Algebraic topology is qualitative analysis of spaces:

solution sets $\{x^2+y^2+z^2=1\}$

$\downarrow$

manifolds.

$\downarrow$

topological spaces

Want to do:

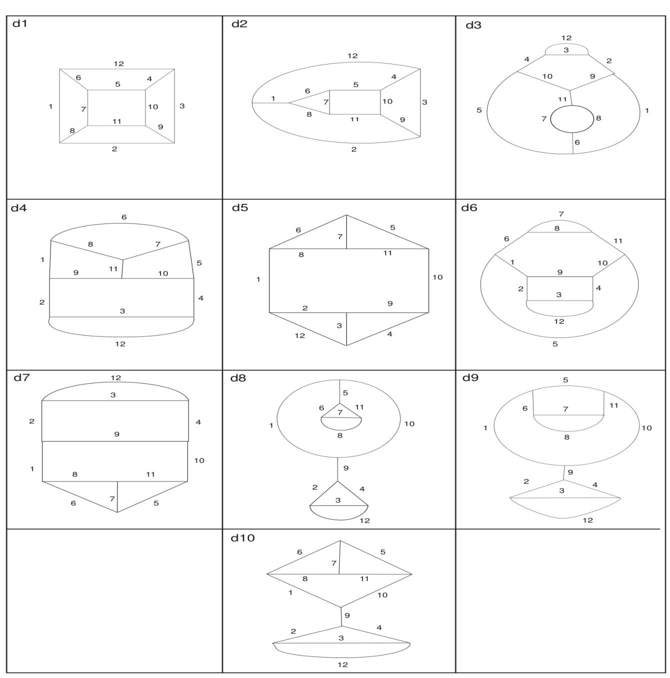
– Have machinery that gives us general info about spaces that is insensitive to collapse, and various other things.

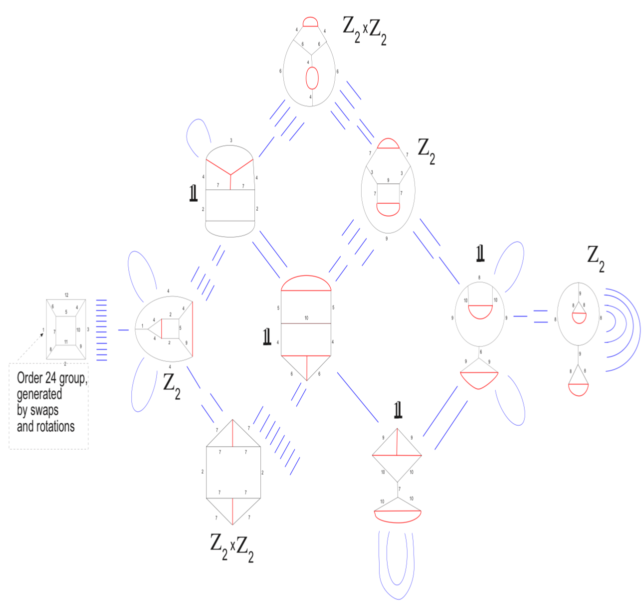
We also need to decide on a family of topological spaces that we are interested in.

Cell complexes and CW complexes

Motivations • combinatorial cell decompositions for moduli spaces

CELL DECOMPOSITIONS OF THE TEICHMÜLLER AND MODULI SPACES OF CLOSED RIEMANN SURFACES OF genus two





Rodado Amaris

- homotopy groups of spheres

Ex (degree maps of spheres)

$$\underbrace{S^k \xrightarrow{p} S^k} \longrightarrow S^k/p =: M(p), \text{ a mod-}p \text{ Moore space}$$

$$= S^k \cup_p e^{k+1}$$

$$\underbrace{S^1 \xrightarrow{2} S^1} \longrightarrow \mathbb{R}P^2$$

inducing isomorphisms in $H^*(-;\mathbb{Q})$

"S^k admits v_0-self maps."

$v_0 = p$ primes

Ex (Adams 1966 J(X), IV; Barratt 1963 unpublished)

Let $d = \begin{cases} 2p-2 & \text{if } p \text{ odd} \\ 8 & \text{if } p=2 \end{cases}$

Then there is a map

$$v : \Sigma^d M(p) \longrightarrow M(p)$$

inducing an isomorphism in K-theory. Hence all iterates of this v are nontrivial.

