entiers = Let x and y be two integers.
Pour que S soit intègre, il faut et il suffit que S soit réduit et irréductible = For S to be integral, it is necessary and sufficient that S be reduced and irreducible.

The **imperative** is used to give an order or an advice. You won't find it much in mathematical texts, except perhaps in exercises.

Verbs VII: Syntax

Except when used at the infinitive or participle mood, all verbs in French have a subject, which in general is the agent of the action (or the thing being in the state) described by the verb. Even when there is no real agent of the action there is still a grammatical subject with no referent, in general il, as in il pleut = it rains. Note in particular the very frequent il y a = there is, there are. Example: Dans le groupe G, il y a quatre classes de conjugaisons.

As we noted above, not all verbs, though many of them, may admit a direct object, that is a group of words referring to the person or the thing upon which the action is performed. Those verbs are called *transitive*. Only those verbs may be put to the passive voice, and when they are, the subject of the verb is what would be the direct object if the verb was at the active voice. (Thus the usage of the passive voice is more restricted than in English, where the indirect object, see below, can also become the subject of the verb at the passive voice: I give her a book = she was given a book by me.

Some verbs in French, transitive or not, may admit an indirect object, which tell to whom or for whom the action is done. In french, it is in general introduced by the proposition à. Example : : J'ai parlé à Serre : il dit qu'il a déjà envoyé les épreuves corrigées au journal = I talked to Serre: he says he has already sent the journal the corrected proofs. In this sentence, à Serre is an indirect object of the intransitive verbe parler, and au journal is an indirect object of the transitive verb envoyer.

Prepositions

Here is a list of the most common prepositions used in French mathematical texts, with their approximate English translation. It is important to know them.

à = at, in, to; après = after; avant = before; au dessus de = above; avec = with; contre = against; dans = in, inside; d'après = according to; de = of, from; depuis = since; en = in; entre = between; environ = about; malgré = despite; par = by, through; parmi = among, amongst; pour = for; sans = without; sauf = except; selon = according to; sur = on; sous = below; vers = towards.

Note: when preposition *à* or *de* is followed by a definite article, a contraction may occur: à + le > au, à + les > aux; de + le > du; de + les > des; Also note: de + un > d'un; de + une > d'une; de + des > des.

The preposition de is used for the very important *genitive construction* in French: le centre d'un groupe = the center of a group; le théorème de Pythagore = Pytahogoras' theorem.

Pronouns, I

Pronouns are the words that are used to replace a noun or a nominal group, called the *referent*. It does not mean that this is it only function: a pronoun may also indicate a question, begin a relative clause, etc. But the fact that they replace a noun or nominal group is what make them a pronoun.

The most important pronoun is **il/elle** and its many forms. It is used only to replace a noun or a nominal group. The form of this pronoun depends of the gender and number of its referent, and of its function in the sentence (exactly like in english). These forms are:

When the pronoun is subject: *il* (masculine singular), *ell* (feminine singular), *les* (plural, both genders). When the pronoun is direct object: *le* (masculine singular), *la* (feminine singular), *les* (plural, both genders). When the pronoun is indirect object: *lui* (singular, both gender), *leur* (plural, both genders).

All those pronouns are placed before the verb, the indirect object the closest to the verb, then the direct object, then the subject.
Exemple: Il la lui donne. = He gives it to her. Il is the guy who gives (masculine), la is the given thing (of feminine gender), lui is the person to whom the thing is given (masculine or feminine, we can't know).

Pronouns, II

The pronouns **en, y**. Those two important pronouns are also placed before the verb.

The pronoun y may replace any complement indicating the place (either where one is or where one goes to). Examples: Je travaille aux États-Unis. J'y vis depuis dix ans = I work in the US. I have lived there for ten years; in this sentence *y* refers to aux États-Unis.

The pronoun en may replace any nominal groups beginning by the preposition de. This is a little bit tricky because the proposition de has many usage, the most egregious of them being in the genitive construction (see above) and to indicate the place where one comes from. Examples: in Je suis à New York depuis trois jours; j'en pars demain = I have been in New York since three days; I leave tomorrow, en = de New York. In Le lemme 2.3 est fondamental. Nous en donnons deux preuves, en = du (= de le) lemme 2.3.

Pronouns, III

The **relative pronouns** in French are used to introduce a relative clause. They replace a noun or a nominal group, which is used in the rest of the sentence, in that relative clause. In French there are two families of relative pronouns.

The relative pronouns **qui, que, où, dont** are the most used. The form chosen depends of the role the noun or nominal group replaced by the pronoun plays in the relative clause. When this nominal group is subject in the relative clause, one uses qui. When it is direct object, que. When it is a complement indicating the place (where one stands or go to), one uses où. And when it replaces a nominal group introduced by the preposition de, one uses dont (For the last two, compare to the description of y, en above.)

