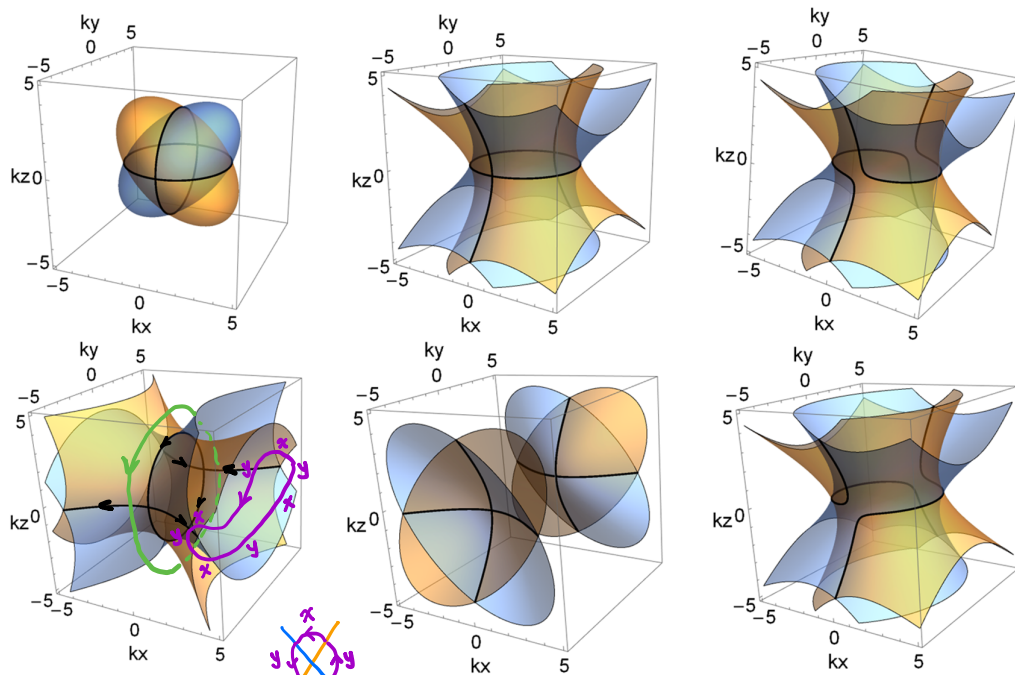
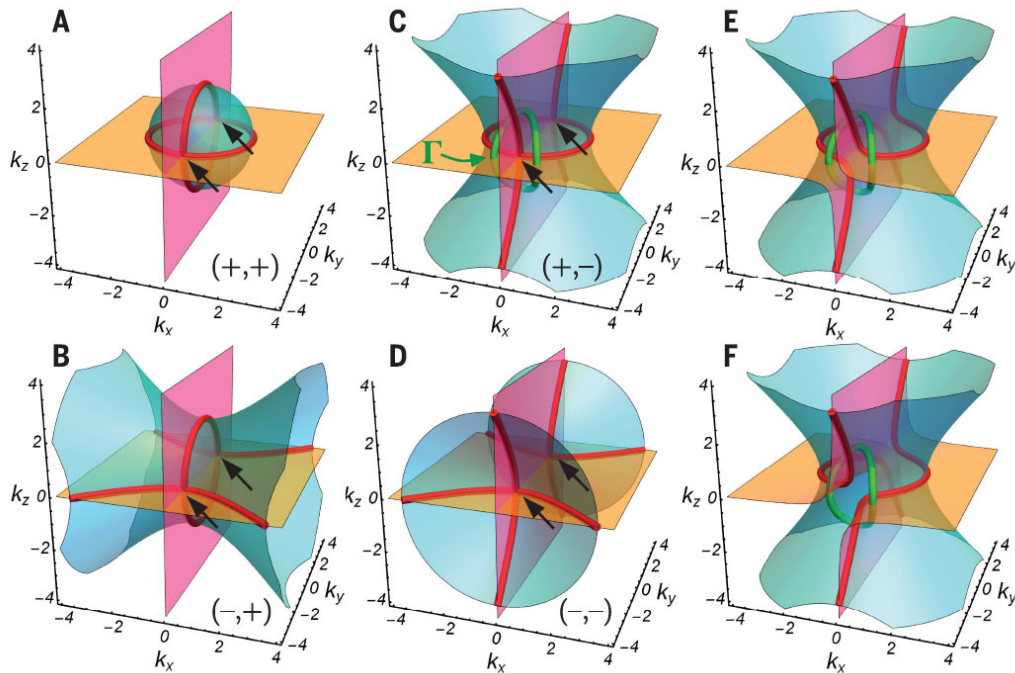


SO(2,1)

The following figures show EP surfaces in two band models

The two band model is nonhermitian and takes the form  $H = f_y i\sigma_y + f_z \sigma_z$ , which is a little different from the hermitian Hamiltonian  $H' = h_x \sigma_x + h_z \sigma_z$ . Here we set the functions  $f_y$  and  $f_z$  according to the form of  $h_x$  and  $h_z$  in the science paper  $f_y = h_x, f_z = h_z$ , and plot the EP surfaces and their intersections



$$y \cdot \bar{x} \cdot y^{-1} \cdot \bar{x}^{-1} \cdot y \cdot x = y \oplus x^{-1}$$

Comparison: The nodal lines (labelled by red curves) in the first figure (from science paper) corresponds to the non-defective intersection lines (labelled by black lines) in the second paper. They are located at the same positions in k-space (i.e.  $h_x = h_z = 0$  or  $f_y = f_z = 0$ ). The pink and yellow surfaces denote  $h_x = 0$ , and green surfaces denote  $h_z = 0$ . It is notable that in the Hermitian Hamiltonian, these surfaces are located in the gap, and there are no degeneracies on the surfaces. In the second figure, the yellow and blue surfaces corresponds to  $f_y = f_z$  and  $f_y = -f_z$ , which are EP surfaces and are degeneracies. Two eigenstates coalesce on the surfaces. The black curves are intersections of these EP surfaces, which are nondefective and have two linearly independent eigenstates.