RESEARCH STATEMENT

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1. Introduction

Even periodic multiplicative generalized cohomology theories are largely controlled by their formal group law. There are three types of formal group laws which come from 1-dimensional algebraic groups:

• the additive group structure corresponding to standard integral Eilenberg-MacLane cohomology;

• the multiplicative group corresponding to complex K-theory

• the formal group law on elliptic curve.

An elliptic cohomology theory is an even periodic multiplicative generalized cohomology theory corresponding to an elliptic curve.

The relation between elliptic cohomology and complex K-theory is close and subtle. The generalized cohomology theory over the Tate curve, by the Goerss-Hopkins-Miller theorem, is Tate K-theory. It can be derived from quasi-elliptic cohomology, the key theory we are studying, which can be expressed by equivariant K-theories in a neat form. It was first introduced by Nora Ganter inspired by Devoto's equivariant Tate K-theory [9].

It is an old idea of Witten, as shown in [20], that the elliptic cohomology of a space X is related to the $\mathbb{T}-$equivariant K-theory of the free loop space $LX = \mathbb{C}^\infty(S^1, X)$ with the circle $\mathbb{T}$ acting on LX by rotating loops. It is surprisingly difficult to make this precise. $QEll$ makes the relationship between Tate K-theory and the loop space explicit.

If G is a Lie group and X is a manifold with a smooth $G-$action, the space of smooth unbased loops in the orbifold $X/\!\!/G$ in principle carries a lot of structure: on the one hand, it includes loops represented by continuous maps $\gamma : \mathbb{R} \longrightarrow X$ such that $\gamma(t+1) = \gamma(t)g$ for some $g \in G$; at the same time the circle acts on the loop space by rotation. One model for the loop space is

$$Loop_1(X/\!\!/G) := Bibun(S^1/\!\!/*, X/\!\!/G),$$

with $Bibun(S^1/\!\!/*, X/\!\!/G)$ the category of bibundles from the trivial groupoid $S^1/\!\!/*$ to the translation groupoid $X/\!\!/G$. The reference for bibundles is [36]. Then we add the circle rotations to this model and formulate the orbifold $Loop_1^{ext}(X/\!\!/G)$.

Quasi-elliptic cohomology $QEll^*_G(X)$ is defined to be the orbifold K-theory of a subgroupoid $\Lambda(X/\!\!/G)$ of $Loop_1^{ext}(X/\!\!/G)$ consisting of constant loops. More explicitly, $QEll^*_G(X)$ can be expressed in term of the equivariant K-theory of X and its subspaces

$$QEll^*_G(X) := \prod_{\sigma \in G^{tors}_{conj}} K^*_{\Lambda_G(\sigma)}(X^\sigma) = \Big(\prod_{\sigma \in G^{tors}} K^*_{\Lambda_G(\sigma)}(X^\sigma) \Big)^G. \tag{1}$$

I will explain more about this definition in Section 2.

In fact, quasi-elliptic cohomology can be defined for any orbifold groupoid X by considering

$$Loop_1(X) := Bibun(S^1/\!\!/*, X)$$

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and formulating $Loop_1^{ext}(X)$. Ganter [13] explores the interaction between elliptic curve and orbifolds from another perspective in the case of the Tate curve, describing K_{Tate} for an orbifold X in term of the equivariant K-theory and the groupoid structure of X. Orbifold quasi-elliptic cohomology can be constructed in a similar way, as shown in Section 2.

This theory has power operations, which was first observed by Nora Ganter, as shown in [12] and [13]. She spelled out the axioms for orbifold theories with power operations in [13], and constructed the power operation for orbifold Tate K-theory in the same paper, which is closely related to the level structure and isogenies on Tate curve. We construct a family of power operations for the orbifold quasi-elliptic cohomology satisfying the axioms that Ganter concluded. It mixes power operation in K-theory with natural operation of dilating and rotating loops. As an application, we prove that the Tate K-theory of symmetric groups modulo a certain transfer ideal classify the finite subgroups of the Tate curve.

