Zhen Huan

University of Illinois at Urbana-Champaign

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Overview

Plan.

- The construction of quasi-elliptic cohomology
- The power operation
- The orthogonal *G*-spectra

An old idea of Witten

[Landweber]

The elliptic cohomology of a space X is related to the \mathbb{T} -equivariant K-theory of $LX=\mathbb{C}^{\infty}(S^1,X)$ with the circle \mathbb{T} acting on LX by rotating loops.

It's surprisingly difficult to make this precise.

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In application, one needs to consider the case that a group G acts on X. In this case the loop space LX has rich structures as an orbifold.

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Bibundles \sim "bimodules" in geometry

Bibundles combine several widely used notions, including smooth maps, Lie homomorphisms, and principal bundles.

A bibundle from \mathbb{H} to \mathbb{G}

Schommer-Pries] [Lerman]

- a smooth manifold P together with
 - the structure maps:
 - $\tau: P \longrightarrow \mathbb{G}_0$;

- a surjective submersion $\sigma: P \longrightarrow \mathbb{H}_0$.
- ullet The action maps in $Man_{G_0 \times H_0}$
 - $\mathbb{G}_{1} \times_{\tau} P \longrightarrow P$;

$$\bullet \ P_{\sigma} \times_{t} \mathbb{H}_{1} \longrightarrow P$$

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$$g_1 \cdot (g_2 \cdot p) = (g_1 g_2) \cdot p$$
; • $(p \cdot h_1) \cdot h_2 = p \cdot (h_1 h_2)$; • $g \cdot (p \cdot h) = (g \cdot p) \cdot h$

- $p \cdot u_H(\sigma(p)) = p$ and $u_G(\tau(p)) \cdot p = p$ for all $p \in P$.
- $\bullet \ \mathbb{G}_{1_S} \times_{_T} P \longrightarrow P_{_{\sigma}} \times_{_{\sigma}} P \qquad \qquad (g,p) \mapsto (g \cdot p,p) \text{ is an isomorphism}.$

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The Loop Space of Interest

Example $(Loop(X//G) := Bibun(S^1//*, X//G))$

• Objects:

$$\mathcal{P} := \{ S^1 \stackrel{\pi}{\longleftarrow} P \stackrel{f}{\longrightarrow} X \}$$

• Morphisms:

$$S^{1} \stackrel{\pi}{\longleftarrow} P \stackrel{f}{\longrightarrow} X$$

$$\downarrow^{\alpha} \qquad \qquad \downarrow^{\alpha}$$

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Example $(Loop^{ext}(X//G))$



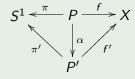
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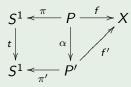
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The isotropy groups in $Loop^{ext}(X//G)$ may be infinite dimensional topological groups when G is not finite.

the subgroupoid $\Lambda(X//G)$ instead

$$\Lambda(X//G) := \coprod_{g \in G_{conj}} X^g // \Lambda_G(g)$$

 G_{coni}^{tors} : a set of representatives of G-conjugacy classes in G^{tors} ;

$$\Lambda_G(g) = C_G(g) \times \mathbb{R}/\langle (g,-1) \rangle$$

QEII as equivariant K—theories

$$QEII_G(X) \cong \prod_{g \in G_{coni}^{tors}} K_{\Lambda_G(g)}(X^g)$$

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Quasi-elliptic cohomology has power operations, which gives it the structure of an " H_{∞} -ring theory" [Ganter 06].

Atiyah's Power Operation

 $[\mathsf{Ganter}]$

V: a vector bundle over $\Lambda(X//G)$.

 $P_n(V) := V^{\bigotimes_{\mathbb{Z}[q^{\pm}]} n}$ defines an operation

$$P_n: QEII_G(X) \longrightarrow QEII_{G\wr \Sigma_n}(X^{\times n})$$

$$\mathbb{P}_{n} = \prod_{(\underline{g},\sigma) \in (G \wr \Sigma_{n})^{tors}_{conj}} \mathbb{P}_{(\underline{g},\sigma)} :$$

$$QEII_{G}(X) \longrightarrow QEII_{G \wr \Sigma_{n}}(X^{\times n}) = \prod_{(\underline{g},\sigma) \in (G \wr \Sigma_{n})^{tors}_{conj}} K_{\Lambda_{G \wr \Sigma_{n}}(\underline{g},\sigma)}((X^{\times n})^{(\underline{g},\sigma)})$$

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Finite Subgroups of Tate Curve

Theorem (Huan)

$$\textit{QEII}(\textit{pt}//\Sigma_\textit{N})/\mathcal{I}^\textit{QEII}_{\textit{tr}} \cong \prod_{\textit{N}=\textit{de}} \mathbb{Z}[\textit{q}^{\pm}][\textit{q}'^{\pm}]/\langle \textit{q}^{\textit{d}} - \textit{q}'^{\textit{e}} \rangle,$$

where \mathcal{I}^{QEII}_{tr} is the transfer ideal and q' is the image of q under the power operation \mathbb{P}_N . The product goes over all the ordered pairs of positive integers (d,e) such that N=de.

Theorem (Huan)

The Tate K-theory of symmetric groups modulo a certain transfer ideal classifies finite subgroups of the Tate curve.

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Goerss-Hopkins-Miller theorem constructs many example of E_{∞} -rings which represent elliptic cohomology theories, including Tate K-theory.

Question

Can we construct $E_{\infty} - G$ —Spectrum which represents equivariant elliptic cohomology theory (e.g. G—equivariant Tate K-theory)?

Orthogonal G—spectra of Quasi-elliptic cohomology

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We construct a commutative \mathcal{I}_G -FSP $(E(G,-),\eta,\mu)$. For each faithful G-representation V, E(G,V) weakly represents $QEll_G^V(-)$ in the sense $\pi_k(E(G,V)) = QEll_G^V(S^k)$, for each k.

Can E(G, -) arise from an orthogonal spectrum?

No.

For a trivial G-representation V, the G-action on E(G, V) is not trivial.

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[Schwede][May]

Observation: It has been noticed since the beginnings of equivariant homotopy theory that certain theories naturally exist not just for a particular group, but in a uniform way for all groups in a specific class.

⇒ global homotopy theory

Prominent examples: equivariant stable homotopy, equivariant K-theory, equivariant bordism.

Almost Global Homotopy Theory

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- an extension of global homotopy theory;
- classifies those theories that are almost "global";
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Combining the orthogonal G-spectra $\{E(G, -)\}$, we get an ultra-commutative global ring spectrum in the new theory.

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Conjecture

There is a global model structure on the almost global spaces that is Quillen equivalent to the global model structure on the orthogonal spaces formulated by Schwede in Global Homotopy Theory.

Thank you.

Some references

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