MA341, Applied and Computational Topology

Assignment 2

Due in-class on Tuesday, November 18

Numbered exercises are from Edelsbrunner and Harer's "Computational topology: An introduction."

1. Homology groups are a fundamental class of topological invariants and provide a powerful tool to classify surfaces. In this exercise, you are required to use `Gudhi` or `scikit-tda` to compute the homology groups for two surfaces: the sphere S^2 and the torus T^2.

   Please follow the steps below and provide a report containing the results.

   (a) Generate two point clouds sampled from a sphere and a torus respectively and visualize them. You may uniformly sample one using the parametric equations, or obtain the datasets directly from here.

   (b) Construct the corresponding simplicial complexes (e.g., Vietoris–Rips complexes) and visualize them.

   (c) Compute the persistent homology of these simplicial complexes and plot the persistence diagrams (or barcodes) for dimensions 0, 1, and 2.

   Note: You are encouraged to explore the homology groups of other topological manifolds by employing computational tools. However, pay attention to the coefficients used for computing homology. Most software packages such as `Gudhi` and `Ripser` compute homology with $\mathbb{Z}/2$-coefficients by default. For example, over $\mathbb{Z}/2$, the torus and the Klein bottle have the same Betti numbers and therefore cannot be distinguished.

2. Robustness is one of the major advantages of topological methods for data science. Generate datasets for the sphere and torus with varied levels of noise (you may refer to the code provided below), and plot the persistence diagrams (or barcodes) following the same steps above.

```python
import numpy as np
P_clean = ... # P_clean: Your N x d point cloud numpy array. N is the number of points and d is the dimension of the points.
sigma = 0.01 # variance of the Gaussian noise
N_noise = np.random.normal(loc=0.0, scale=sigma, size=P_clean.shape)
P_noisy = P_clean + N_noise
```

   Compare them with those obtained in Exercise 1, provide a brief analysis of your observations on the changes in the persistence diagrams (or barcodes) before and after adding noise, and explain the robustness of the topological method.

3. Edelsbrunner–Harer, Exercise 2 on page 74.

4. Edelsbrunner–Harer, Exercise 3 on page 74.

5. Edelsbrunner–Harer, Exercise 7 on page 74.