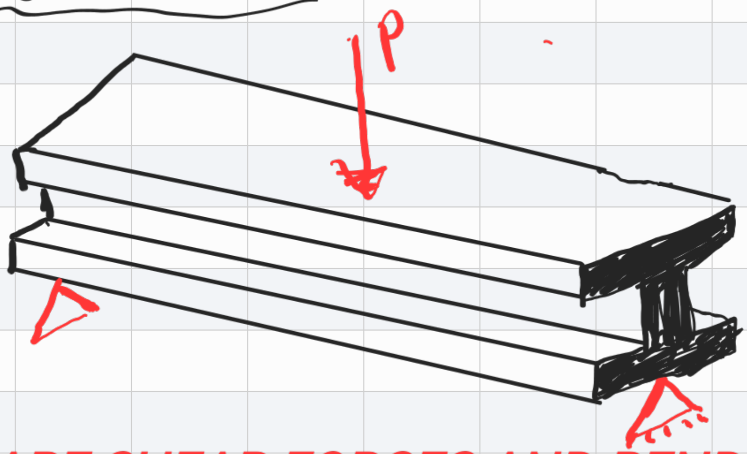
Beams

- a Beam is a structural part upon which a system of external loads acting at right angles (transverse) to the longitudinal axis.

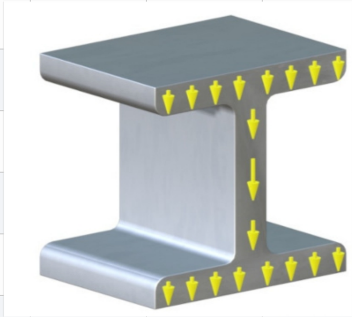


WHAT ARE SHEAR FORCES AND BENDING MOMENTS?

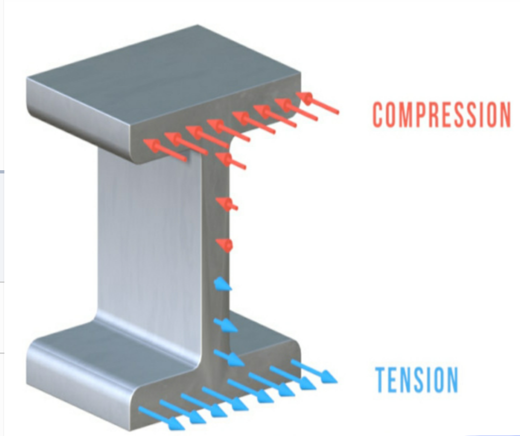
- When loads are applied to a beam, internal forces develop within the beam in response to the loads.
- We can visualise these forces by making an imaginary cut through the beam and considering the internal forces acting on the cross-section.

These internal forces have two components:

- ***Shear forces***, that are oriented in the vertical direction, **parallel** to the Beam cross-section

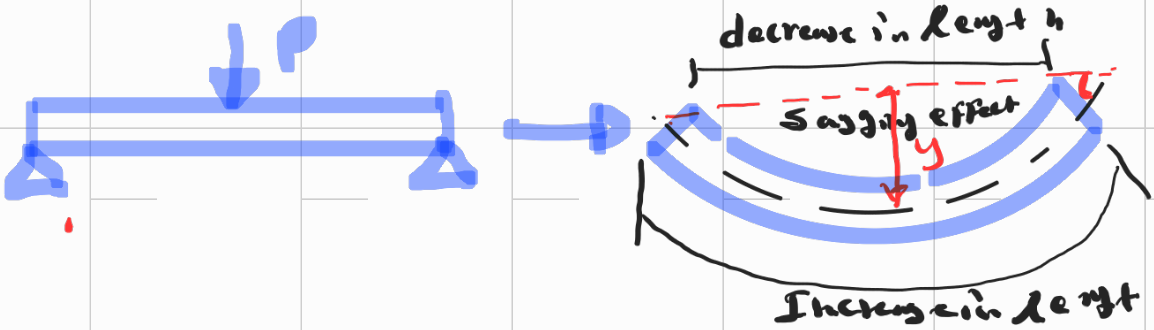


- ***Normal forces***, that are oriented along the axis of the beam, **perpendicular** to the beam cross-section.
- The normal stresses will be tensile on one side of the cross-section, and compressive on the other.



