MAT8201, Algebraic Topology

Assignment 2

Due in-class on Tuesday, March 2

Numbered exercises are from Hatcher's "Algebraic Topology."

1. Compute the simplicial homology groups $H_n(\mathbb{RP}^2;\mathbb{Z})$ using the Δ-complex structure given in class.
2. Hatcher, Exercise 4 on page 131.
3. Suppose $f\colon A \to B$ and $g\colon B \to C$ are homomorphisms of abelian groups. Show that there is an exact sequence

$$0 \to \ker(f) \to \ker(gf) \to \ker(g) \to \operatorname{coker}(f) \to \operatorname{coker}(gf) \to \operatorname{coker}(g) \to 0$$

4. In class, we defined subdivision maps $s_n^i : \Delta[n+1] \to \Delta[n] \times [0,1]$ for $0 \le i \le n$ by

$$s_n^i(t_1,\ldots,t_{n+1}) = \big((t_1,\ldots,\widehat{t_{i+1}},\ldots,t_{n+1}),t_{i+1}\big)$$

Show that these satisfy the relations

- $s_n^i d_{n+1}^j = \begin{cases} (d_n^{j-1},\mathrm{id}) \circ s_{n-1}^i & \text{if } i < j-1 \\ (d_n^j,\mathrm{id}) \circ s_{n-1}^{i-1} & \text{if } i > j \end{cases}$
- $s_n^0 d_{n+1}^0 = i_0$
- $s_n^n d_{n+1}^0 = i_1$
- $s_n^{i-1} d_{n+1}^i = s_n^i d_{n+1}^i$ for $i \ge 1$

Use this to show that the operator $h\colon C_n(X) \to C_{n+1}(X \times [0,1])$ given by

$$h\left(\sum a_\sigma \sigma\right) = \sum a_\sigma \sum_{i=0}^{n} (-1)^i (\sigma \circ \mathrm{id}) \circ s_n^i$$

satisfies $\partial\, h(x) + h\,\partial\sigma(x) = i_0(x) - i_1(x)$.