One advantage of quasi-elliptic cohomology is it's built using equivariant topological K-theory, each aspect of which has been studied thoroughly. Some contructions on quasi-elliptic cohomology can be made simpler than most elliptic cohomology theories, including the Tate K-theory. Applying equivariant homotopy theory, in section 4, we construct a G-orthogonal spectra weakly representing quasi-elliptic cohomology. Unfortunately, our construction does not arise from a global spectra; thus, we consider a new formulation of global stable homotopy theory that contains quasi-elliptic cohomology, as shown in Section 6.

Moreover, we define an involution for the theory which is compatible with its geometric interpretation. As equivariant K-theory, quasi-elliptic cohomology also has the Real and real version, which is shown in Section 5.

Devoto's equivariant K-theory has a Hopkins-Kuhn-Ravenel (HKR) character theory [19]. Ganter expected the HKR theory for orbifold Tate K-theory to be formulated via Stapleton's framework of tanschromatic character map. Stapleton constructed in his paper [39] and [40], for each finite $G-$CW complex X and each positive integer n, a topological groupoid $Twist_n(X)$, whose construction is analogous to that of the groupoid $\Lambda(X/\!\!/G)$ in Section 2. With the topological groupoid $Fix_n(X)$ in [19], he constructed in [39] extensions of the generalized character map of Hopkins, Kuhn, and Ravenel [19] for Morava E-theory to every height between 0 and n. And with $Twist_n(X)$, in [40] he constructed the twisted character map which can canonically recover the transchromatic generalized character map in [39]. Apply this transchromatic character theory, he and Schlank provided a new proof of Strickland's theorem in [41] that the Morava E-theory of the symmetric group has an algebro-geometric interpretation after taking the quotient by a certain transfer ideal. Moreover, he showed in [35] this extended character theory can be used to compute the total power operation for the Morava E-theory of any finite group, up to torsion.

Stapleton's theory may also be applied to formulate the HKR charactor theory for quasi-elliptic cohomology.

2. What is this theory $QEll$?

The main reference for quasi-elliptic cohomology is Rezk [30].

Let G be a compact Lie group and X a $G-$space. First we explain explicitly the definition in (1). For any torsion element $g \in G$, define

$$\Lambda_G(g) = C_G(g) \times \mathbb{R}/\langle (g, -1)\rangle.$$

Let $\mathbb{T}$ be the circle $\mathbb{R}/\mathbb{Z}$. We have an exact sequence

$$1 \longrightarrow C_G(g) \longrightarrow \Lambda_G(g) \xrightarrow{\pi} \mathbb{T} \longrightarrow 0$$

where the first map is $g \mapsto [g, 0]$ and the second map is $\pi : [g, t] \mapsto t \text{ mod } \mathbb{Z}$.

Let X be a $G-$space. Let $G^{tors} \subseteq G$ be the set of torsion elements of G. Let G^{tors}_{conj} denote a set of representatives of $G-$conjugacy classes in G^{tors}. Let $\sigma \in G^{tors}$. The fixed point space X^σ is a $C_G(\sigma)-$space. We can define a $\Lambda_G(\sigma)-$action on X^σ by

$$[g,t] \cdot x := g \cdot x.$$

As shown in (1), the quasi-elliptic cohomology $QEll$ of $X/\!\!/G$ is given by

$$QEll^*_G(X) := \prod_{\sigma \in G^{tors}_{conj}} K^*_{\Lambda(\sigma)}(X^\sigma) = \Big(\prod_{\sigma \in G^{tors}} K^*_{\Lambda(\sigma)}(X^\sigma)\Big)^G.$$

Geometrically, $QEll^*_G(X)$ classifies all the $\Lambda(X/\!\!/G)-$vector bundler over the orbifold groupoid $\Lambda(X/\!\!/G)$.

