Homework 6.

1. $X_1, \cdots X_n \sim Exp(\beta)$ $\quad f(x;\beta) = \frac{1}{\beta} e^{-\frac{x}{\beta}}$

$$F(x;\beta) = 1 - e^{-\frac{x}{\beta}}$$

Test $\quad H_0: \beta \geq 1$

$\qquad\quad H_1: \beta < 1$

So we know that: $B(\beta) = P(X \in R) = P(X \geq 1 \mid \beta)$

$$= 1 - P(X < 1 \mid \beta)$$

$$= 1 - F(1) = e^{-\frac{1}{\beta}}$$

$$\alpha = \sup_{\beta \geq 1} B(\beta) = e^{-1}$$

2. $y_1 \cdots y_n \sim B(p)$

So $f(y, n, p) = \binom{20}{y} p^y (1-p)^{20-y}$

Test $\quad H_0: P = 0.2$

$\qquad\quad H_1: P \neq 0.2$

If $Y \geq 7$ or $Y \leq 1$, then we will reject H_0

$B(p) = P(y \in R)$

$$= P(Y \leq 1 \mid p) + 1 - P(Y < 7 \mid R)$$

$$= \Sigma \binom{20}{y} p^y (1-p)^{20-y}$$

$$= 1 - \sum_{y=} \binom{20}{y} p^y (1-p)^{20-y}$$

$$\alpha = \sup_{p=0.2} B(p) = \binom{20}{1} 0.2 \times 0.8^{19} + 1 - \binom{20}{7} \times 0.2^7 \times 0.8^{13}$$

$$= 0.156.$$

3. $X_1, \cdots, X_n \sim N(\mu, \sigma^2 = 1)$

$$f(x;\mu) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(x-\mu)^2}$$

As we know: $n = 25$ $\alpha = 0.05$

Test $H_0: \mu = \mu_0$

$H_1: \mu \neq \mu_0$.

$$L(\mu; X_1, \cdots, X_n) = \left(\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}}\right)^n \cdot e^{\sum_{i=1}^{n}(X_i - \mu)^2}$$

$$\alpha = \sup_{\mu = \mu_0} B(\mu) = B(\mu_0) = 0.05$$

$$B(\mu) = P(T(x) > c) = P\left(\frac{\sqrt{n}|\bar{X} - \mu_0|}{\sigma} > \frac{\sqrt{n}\, c}{\sigma}\right) = 0.05$$

Thus, $\frac{\sqrt{n}c}{\sigma} = 1.95$

$\Downarrow$

$$c = \frac{1.95\sigma}{\sqrt{n}} = \frac{1.95}{\sqrt{25}} = 0.39$$

4. $X_1, \cdots X_n \sim B(p)$ $n = 9$

$$Y = \sum_{i=1}^{n} X_i$$

$$f(x,p) = p^x (1-p)^{1-x}$$

$$f(y,p) = \binom{9}{y} p^y (1-p)^{9-y}$$

Test $H_0: p = 0.4$

$H_1: p \neq 0.4$

(a) if $C_1 \geq 2$, $P(Y \leq C_1 | p = 0.4) > 0.1$ $\rightarrow C_1 \leq 1$

$C_2 \leq 5$, $P(Y \geq C_2 | p = 0.4) > 0.1$ $\rightarrow C_2 \geq 6$.

So $C_1 = 0$, $C_2 = 6$.

(b) If $Y \leq C_1$ or $Y \geq C_2$, we will reject H_0

$$B(p) = P((Y \leq C_1 \text{ or } Y \geq C_2) | p) = 1 - \sum_{y \in [a+1, a_4)} \binom{9}{y} p^y (1-p)^{9-y}$$

$$\alpha = \sup_{p = 0.4} B(p) = 1 - \sum \binom{9}{y} 0.4^y \cdot 0.6^{9-y}$$

$= 0.1$