

# MAT8021, Algebraic Topology

## Assignment 7

Due in-class on Tuesday, April 6

Numbered exercises are from Hatcher's "Algebraic Topology."

1. Find all  $(2, 3)$ -shuffles  $\alpha$  and give formulas for the associated shuffle maps  $f_\alpha: \Delta[5] \rightarrow \Delta[2] \times \Delta[3]$ .
2. Find recursive formulas for  $\dim_{\mathbb{Z}/2} H_k((\mathbb{RP}^2)^n; \mathbb{Z}/2)$  in terms of  $k$  and  $n$ .
3. Find a pair of chain complexes  $C_*$  and  $D_*$  such that the tensor product chain complex  $C_* \otimes D_*$  does not satisfy the Künneth formula, i.e., there is some  $n$  such that

$$H_n(C_* \otimes D_*) \not\cong \bigoplus_{p+q=n} H_p(C_*) \otimes H_q(D_*) \oplus \bigoplus_{p+q=n-1} (H_p(C_*), H_q(D_*))$$

4. Suppose  $G$  is a topological group and  $X$  is a topological space with a continuous map  $G \times X \rightarrow X$  which is an action of  $G$ . Show that  $H_*(X)$  becomes a left module over the Pontrjagin ring  $H_*(G)$ .
5. Find the homology of the complex Grassmannian  $\text{Gr}_{\mathbb{C}}(3, 5)$ .
6. There is a continuous map from one Grassmannian  $\text{Gr}(k, n)$  to the next  $\text{Gr}(k, n+1)$  by sending a plane  $V \subset \mathbb{R}^n$  to the plane

$$\{(0, x_1, \dots, x_n) \mid (x_1, \dots, x_n) \in V\}$$

Show that the image consists of a union of Schubert cells, and find the dimension of the smallest cell not in the image.