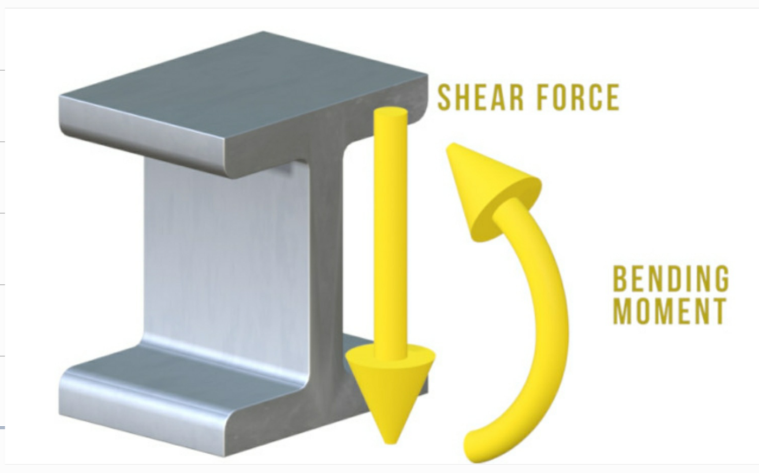
Normal internal forces are either tensile or compressive

- If the beam is sagging the top of the beam will get shorter, and so the normal forces acting at the top will be compressive.



- These forces cancel each other out so they don't produce a net force perpendicular to the beam cross-section, but they do produce a **moment**.
- This means that the internal forces acting on the cross-section of the beam can be represented by one resultant force, called a **shear force and bending moment.**

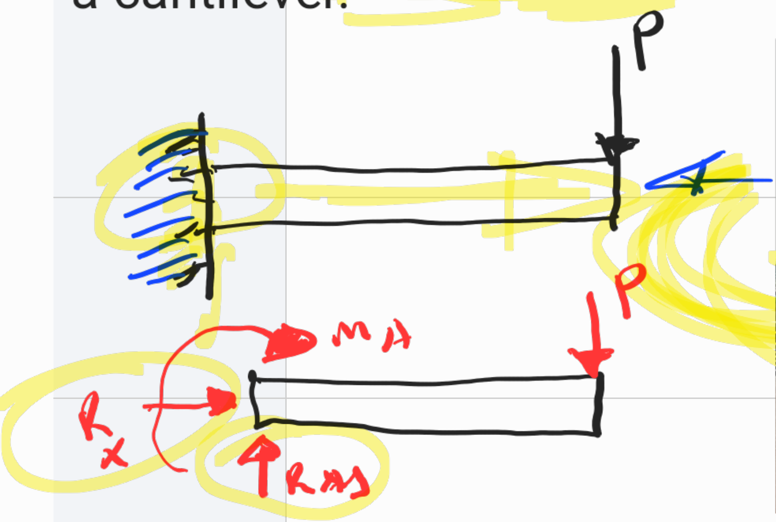
shear force :the result of the vertical internal forces) and
bending moment:the Resultant of the normal internal forces



Bending moment and shear force depends up on the *types of Beam* and loading type .

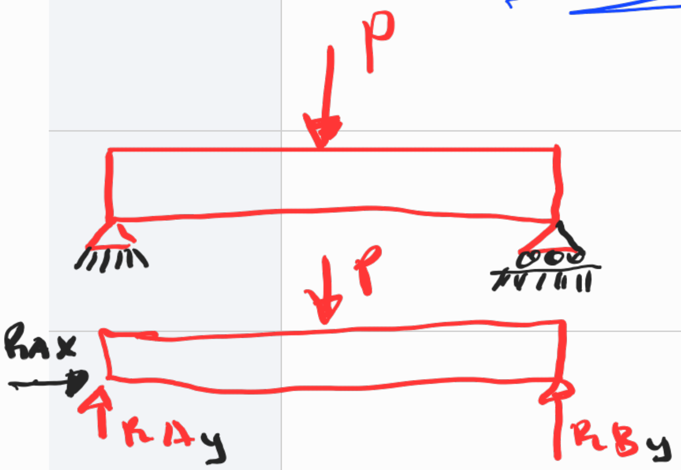
A) Cantilever Beam

A beam with one end fixed and the other free is referred to as a cantilever.



B) Simply Supported Beam

It is a beam having its ends freely resting on supports.