Examples: Un groupe qui agit librement sur un arbre n'a pas la propriété (FA). Here qui = le groupe. Une variété algébrique projective dont le fibré canonique est ample est ample est dite *de type général*. Here dont = de la variété algébrique, genitive complement of fibré canonique.

The relative pronouns **lequel, laquelle, lesquelles** are often preceded by a proposition. The chosen form depends on the gender and number of the noun the pronoun represents: lequel if the noun is masculine and singular, laquelle if it is feminine and singular, lesquelles if it is plural; the pronoun is preceded by the proposition that would precede the noun it represents in the relative clause.

Example: Une variété sur laquelle un groupe de Lie agit transitivement est dite *homogène.*

Pronouns, IV

Interrogative pronouns.

qui (subject) = who, what, which .
que, quoi (object) = who (or whom in pedantic English), what, which.

Note that the pairs qui/who and quoi/what are Indo-European cognates. This is clearer if we replace the French pronouns by their Latin origin: qui comes from Latin quo and quoi comes from Latin quod.

où = where
quand = when
pourquoi = why
de qui = whose

Possessive pronouns vary according to the same categories that the possessive adjectives.

le mien, la mienne, les miens, les miennes = mine
le tien, la tienne, les tiens, les tiennes = yours
le sien, la sienne, les siens, les siennes = his, hers
le nôtre, la nôtre, les nôtres = ours
le vôtre, la vôtre, les vôtres = yours
le leur, la leur, les leurs = their

Used without the article at the end of a letter: sincèrement vôtre = sincerely yours.

Pronouns, V

Demonstrative pronouns
celui-ci (masculine singular), celle-ci (feminine singular), ceci (for a thing, not a person, of any gender; singular) = this
ceux-ci (masculine plural), celles-ci (feminine plural) = these
celui-là (masculine singular), celle-là (feminine singular), cela (for a thing, not a person, any gender; singular) = that
ceux-là (masculine plural), celles-là (feminine plural) = those

Example: On suppose que $\dim A \geq 2$. La réduction au cas général à celui-là est triviale. Here the demonstrative pronoun celui-là means le cas où $\dim A \geq 2$.

Remark: The difference between the -ci and the -là pronouns is not very clear in French, and many people tends to employ them indifferently. There are also variants without either -ci or là, that is celui, celle, etc. but they are used only before a genitive or a relative clause. Example: Exercice: Alice and Bob misent chacun un euro, et tirent une carte. Celui qui a la plus forte carte reprend sa mise et prend celle de l'autre. Calculez l'espérance de gain d'Alice. Here celle means la mise and is followed by a genitive de l'autre; celui means Alice or Bob (whoever has the highest card) and is followed by a relative clause.

ce (c' before a vowel), ça are neutral pronouns that may represent a nominal group of any gender, but also an infinitive, or a whole sentence.
Example: Ce Qu'il Fallait Démontrer (CQFD) = Quod Erat Demonstrandum (QED).

Adverbs

Adverbs are words that serves to modify or qualify a verb or an adjective. Most adverbs in French are obtained from adjectives by adding the suffix -ment to the feminine form of the adjective. This is parallel to the English construction, where one adds the suffix ly.

Example: fidèle > fidèlement = faithfully. L'action de G sur X est fidèle = G agit fidèlement sur X. Also: un morphisme fidèlement plat, where the adverb fidèlement modifies the adjective plat.

The construction of adverbs with -ment is still *productive* in French, which means that anyone can construct a new adverb using it (from a recently coined or borrowed adjective, for instance), use it, and be understood by anyone else. Thus, fonctoriellement was coined in the 1960's and is used frequently in mathematics, but is not yet in a French dictionnary.
However, certain authors are reluctant to form a new adverb and prefers to use instead the adverbial locution de manière + adjective or de façon + adjective.

Here is a list of adverbs that do not come from adjectives and that you may meet: vite = fast; partout = everywhere; ailleurs = elsewhere; bien = well; mal = badly; environ = about, approximatively. très = very and can be used only to modify adjectives, not verbs (in both French and English); beaucoup = very much, a lot only modifies verbs.

Part IV.

Training

This part contains extracts from the mathematical literature, for practice. Because they are given without a larger context, and because they are chosen from parts of the texts that contains more words than mathematical symbols, those extracts are probably much more difficult to read than the typical mathematical French article. They are ordered roughly by increasing difficulty.