Let X be a topological groupoid. We can construct $Loop^{ext,tors}_1(X)$ similarly. The orbifold quasi-elliptic cohomology is defined to be the orbifold K-theory of the full subcategory of $Loop^{ext,tors}_1(X)$ consisting of constant loops. In the global quotient case,

$$QEll^*(M/\!\!/G) = QEll^*_G(M).$$

In [13], Ganter explains that Tate K-theory is really a cohomology theory for orbifolds, which is based on Devoto's definition of the equivariant theory. Orbifold quasi-elliptic cohomology can be constructed in a similar way, as shown below.

Let X be a topological groupoid. The inertia groupoid of X

$$\Lambda X := \mathrm{Fun}(\mathrm{pt}/\!\!/\mathbb{Z}, X).$$

$$QEll^*(X) := K^*_{orb}(\mathrm{pt}/\!\!/\mathbb{R} \times_{1\sim\xi} \Lambda(X))$$

where $\mathrm{pt}/\!\!/\mathbb{R} \times_{1\sim\xi} \Lambda(X)$ is the groupoid

$$(\mathrm{pt}/\!\!/\mathbb{R}) \times \Lambda(X)/\sim$$

with $\xi \in \mathrm{Center}(X)$ and $\sim$ generated by $1 \sim \xi$.

3. Power operations

Cohomology operations are essential calculating tools in algebraic topology. Nora Ganter established in her paper [13] the axioms for power operation on orbifold theory and constructed in her paper [11] and [12] power operations for Tate K-theory, and in [13] that for orbifold Tate K-theory, which is compatible with the level structure of elliptic curves.

Quasi-elliptic cohomology has power operations, which gives the theory an equivariant $H_\infty-$structure, as formulated in [11]. The tensor product of it defines the Atiyah power operation of it. Moreover, it has a power operation with closer relation to the Tate elliptic curve.

THEOREM 3.1. *Quasi-elliptic cohomology has a power operation*

$$\mathbb{P}_n : QEll_G(X) \longrightarrow QEll_{G\wr\Sigma_n}(X^{\times n})$$

that is elliptic in the sense: $\mathbb{P}_n$ *can be uniquely extended to the stringy power operation*

$$P^{string}_n : K_{Tate}(X/\!\!/G) \longrightarrow K_{Tate}(X^{\times n}/\!\!/(G \wr \Sigma_n))$$

of the Tate K-theory in [12]*, which is elliptic.*

We construct $\mathbb{P}_n$ via explicit formulas that interwine the power operation in K-theory and natural symmetries of the free loop space.

In general, we can define an elliptic power operation for orbifold quasi-elliptic cohomology.

In fact it is already illuminating to study the power operation $\{\mathbb{P}_n\}_{n\geqslant 0}$ when X is a point pt with trivial group action. A big ingredient then is understanding $QEll(\mathrm{pt}/\!\!/\Sigma_N)$. Applying that we prove that the Tate K-theory of symmetric groups modulo a certain transfer ideal classifies finite subgroups of the Tate curve, which is analogous to the principal result in Strickland [41] that the Morava $E-$theory of the symmetric group Σ_n modulo a certain transfer ideal classifies the power subgroups of rank n of the formal group $\mathbb{G}_E$.

First we prove

THEOREM 3.2.

$$QEll(pt/\!\!/\Sigma_N)/\mathcal{I}^{QEll}_{tr} \cong \prod_{N=de} \mathbb{Z}[q,q^{-1}][q']/\langle q^d - q'^e\rangle, \tag{2}$$

where $\mathcal{I}^{QEll}_{tr}$ is the transfer ideal

$$\mathcal{I}^{QEll}_{tr} := \sum_{\substack{i+j=N,\\ N>j>0}} Image[\mathcal{I}^{\Sigma_N}_{\Sigma_i\times\Sigma_j} : QEll(pt/\!\!/\Sigma_i\times\Sigma_j) \longrightarrow QEll(pt/\!\!/\Sigma_N)] \tag{3}$$

with $\mathcal{I}^G_H$ the transfer map of $QEll$ along $H \hookrightarrow G$ and q' is the image of q under the power operation $\mathbb{P}_N$. The product goes over all the pairs of positive integers (d,e) such that $N = de$ whose order matters. For example, $(1,2)$ and $(2,1)$ are considered as different pairs.