"$M(p)$ admits v_1-self maps."

Note The stable composite

pinch to top cell

$$S^d \longrightarrow \Sigma^d M(p) \xrightarrow{v} M(p) \longrightarrow S^1$$

inclusion of bottom cell

the Moore spectrum with bottom cell in dimension 0

is α_1 (the first element of order p in π_*^s) for p odd and 8σ (σ is the generator of π_7^s) for $p=2$.

- refined / realistic structures in math modeling

shape of networks :

SIAM REVIEW
Vol. 62, No. 2, pp. 353–391

Random Walks on Simplicial Complexes and the Normalized Hodge 1-Laplacian*

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Paul Horn§
Gabor Lippner¶
Ali Jadbabaie||

Abstract. Using graphs to model pairwise relationships between entities is a ubiquitous framework for studying complex systems and data. Simplicial complexes extend this dyadic model of graphs to polyadic relationships and have emerged as a model for multinode relationships occurring in many complex systems. For instance, biological interactions occur between sets of molecules and communication systems include group messages that are not pairwise interactions. While Laplacian dynamics have been intensely studied for graphs, corresponding notions of Laplacian dynamics beyond the node-space have so far remained largely unexplored for simplicial complexes. In particular, diffusion processes such as random walks and their relationship to the graph Laplacian—which underpin many methods of network analysis, including centrality measures, community detection, and contagion models—lack a proper correspondence for general simplicial complexes.

Focusing on coupling between edges, we generalize the relationship between the normalized graph Laplacian and random walks on graphs by devising an appropriate normalization for the Hodge Laplacian—the generalization of the graph Laplacian for simplicial complexes—and relate this to a random walk on edges. Importantly, these random walks are intimately connected to the topology of the simplicial complex, just as random walks on graphs are related to the topology of the graph. This serves as a foundational step toward incorporating Laplacian-based analytics for higher-order interactions. We demonstrate how to use these dynamics for data analytics that extract information about the edge-space of a simplicial complex that complements and extends graph-based analysis. Specifically, we use our normalized Hodge Laplacian to derive spectral embeddings for examining trajectory data of ocean drifters near Madagascar and also develop a generalization of personalized PageRank for the edge-space of simplicial complexes to analyze a book copurchasing dataset.

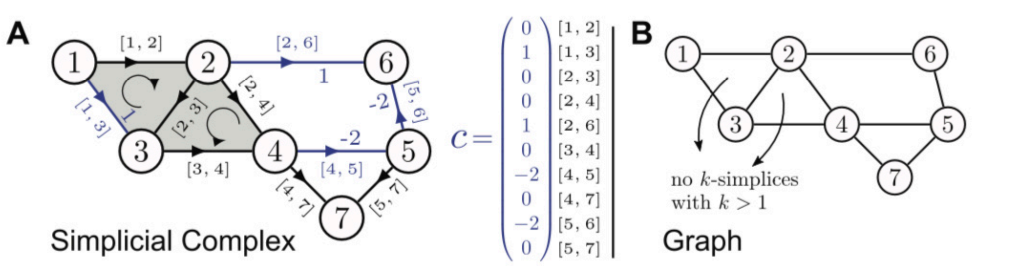
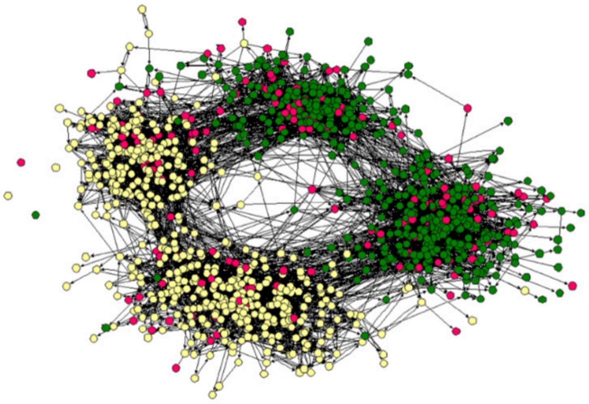
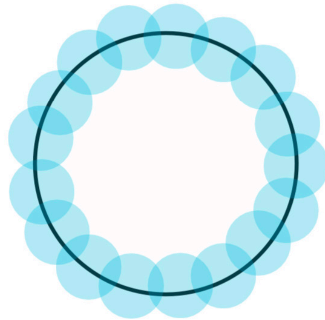
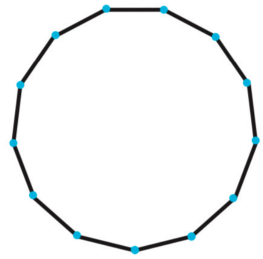