On note Δ la catégorie dont les objets sont les ensembles finis totalement ordonnés non vides, les flèches les applications croissantes au sens large. Pour $n \in \mathbb{N}$, on désigne par $d_n^i : [0, n-1] \to [0, n]$ l'injection croissante qui oublie i, et par $s_n^i : [0, n+1] \to [0, n]$ la surjection croissante qui répète i. Soit C une catégorie. Un *objet simplicial* de C est un foncteur de Δ^o dans C, un morphisme d'objets simpliciaux est un morphisme des foncteurs correspondant. (Illusie)

Pour énoncer le théorème principal nous avons besoin de quelques définitions.

Définition

Soit $\delta \geq 0$. Un espace métrique (X, d) est dit δ-hyperbolique si pour tout quadruplet (x, y, z, t) de points de X on a

$$d(x, z) + d(y, t) \leq \max\left(d(x, y) + d(z, t), d(x, t) + d(y, z)\right) + \delta. \quad (H_\delta(x, y, z, t))$$

Définition

Soit $\delta \geq 0$. Un espace métrique (X, d) est dit faiblement δ-géodésique si pour tous $x, y \in X$ et pour tout $s \in [0, d(x, y) + \delta]$ il existe $z \in X$ tel que $d(x, z) \leq s$ et $d(z, y) \leq d(x, y) - s + \delta$.

Un espace métrique (X, d) est dit hyperbolique (resp. faiblement géodésique) s'il existe $\delta \geq 0$ tel que (X, d) soit δ-hyperbolique (resp. faiblement δ-géodésique). Lorsque $x \in X$ et $r \in \mathbb{R}_+$, on note $B(x, r) = \{y \in X, d(x, y) \leq r\}$.

Définition

Un espace métrique (X, d) est dit uniformément localement fini si pour tout $r \in \mathbb{R}_+$ il existe $K \in \mathbb{N}$ tel que, pour tout $x \in X$, $B(x, r)$ contienne au plus K points.

Le théorème principal est le suivant.

Théorème

Soit G un groupe localement compact agissant de façon isométrique, continue et propre sur un espace métrique hyperbolique, faiblement géodésique et uniformément localement fini. Alors G vérifie la conjecture de Baum-Connes à coefficients, c'est-à-dire que pour toute G-C^*-algèbre A, $\mu_{red}^{G,A} : K_*^\top(G, A) \to K_*(C_{red}^*(G, A))$ est une bijection.

(V. Lafforgue)

En caractéristique zéro, les seuls corps pour lesquels la conjecture d'annulation de Beilinson–Soulé soit démontrée sont les corps de nombres ($=$ extensions finies de $\mathbb{Q}$), les corps de fonctions rationnelles d'une courbe de genre 0 sur un corps de nombres, et les limites inductives de tels corps. (Deligne-Goncharov)

Le contenu de cet exposé est le suivant : le §1 reproduit la démonstration du théorème de Lang, dans le cas p-adique; le §2 en donne une généralisation à plusieurs variables, sous certaines hypothèses de répartition. Dans les deux cas, on a besoin de variantes p-adiques du lemme de Schwartz; elles sont démontrées en Appendice. (Serre)

Les sous-groupes finis du groupes des rotations $SO_3(\mathbb{R})$ sont bien connus. [...]. On aimerait avoir une liste analogue pour d'autres groupes compacts [...]. Plutôt que de travailler dans la catégorie des groupes de Lie compacts, on préfère se placer dans celle des groupes réductifs connexes. Cela ne change rien: on sait que, si K est un groupe de Lie compact, il possède un complexifié G qui est un groupe réductif sur $\mathbb{C}$; le groupe K est un sous-groupe compact maximal de $G(\mathbb{C})$. Tous sous-groupe fini de $G(\mathbb{C})$ est conjugué à un sous-groupe de K; de plus, K "contrôle la fusion de K dans $G(\mathbb{C})$" au sens suivant: si A et B sont deux sous-groupes de K, et si $g \in G(\mathbb{C})$ est tel que $gAg^{-1} = B$, il existe un élément g_0 de K tel que $g_0ag_0^{-1} = gag^{-1}$ pour tout $a \in A$ (cela se déduit de la décomposition de Cartan de $G(\mathbb{C})$). (Serre)