Then by the relation between $QEll^*$ and Tate K-theory is for any topological groupoid X,

$$QEll^*(X)\otimes_{\mathbb{Z}[q^{\pm}]}\mathbb{Z}((q)) = K^*_{Tate}(X)$$

and the corresponding relation between the power operations of each theory, we have the theorem below.

THEOREM 3.3. *The Tate K-theory of symmetric groups modulo a certain transfer ideal classifies finite subgroups of the Tate curve. Explicitly,*

$$K_{Tate}(pt/\!\!/\Sigma_N)/I^{Tate}_{tr} \cong \prod_{N=de}\mathbb{Z}((q))[q']/\langle q^d - q'^e\rangle, \tag{4}$$

where I^{Tate}_{tr} is the transfer ideal

$$I^{Tate}_{tr} := \sum_{\substack{i+j=N,\\ N>j>0}} Image[I^{\Sigma_N}_{\Sigma_i\times\Sigma_j} : K_{Tate}(pt/\!\!/\Sigma_i\times\Sigma_j) \longrightarrow K_{Tate}(pt/\!\!/\Sigma_N)] \tag{5}$$

with I^G_H the transfer map of K_{Tate} along $H\hookrightarrow G$ and q' is the image of q under the power operation P^{string}_N constructed in [13]. The product goes over all the pairs of positive integers (d,e) such that $N = de$ whose order matters.

Moreover, via the isomorphism in Theorem 3.2, we can define an operation via the power operation

$$\begin{aligned}\overline{P}_N : & QEll_G(X) \xrightarrow{\mathbb{P}_N} QEll_{G\wr\Sigma_N}(X^{\times N}) \xrightarrow{res} QEll_{G\times\Sigma_N}(X^{\times N})\\ & \xrightarrow{diag^*} QEll_{G\times\Sigma_N}(X) \cong QEll_G(X)\otimes_{\mathbb{Z}[q^{\pm}]} QEll_{\Sigma_N}(\mathrm{pt})\\ & \longrightarrow QEll_G(X)\otimes_{\mathbb{Z}[q^{\pm}]} QEll_{\Sigma_N}(\mathrm{pt})/\mathcal{I}^{QEll}_{tr} \cong QEll_G(X)\otimes_{\mathbb{Z}[q^{\pm}]}\prod_{N=de}\mathbb{Z}[q,q^{-1}][q']/\langle q^d-q'^e\rangle,\end{aligned}$$

which is a ring homomorphism and analogous to the Adams operation of equivariant K-theories. It uniquely extends to an additive operation of Tate K-theory

$$\overline{P^{string}}_n : K_{Tate}(X/\!\!/G) \longrightarrow K_{Tate}(X/\!\!/G)\otimes_{\mathbb{Z}((q))} K_{Tate}(\mathrm{pt}/\!\!/\Sigma_N)/I^{Tate}_{tr}.$$

The trace of $\overline{P^{string}}_n(x)$ is $nT_n(x)$ with T_n the $n-$th Hecke operator of Tate K-theory defined by the stringy power operation P^{String}_n in [12].

Via the operation $\overline{P}_N$, we construct the universal finite subgroup of the Tate curve.

4. Spectrum

Goerss-Hopkins-Miller theorem constructs many examples of $E_\infty-$rings which represent elliptic cohomology theories, including Tate K-theory.

QUESTION 4.1. *Can we construct $E_\infty - G-$spectrum which represents equivariant elliptic cohomology theory (e.g. $G-$equivariant Tate K-theory)?*

We try to answer this question by studying the spectrum of quasi-elliptic cohomology. It is closed to generalized elliptic cohomology and can be expressed by equivariant K-theories, which has been thoroughly studied.