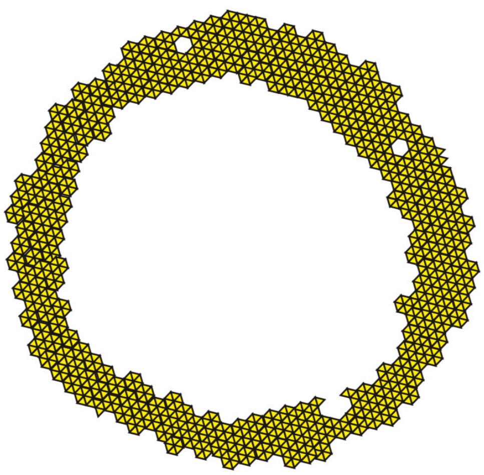
Fig. 2.1 Simplicial complexes and graphs. (A) *Schematic of an SC with a prescribed orientation. This is the running example of an SC in the text. Shaded areas correspond to the 2-simplices* $\{1,2,3\}$ *and* $\{2,3,4\}$. *An edge flow* c *along the paths* $2 \to 6 \to 5 \to 4$ *and* $1 \to 3$ *as well as its corresponding vector representation are depicted in blue.* (B) *Schematic of a graph, corresponding to the 1-skeleton of the SC in* (A). *There are no* k*-simplices with* $k > 1$ *in the graph.*

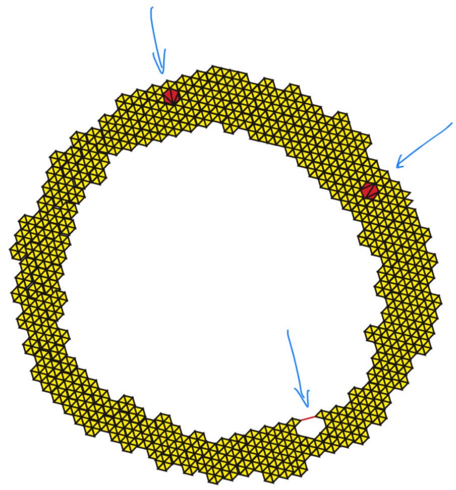
shape of data:

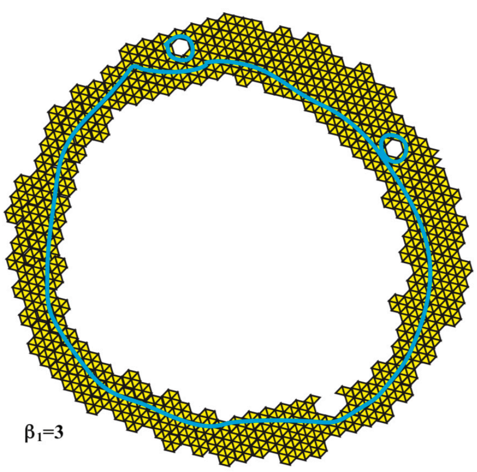




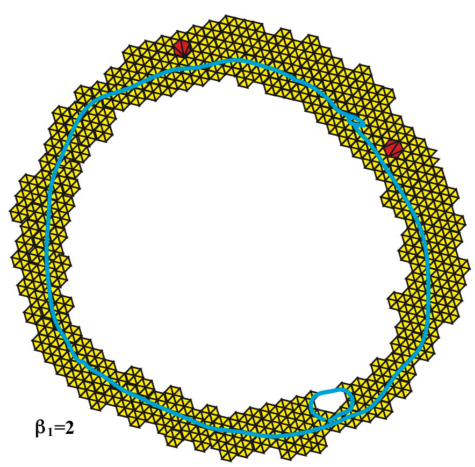








$\beta_1=3$



$\beta_1=2$

Carlsson