Le *modèle standard* de la physique des plasmas classique est l'équation de Vlasov–Poisson–Landau, ici écrite avec des conditions aux limites périodiques et en unités adimensionnées:

$$\frac{\partial f}{\partial t} + v \cdot \nabla_x f + F[f] \cdot \nabla_v f = \frac{\log \Lambda}{2\pi\Lambda} \, Q_L(f, f), \tag{1}$$

où $f = f(t, x, v)$ est la fonction de distribution des électrons ($t \geq 0$, $v \in \mathbb{R}^3$, $x \in \mathcal{T}^3 = \mathbb{R}^3/\mathbb{Z}^3$),

$$F[f](t, x) = - \iint \nabla W(x - y) \, f(t, y, w) \, dw \, dy \tag{2}$$

est la force auto-induite, $W(x) = 1/|x|$ est le potentiel d'interaction coulombien, et Q_L est l'opérateur de collision de Landau, décrit par exemple dans ... ou ... . Le paramètre Λ est très grand, variant typiquement entre 10^2 et 10^{30}.
Sur de très grandes échelles de temps (disons $O(\Lambda/\log \Lambda)$), les phénomènes dissipatifs jouent un rôle non négligeable, et l'augmentation de l'entropie est supposée forcer la convergence (lente) vers une maxwellienne. Grâce aux progrès récents sur l'hypocoercivité, ce mécanisme est maintenant assez bien compris mathématiquement parlant, dès que l'on dispose d'estimations de régularité globale. (Mouhaut-Villani)

Paris, le 20 Septembre 1961

Cher Grothendieck,

Auslander vient de trouver une démonstration "homologique" du théorème de pureté de Nagata. Voici en gros l'idée de la démonstration :

Soit A régulier, et soit B une extension finie de A, normale, galoisienne, et non ramifiée en dehors de l'origine. On suppose que $\dim A \geq 2$ et il s'agit de prouver que B est libre (ce qui entraînera que B est non ramifiée).

[La réduction au cas général ("quasi-fini", non galoisien) à celui-là est triviale, sans même que l'on ait à passer aux complétés.]

Soit G le groupe de Galois, n son ordre. À $s \in G$ et $b \in B$, on fait correspondre l'endomorphisme $x \mapsto b \cdot s(x)$ de B. On envoie ainsi $B \times \cdots \times B$ (n fois) dans $\mathrm{Hom}_A(B, B)$; comme les deux modules sont réflexifs, et que l'application est un isom. localisée en codim. 1, c'est un isom. D'où $\mathrm{Hom}_A(B, B) = B^*$. À partir de là, c'est une question de modules ; en effet, Auslander démontre :

<u>Théorème</u>. Soit A régulier, soit M un A-module réflexif, <u>libre en dehors de l'origine</u> [inutile (récurrer)], et tel que $\mathrm{Hom}(M, M)$ soit isomorphe à M^n. Alors M est libre.
C'est trivial en dimension 2. En dimension 3, il donne une démonstration très astucieuse, que j'ai la flemme de reproduire (mais je l'ai vérifiée) ; le cas de la dimension ≥ 4 se ramène assez facilement (mais oui!) à celui de la dimension $n - 1$.

(Note que le th. ci-dessus, appliqué à un module de rang 1, redémontre que A est factoriel.)

Salut et fraternité

P-S. Je prends le bateau après-demain pour New-York. (Serre)

La motivation physique de la renormalisation est très claire et remonte aux travaux de Green au dix-neuvième siècle sur l'hydrodynamique. Pour prendre un exemple simple: si l'on calcule l'accélération initiale d'une balle de ping-pong plongée à quelques mètres sous l'eau, l'on obtient en appliquant la loi de Newton $F = ma$ et la poussée d'Archimède $F = (M - m)g$, où m est la masse inerte, et M la masse d'eau occupée, une accélération initiale de l'ordre de $11g$! En réalité, si l'on réalise l'expérience, l'accélération est de l'ordre de $2g$. En fait la présence du fluide autour de la balle oblige à corriger la valeur m de la masse inerte dans la loi de Newton et à la remplacer par une "masse effective" qui en l'occurrence vaut $m + \frac{M}{2}$. (Connes)

La théorie moderne des formes automorphes, cependant, a conduit – depuis l'introduction par Maass des formes qui portent son nom – à l'étude de nombreux objets qui, à la différence des formes modulaires classiques, n'ont qu'une relation ténue avec des objets arithmétiques. (Clozel)

Dix choses soupçonnées seulement, dont aucune (la conjecture de Hodge, disons) n'entraîne conviction, mais qui mutuellement s'éclairent et se complètent et semblent concourir à une même harmonie encore mystérieuse, acquièrent dans cette harmonie force de vision. Alors même que toutes les dix finiraient par se révéler fausses, le travail qui a abouti à cette vision provisoire n'a pas été fait en vain, et l'harmonie qu'il nous a fait entrevoir et qu'il nous a permis de pénétrer tant soit peu n'est pas une illusion, mais une réalité, nous appelant à la connaître. (Grothendieck)