

# MAT8021, Algebraic Topology

## Assignment 13

Due in-class on Tuesday, May 25

1. Show that if a principal bundle  $P \rightarrow B$  has a section, then there is a homeomorphism to the trivial principal bundle:  $P \cong B \times G$  as right  $G$ -spaces.
2. Let  $G$  and  $H$  be topological groups. Suppose  $P_1 \rightarrow B$  is a principal  $G$ -bundle and  $P_2 \rightarrow B$  is a principal  $H$ -bundle. Show that the pullback  $P_1 \times_B P_2$  is a principal  $G \times H$ -bundle.
3. Suppose  $G \rightarrow H$  is a homomorphism of topological groups, and  $P \rightarrow B$  is a principal  $G$ -bundle. Show that the mixing construction gives a *principal*  $H$ -bundle  $P \times_G H \rightarrow B$ .
4. We identified  $\mathbb{CP}^1$  with the space of lines in  $\mathbb{C}^2$ . Associated to this, there is a vector bundle  $\xi \rightarrow \mathbb{CP}^1$ :

$$\xi = \{(L, v) | L \in \mathbb{CP}^1, v \in L\}$$

Find an open cover  $\{U_\alpha\}$  together with transition functions  $\{h_{\alpha,\beta}: U_\alpha \cap U_\beta \rightarrow \mathrm{GL}_1(\mathbb{C})\}$  to reconstruct the associated principal  $\mathrm{GL}_1(\mathbb{C})$ -bundle.