We construct a space $QEll_{G,n}$ explicitly for each G and each n representing $QEll^n_G(-)$ in the sense of (6).

$$\pi_0(QEll_{G,n}) = QEll^n_G(S^0), \text{ for each } k. \tag{6}$$

Moreover, we consider the question

QUESTION 4.2. *Is there an orthogonal $G-$spectrum representing $QEll^*_G$? And can they arise from an orthogonal spectrum?*

The answer to the first question is yes. Based on the construction of $QEll_{G,n}$, we construct a space $E(G,V)$ for each faithful $G-$representation V that weakly represents $QEll^V_G(-)$ in the sense of (7),

$$\pi_k(E(G,V)) = QEll^V_G(S^k), \text{ for each } k. \tag{7}$$

The construction is sketched below.

Recall for any $G-$representation V, let $Sym(V) := \bigoplus_{n\geq 0} Sym^n(V)$ denote the total symmetric power.

Let $S(G,V)_g$ denote the space

$$Sym(V) \setminus Sym(V)^g. \tag{8}$$

Let $F_g(G,V)$ denote the space $\mathrm{Map}_{\mathbb{R}}(S^{(V)_g}, KU((V)_g \oplus V^g))$ where KU is the global complex $K-$spectrum and $(V)_g$ is a specific $\Lambda_G(g)-$representation. The basepoint c_0 of it is the constant map from $S^{(V)_g}$ to the basepoint of $KU((V)_g \oplus V^g)$.

Let $E_g(G,V)$ denote

$$\{t_1 a + t_2 b \in F_g(G,V) * S(G,V)_g \mid \|b\| \leqslant t_2\}/\{t_1 c_0 + t_2 b\}.$$

It is the quotient space of a closed subspace of the join $F_g(G,V) * S(G,V)_g$ with all the points of the form $t_1 c_0 + t_2 b$ collapsed to one point, which we pick as the basepoint of $E_g(G,V)$. $E_g(G,V)$ has the evident $C_G(g)-$action. When V is a faithful $G-$representation,

$$E(G,V) := \prod_{g\in G^{tors}_{conj}} \mathrm{Map}_{C_G(g)}(G, E_g(G,V)) \tag{9}$$

weakly represents $QEll^V_G(-)$ in the sense

$$\pi_0(E(G,V)) \cong QEll^V_G(S^0). \tag{10}$$

Moreover, we construct the structure maps making E an orthogonal $G-$spectra and an $\mathcal{I}_G-$FSP.

In addition, we construct the restriction maps $E(G,V) \longrightarrow E(H,V)$ for each group homomorphism $H \longrightarrow G$, which is, as expected, is not a homeomorphism, but an $H-$weak equivalence.

The orthogonal $G-$spectra $E(G,-)$, however, cannot arise from an orthogonal spectrum.

Note that the constructions above can be applied to the construction of the $G-$spectrum and the orthogonal $G-$spectrum up to weak equivalence of any theory of the form

$$QE^*_G(X) := \prod_{\sigma\in G^{tors}_{conj}} E^*_{\Lambda(\sigma)}(X^\sigma) = \Big(\prod_{\sigma\in G^{tors}} E^*_{\Lambda(\sigma)}(X^\sigma)\Big)^G \tag{11}$$

with E any equivariant cohomology theory having the same key features as equivariant K-theory. Generalized Morava E-theories and equivariant Tate K-theory are in this family of theories.

5. Real Quasi-elliptic cohomology

Considering the relation between equivariant K-theory and the quasi-elliptic cohomology, it's natural to expect Real and real quasi-elliptic cohomology to be defined.

Let G be a Real Lie group and X a Real $G-$space. The Real quasi-elliptic cohomology $QEllR^*_G(X)$ should be the cohomology theory classifying all the Real $\Lambda(X/\!\!/G)-$vector bundler over the Real orbifold groupoid $\Lambda(X/\!\!/G)$, which inherits a Real structure from that on X. First we formulate the definition of Real orbifold vector bundle 5.1 from that of orbifold vector bundle in [2].