Simply supported deck

C) Fixed Beam

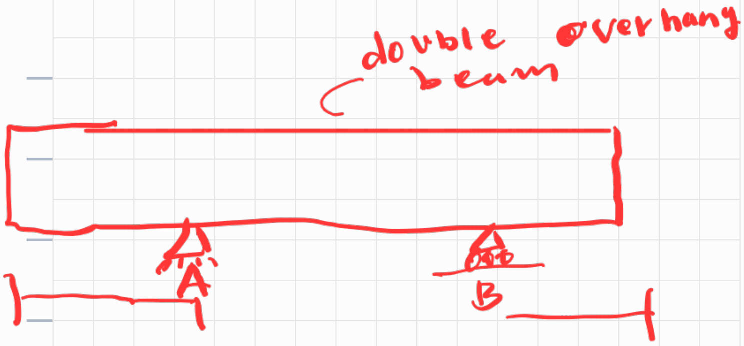
A beam having its both ends rigidly fixed or built in to the supporting walls or colums is known as fixed beam.

D) Continuous Beam

There are more than two supports for a continuous beam.

E) Overhanging Beam

In this, one or both ends of the beam extend beyond the supports.

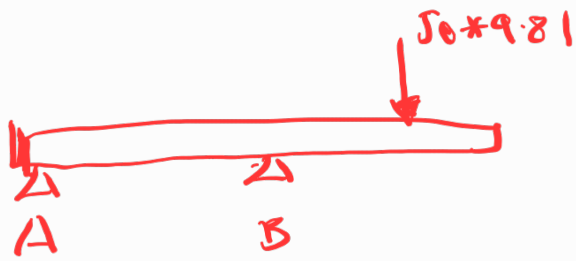


F) Propped Cantilever Beam

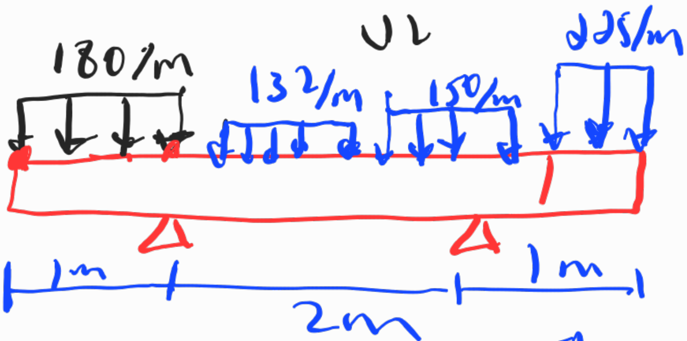
One end of the beam is constrained in all degrees of freedom, whereas the opposite end is simply supported.

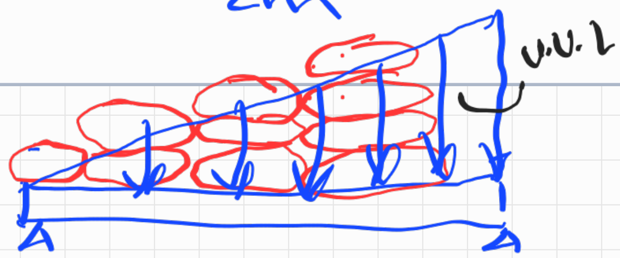
Types of Loading

Point load

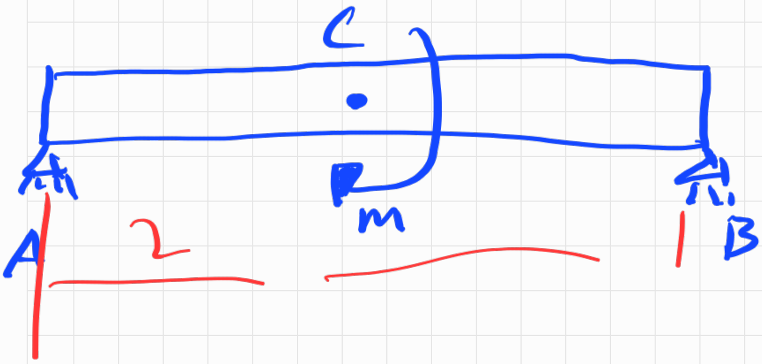


Distributed loads.





