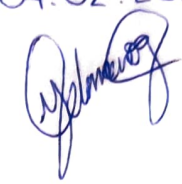
ME 303 - Final Exam

Yiğit Günsür Elmacıoğlu
2017405120

I pledge on my honor that I have neither given nor received unauthorized assistance in this exam. I have not done anything that would put me at an unfair advantageous position and equally I have not done anything that would put my classmates at an unfair disadvantageous position. I have refrained from any kind of cheating not only because of my respect for my classmates and for my instructor, but also for out of respect for myself.

Yiğit Günsür Elmacıoğlu
04.02.2021



Question 1

$$m\,y''(t) = mg - k\,y(t) - c\,y'(t)$$

I will divide this equation into 2 first order DE.

$$y'(t) = z(t)$$
$$z'(t) = g - \frac{k}{m}\,y(t) - \frac{c}{m}\,z(t) \quad \text{where} \quad \begin{matrix} m = 4\,kg \\ k = 2\,N/m \\ c = 0{,}4\,Ns/m \end{matrix}$$

$$\Rightarrow \begin{matrix} y'(t) = z(t) \\ z'(t) = 9{,}81 - 0{,}5\,y(t) - 0{,}1\,z(t) \end{matrix} \quad \text{initial conditions} \quad \begin{matrix} y(0) = 0 \\ y'(0) = z(0) = 0 \end{matrix}$$

at $t=0$ $y=0$, we want position at $t=2s$ and $h=1s$.

For runge-kutta we need functions; for $i = 1:2$

$y1 = y'(z(i))$; $\Longrightarrow$ for $i=1$ $y1 = 0$; $i=2$ $y1 = 8{,}5588$

$z1 = z'(y(i), z(i))$; $\Longrightarrow$ for $i=1$ $z1 = 9{,}81$; $i=2$ $z1 = 6{,}6835$

$y2 = y'(z(i) + h/2 \cdot z1)$ $\Longrightarrow$ for $i=1$ $y2 = 4{,}905$; $i=2$ $y2 = 11{,}9006$

$z2 = z'(y(i) + h/2 \cdot y1,\ z(i) + h/2 \cdot z1)$ $\Rightarrow$ for $i=1$ $z2 = 9{,}3195$; $i=2$ $z2 = 4{,}2096$

$y3 = y'(z(i) + h/2 \cdot z2)$ $\Longrightarrow$ for $i=1$ $y3 = 4{,}6597$; $i=2$ $y3 = 10{,}6636$

$z3 = z'(y(i) + h/2 \cdot y2,\ z(i) + h/2 \cdot z2)$ for $i=1$ $z3 = 8{,}1178$; $i=2$ $z3 = 3{,}4979$

Question 1 continued

$y4 = y'(z(i) + h.z3)$ ⟹ for i=1 $y4 = 8,1178$, i=2 $y4 = 12,0567$

$z4 = z'(y(i) + h.y3, z(i) + h.z3)$ ⟹ for i=1 $z4 = 6,6683$, i=2 $z4 = 1,0019$

Finally

$$y(i+1) = y(i) + \frac{h}{6}(y1 + 2.y2 + 2.y3 + y4)$$
$$z(i+1) = z(i) + \frac{h}{6}(z1 + 2.z2 + 2.z3 + z4)$$

to show better

	i=1	i=2
y1	0	8,5588
z1	9,81	6,6835
y2	4,905	11,9006
z2	9,3195	4,2096
y3	4,6597	10,6636
z3	8,1178	3,4979
y4	8,1178	12,0567
z4	6,6683	1,0019

from these values we get

$y(1) = 4,5412$
$z(1) = 8,5588$

$y(2) = 15,4985$
$z(2) = 12,4089$

Question 2

a) $2x''(t) - 5x'(t) - 3x(t) = 45 \cdot e^{2t}$

$\{y(t) = x'(t)\}$

$$x''(t) = \frac{5x'(t) + 3x(t) + 45e^{2t}}{2}$$

$$\Rightarrow y'(t) = \frac{5y(t) + 3x(t) + 45e^{2t}}{2}$$

$$x'(t) = y(t)$$

initial conditions $x(0) = 2$, $x'(0) = 1$

Question 3

To find a curve fit, at first I have to make some variable change.

$$y = \frac{x}{A + Bx}$$ divide both sides by x and take reciprocal

$$\frac{x}{y} = A + Bx$$ I will fit given data to this form