DEFINITION 5.1. *A Real $\mathcal{G}-$vector bundle over a Real orbifold groupoid $\mathcal{G}$ is a $\mathcal{G}-$vector bundle over $\mathcal{G}$, a Real vector bundle over G_0, and also a Real $\mathcal{G}-$space, so that the involutions and the $\mathcal{G}-$structure are compatible with each other.*

In light of the geometric interpretation, Real and real quasi-elliptic cohomology should be defined in this way:

$$QEllR^*_G(X) := \prod_{g\in G^{tors}_{conj}} KR^*_{\Lambda_G(g)}(X^g).$$

And real quasi-elliptic cohomology

$$QEllr^*_G(X) := \prod_{g\in G^{tors}_{conj}} KO^*_{\Lambda_G(g)}(X^g).$$

We construct the spectrum of both theories in a way similar to that of complex quasi-elliptic cohomology. Moreover, for each augmented Lie group G, We construct a $\mathcal{I}_G-$FSP $ER(G,-)$ weakly representing $QEllR_G$ and a $\mathcal{I}_G-$FSP $EO(G,-)$ weakly representing $QEllr_G$. The construction is analogous to that of $E(G,-)$ in Section 4. We construct a Real structure on each $ER(G,V)$ explicitly so that the involution is compatible with the homotopy adjunction and the structure maps of the $\mathcal{I}_G-$FSP.

6. Global Quasi-elliptic cohomology

The idea of global orthogonal spectra was first inspired in Greenlees and May [16]. Many classical theories, equivariant stable homotopy theory, equivariant bordism, equivariant K-theory, etc, naturally exist not only for a single group but a specific family of groups in a uniform way. Several models of global homotopy theories have been established, including that by Bohmann [6] and Schwede [37]. The two model categories of global spectra are Quillen equivalent, as shown in [6].

However, there are even more equivariant theories that cannot fit into this global world. Quasi-elliptic cohomology is one of them, as indicated in Section 4. Ganter showed that $\{QEll^*_G\}$ have the change-of-group isomorphism. We establish a more flexible global homotopy theory and show quasi-elliptic chomomology can fit into it.

Recall an orthogonal space is a continuous functor $Y : \mathbb{L} \longrightarrow \mathcal{T}$ to the category of topological spaces where $\mathbb{L}$ is the category whose objects are inner product real spaces and whose morphism set between two objects V and W are the linear isometric embeddings $L(V, W)$. We formulate a category D_0 whose objects are (G, V, ρ) with V an inner product vector space, G a compact group and ρ a faithful group representations

$$\rho : G \longrightarrow O(V),$$

and whose whose morphism $\phi = (\phi_1, \phi_2) : (G, V, \rho) \longrightarrow (H, W, \tau)$ consists of a linear isometric embedding $\phi_2 : V \longrightarrow W$ and a group homomorphism $\phi_1 : \tau^{-1}(O(\phi_2(V))) \longrightarrow G$, which makes the diagram commute.

$$\begin{tikzcd} G \arrow[r, "\rho"] & O(V) \arrow[d, "\phi_{2*}"] \\ \tau^{-1}(O(\phi_2(V))) \arrow[u, "\phi_1"] \arrow[r, "\tau"] & O(W) \end{tikzcd} \tag{12}$$

In other words, the group action of H on $\phi_2(V)$ is induced from that of G. Intuitively, the category D_0 is obtained by adding the restriction maps between representations.

A D_0-space is a continuous functor $X : D_0 \longrightarrow T$ to the category of compactly generated weak Hausdorff spaces. A morphism of D_0-spaces is a natural transformation. The category of orthogonal spaces is a full subcategory of the category of D_0-spaces. We use D_0T to denote the category of D_0-spaces.

Apply the idea of diagram spectra in [32], we can also define D_0-spectra and D_0-FSP.

