MAT7064, Topics in Geometry and Topology

Assignment 8

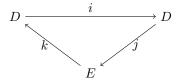
Due in-class on Friday, November 29

One way that spectral sequences arise is through what is called an *exact couple*. It arises algebraically in the following situation (which is a simplification of one we would actually use).

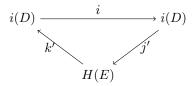
Suppose that we have abelian groups D and E, and maps $i:D\to D$, $j:D\to E$, and $k:E\to D$ such that

$$\cdots \to D \to D \to E \to D \to D \to E \to \cdots$$

is exact. We often write this in a triangle:



- 1. Show that the map d = jk satisfies $d^2 = 0$, and so we get a homology group $H(E) = \ker(d)/\operatorname{im}(d)$.
- 2. Let $i(D) = \operatorname{im}(i)$. Show that the map k induces a well-defined map $k' \colon H(E) \to i(D)$.
- 3. Show that the description $j'(x) = ji^{-1}(x)$ gives us a well-defined map $i(D) \to H(E)$.
- 4. Show that these maps give us a new exact couple:



In other words, show that these three maps give a new long exact sequence. This is called the *derived* exact couple, and this procedure can be iterated again, and again, and again...