Zhen Huan

University of Illinois at Urbana-Champaign

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Overview

Plan.

- The construction of quasi-elliptic cohomology
- The power operation
- The orthogonal *G*-spectra

An old idea of Witten

[Landweber]

The elliptic cohomology of a space X is related to the \mathbb{T} -equivariant K-theory of $LX=\mathbb{C}^{\infty}(S^1,X)$ with the circle \mathbb{T} acting on LX by rotating loops.

It's surprisingly difficult to make this precise.

Why?

In application, one needs to consider the case that a group G acts on X. In this case the loop space LX has rich structures as an orbifold.

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Bibundles \sim "bimodules" in geometry

Bibundles combine several widely used notions, including smooth maps, Lie homomorphisms, and principal bundles.

A bibundle from \mathbb{H} to \mathbb{G}

Schommer-Pries] [Lerman

- a smooth manifold P together with
 - the structure maps:
 - $\tau: P \longrightarrow \mathbb{G}_0$:

- a surjective submersion $\sigma: P \longrightarrow \mathbb{H}_0$.
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 - $\mathbb{G}_{1_s} \times_{\tau} P \longrightarrow P$;

$$\bullet \ P_{\sigma} \times_{t} \mathbb{H}_{1} \longrightarrow P$$

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$$g_1 \cdot (g_2 \cdot p) = (g_1 g_2) \cdot p$$
; • $(p \cdot h_1) \cdot h_2 = p \cdot (h_1 h_2)$; • $g \cdot (p \cdot h) = (g \cdot p) \cdot h$

- $p \cdot u_H(\sigma(p)) = p$ and $u_G(\tau(p)) \cdot p = p$ for all $p \in P$.
- $\bullet \ \mathbb{G}_{1_S} \times_{_T} P \longrightarrow P_{_{\sigma}} \times_{_{\sigma}} P \qquad \qquad (g,p) \mapsto (g \cdot p,p) \text{ is an isomorphism}.$

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The Loop Space of Interest

Example $(Loop(X//G) := Bibun(S^1//*, X//G))$

Objects:

$$\mathcal{P} := \{ S^1 \stackrel{\pi}{\longleftarrow} P \stackrel{f}{\longrightarrow} X \}$$

• Morphisms:

$$S^{1} \stackrel{\pi}{\longleftarrow} P \stackrel{f}{\longrightarrow} X$$

$$\downarrow^{\alpha} \qquad \downarrow^{\alpha} \qquad \downarrow^{f'} \qquad \downarrow^{\rho'}$$

Example $(Loop^{ext}(X//G))$



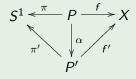
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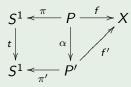
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The isotropy groups in $Loop^{ext}(X//G)$ may be infinite dimensional topological groups when G is not finite.

the subgroupoid $\Lambda(X//G)$ instead

$$\Lambda(X//G) := \coprod_{g \in G_{conj}^{tors}} X^g //\Lambda_G(g)$$

 G_{coni}^{tors} : a set of representatives of G-conjugacy classes in G^{tors} ;

$$\Lambda_G(g) = C_G(g) \times \mathbb{R}/\langle (g,-1) \rangle$$

QEII as equivariant K—theories

$$QEII_G(X) \cong \prod_{g \in G_{conj}^{tors}} K_{\Lambda_G(g)}(X^g)$$

$$QEll_G^*(X) \otimes_{\mathbb{Z}[q^{\pm}]} \mathbb{Z}((q)) \cong K_{Tate}^*(X//G).$$

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Quasi-elliptic cohomology has power operations, which gives it the structure of an " H_{∞} -ring theory" [Ganter 06].

Atiyah's Power Operation

[Ganter

V: a vector bundle over $\Lambda(X//G)$.

 $P_n(V) := V^{\otimes_{\mathbb{Z}[q^{\pm}]}^n}$ defines an operation

$$P_n: QEII_G(X) \longrightarrow QEII_{G\wr \Sigma_n}(X^{\times n})$$

$$\begin{split} \mathbb{P}_{n} &= \prod_{(\underline{g},\sigma) \in (G \wr \Sigma_{n})^{tors}_{conj}} \mathbb{P}_{(\underline{g},\sigma)} : \\ & QEII_{G}(X) \longrightarrow QEII_{G \wr \Sigma_{n}}(X^{\times n}) = \prod_{(\underline{g},\sigma) \in (G \wr \Sigma_{n})^{tors}_{conj}} K_{\Lambda_{G \wr \Sigma_{n}}(\underline{g},\sigma)}((X^{\times n})^{(\underline{g},\sigma)}) \\ & \mathbb{P}_{(\underline{g},\sigma)} : QEII_{G}(X) \stackrel{U^{*}}{\longrightarrow} K_{orb}(\Lambda_{(\underline{g},\sigma)}(X)) \stackrel{() \land}{\longrightarrow} K_{orb}(\Lambda_{(\underline{g},\sigma)}^{var}(X)) \end{split}$$