We formulate several model structures on D_0T. First by the theory in [32], there is a level model structure on D_0T.

THEOREM 6.1. *The category of D_0-spaces is a compactly generated topological model category with respect to the level equivalences, level fibrations and $q-$cofibrations. It is right proper and left proper.*

D_0 is a generalized Reedy category in the sense of [5]. And we can formulate a Reedy model structure on D_0T.

THEOREM 6.2. *The Reedy cofibrations, Reedy weak equivalences and Reedy fibrations form a model structure, the Reedy model structure, on the category of D_0-spaces.*

I'm formulating a global model structure on D_0T Quillen equivalent to the global model structure on the orthogonal spaces formulated by Schwede in [37].

Combining the orthogonal $G-$spectra of quasi-elliptic cohomology together, we get a well-defined D_0-spectra and D_0-FSP. Thus, we can define global quasi-elliptic cohomology in the category of D_0-spectra.

THEOREM 6.3. *There is a D_0-FSP weakly representing quasi-elliptic cohomology.*

7. Future Problems

QUESTION 7.1. *In* [32]*, Mandell, May, Schwede and Shipley showed there is a Quillen adjoint pair* $(\mathbb{P}, \mathbb{U})$ *between the category of symmetric spectra and the category of orthogonal spectra.* $\mathbb{U}$ *preserves smash product while* $\mathbb{P}$ *doesn't. So* $\mathbb{U}$ *gives a functor from commutative orthogonal ring spectra to commutative symmetric ring spectra. One may ask whether there is a functor from the category of ultra-commutative symmetric ring spectra to the category of ultra commutative orthogonal ring spectra.*

QUESTION 7.2. *I will formulate the Hopkins-Kuhn-Ravenel (HKR) character theory for QEll and that for Tate K-theory. It can probably be fit into Stapleton's transchromatic character theory.*

QUESTION 7.3. *As I indicate in Section 6, I'm formulating a global model structure on D_0T Quillen equivalent to the global model structure on the orthogonal spaces formulated by Schwede in* [37]*.*

QUESTION 7.4. *We can generalize quasi-elliptic cohomology. Let n be any positive integer. Let G be a compact Lie group and X a $G-CW$ complex. Let $\sigma=(\sigma_1, \sigma_2, \cdots \sigma_n)$ with each $\sigma_i\in G^{tors}_{conj}$ and $[\sigma_i, \sigma_j]=e$. Let G^n_z be the set of of all such $(\sigma_1, \cdots \sigma_n)$s. Let $\Lambda^n(\sigma)$ be the quotient of $C_G(\sigma)\times\mathbb{R}^n$ by the normal subgroup generated by elements $(\sigma_i, -e_i)$ where $e_i=(0, \cdots 0, 1, 0, \cdots 0)\in\mathbb{R}^n$.*

A topological groupoid $\Lambda^n(X/\!\!/G)$ analogous to $\Lambda(X/\!\!/G)$ can also be constructed. It's analogous to the topological groupoid $Fix_n(X)$ in Hopkins, Kuhn and Ravenel [19].

Let E^* be any equivariant cohomology theory. We can define

$$\boldsymbol{E}^*(X):=\prod_{\sigma\in G^n_z}E_{\Lambda^n(\sigma)}(X^{\sigma}).$$

Those results for quasi-elliptic cohomology on power operation and spectra can be formulated analogous to $\boldsymbol{E}^*$. I'm also working on this.

QUESTION 7.5. *David Gepner constructed a global homotopy theory via orbispaces in* [14]. *I'd like to see whether quasi-elliptic cohomology can fit into that theory.*

QUESTION 7.6. *I will study the relation between the global homotopy theory defined in Section 6 and David Gepner's global homotopy theory.*

QUESTION 7.7. *Via quasi-elliptic cohomology, I will give a classification of $\Gamma(N)-$level structure of the Tate curve and see whether the conclusion can be generalized to elliptic curves.*

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