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$$QEII_G^*(X) = K_{\mathbb{T}}^*(X)$$
. For each $\sigma \in \Sigma_n$, $\mathbb{P}_{(\underline{1},\sigma)}(x) = \boxtimes_k \boxtimes_{(i_1,\cdots i_k)} (x)_k$. When $n = 2$,

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When
$$n = 3$$
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$$\begin{array}{l} \overline{P}_N: \mathit{QEII}_G(X) \xrightarrow{\mathbb{P}_N} \mathit{QEII}_{G\wr \Sigma_N}(X^{\times N}) \xrightarrow{\mathit{res}} \mathit{QEII}_{G\times \Sigma_N}(X^{\times N}) \xrightarrow{\mathit{diag}^*} \\ \mathit{QEII}_{G\times \Sigma_N}(X) \cong \mathit{QEII}_G(X) \otimes_{\mathbb{Z}[q^\pm]} \mathit{QEII}_{\Sigma_N}(\mathsf{pt}) \longrightarrow \\ \mathit{QEII}_G(X) \otimes_{\mathbb{Z}[q^\pm]} \mathit{QEII}_{\Sigma_N}(\mathsf{pt}) / \mathcal{I}_{tr}^{\mathit{QEII}} \end{array}$$

- analogous to the Adams operations of equivariant K-theories.
- but different and new.

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$$\begin{array}{l} \overline{P}_N: \mathit{QEII}_G(X) \stackrel{\mathbb{P}_N}{\longrightarrow} \mathit{QEII}_{G \wr \Sigma_N}(X^{\times N}) \stackrel{\mathit{res}}{\longrightarrow} \mathit{QEII}_{G \times \Sigma_N}(X^{\times N}) \stackrel{\mathit{diag}^*}{\longrightarrow} \\ \mathit{QEII}_{G \times \Sigma_N}(X) \cong \mathit{QEII}_G(X) \otimes_{\mathbb{Z}[q^\pm]} \mathit{QEII}_{\Sigma_N}(\mathsf{pt}) \longrightarrow \\ \mathit{QEII}_G(X) \otimes_{\mathbb{Z}[q^\pm]} \mathit{QEII}_{\Sigma_N}(\mathsf{pt}) / \mathcal{I}_{tr}^{\mathit{QEII}} \end{array}$$

- analogous to the Adams operations of equivariant K-theories.
- but different and new.

Finite Subgroups of Tate Curve

Theorem (Huan)

$$\textit{QEII}(\textit{pt}//\Sigma_\textit{N})/\mathcal{I}^\textit{QEII}_{tr} \cong \prod_{\textit{N}=\textit{de}} \mathbb{Z}[\textit{q}^{\pm}][\textit{q}'^{\pm}]/\langle \textit{q}^{\textit{d}} - \textit{q}'^{\textit{e}} \rangle,$$

where \mathcal{I}^{QEII}_{tr} is the transfer ideal and q' is the image of q under the power operation \mathbb{P}_N . The product goes over all the ordered pairs of positive integers (d,e) such that N=de.

Theorem (Huan)

The Tate K-theory of symmetric groups modulo a certain transfer ideal classifies finite subgroups of the Tate curve.

$$K_{Tate}(pt//\Sigma_N)/I_{tr}^{Tate} \cong \prod_{N=de} \mathbb{Z}((q))[q_s'^{\pm}]/\langle q^d - q_s'^e \rangle,$$

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Goerss-Hopkins-Miller theorem constructs many example of E_{∞} -rings which represent elliptic cohomology theories, including Tate K-theory.

Question

Can we construct $E_{\infty} - G$ —Spectrum which represents equivariant elliptic cohomology theory (e.g. G—equivariant Tate K-theory)?

Orthogonal G-spectra of Quasi-elliptic cohomology

Huan

We construct a commutative \mathcal{I}_G -FSP $(E(G,-),\eta,\mu)$. For each faithful G-representation V, E(G,V) weakly represents $QEll_G^V(-)$ in the sense $\pi_k(E(G,V)) = QEll_G^V(S^k)$, for each k.

Can E(G, -) arise from an orthogonal spectrum?

No.

For a trivial G-representation V, the G-action on E(G, V) is not trivial.

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Global Homotopy Theory

[Schwede][May]

Observation: It has been noticed since the beginnings of equivariant homotopy theory that certain theories naturally exist not just for a particular group, but in a uniform way for all groups in a specific class.

 \Rightarrow global homotopy theory

Prominent examples: equivariant stable homotopy, equivariant K-theory, equivariant bordism.

Almost Global Homotopy Theory

[Huan

- an extension of global homotopy theory;
- classifies those theories that are almost "global";
- the restriction maps are equivariant weak equivalence.

We can define global quasi-elliptic cohomology.

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Combining the orthogonal G-spectra $\{E(G, -)\}$, we get an ultra-commutative global ring spectrum in the new theory.

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Model Structure on the almost global spaces

We formulate several model structures and are formulating the one below.

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Thank you.

Some references

http://www.math.uiuc.edu/~huan2/Zhen-AMS-2017-Slides.pdf

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