Point moment

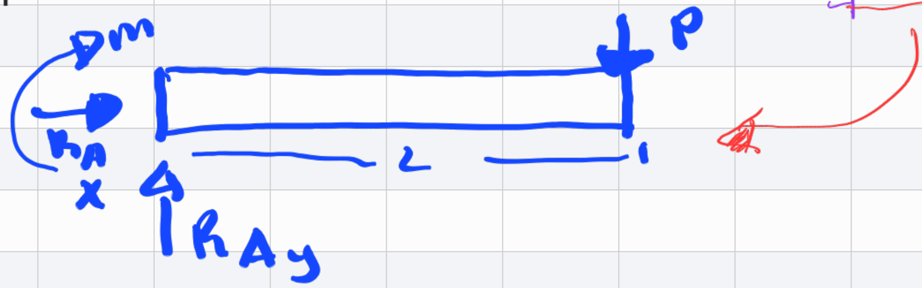


WHY ARE S.F AND B. M DIAGRAMS USEFUL?

- By showing how the shear force and bending moment vary along the length of a beam, they allow the loading on the beam to be quantified.
- They are often used as a starting point for performing more detailed analysis, which might include calculating stresses in beams or determining how beams will deflect.

HOW TO DRAW THE SHEAR FORCE AND BENDING MOMENT DIAGRAM

STEP 1 | DRAW A FREE BODY DIAGRAM



STEP 2 | CALCULATE THE REACTION FORCES AND MOMENTS AT THE BEAM SUPPORTS

- The reaction forces and moments at the supports can be calculated using equilibrium equations.

$\Sigma F_y = 0$, $R_{Ay} - P = 0$ $\quad R_{Ay} = P$

$\Sigma F_x = 0$ $\quad R_{Ax} = 0$

$(+\circlearrowleft)\ \Sigma M_A = 0$ $\quad m - PL = 0$ $\quad m = PL$

STEP 3 | DETERMINE THE INTERNAL SHEAR FORCES AND BENDING MOMENTS AT EVERY LOCATION ALONG THE BEAM

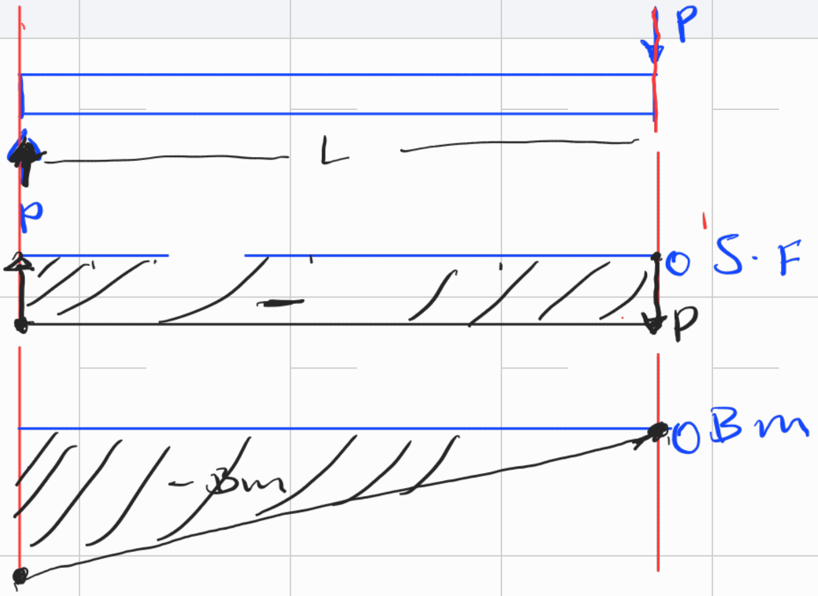
$\Sigma F_{y\,x} = 0$ $\quad -P + V_x = 0$

$V_x = P$

$+\Sigma M_{xx} = 0$ $\quad Px + m_x = 0$

$m_x = -Px$

STEP 4 | DRAW SHEAR FORCES AND BENDING MOMENTS DIAGRAM BASED THE RESULTS FROM STEP 3



$0 < x < L$

$-P + V_x = 0$

$V_x = P$

$Px + m_x = 0$

$m_x = -Px$ 3

SIGN CONVENTIONS

- If a bending moment causes sagging (concavity at the top layer) then it is positive, and if it causes hogging (convexity at the top layer) then it is negative.

Saggng

hogging

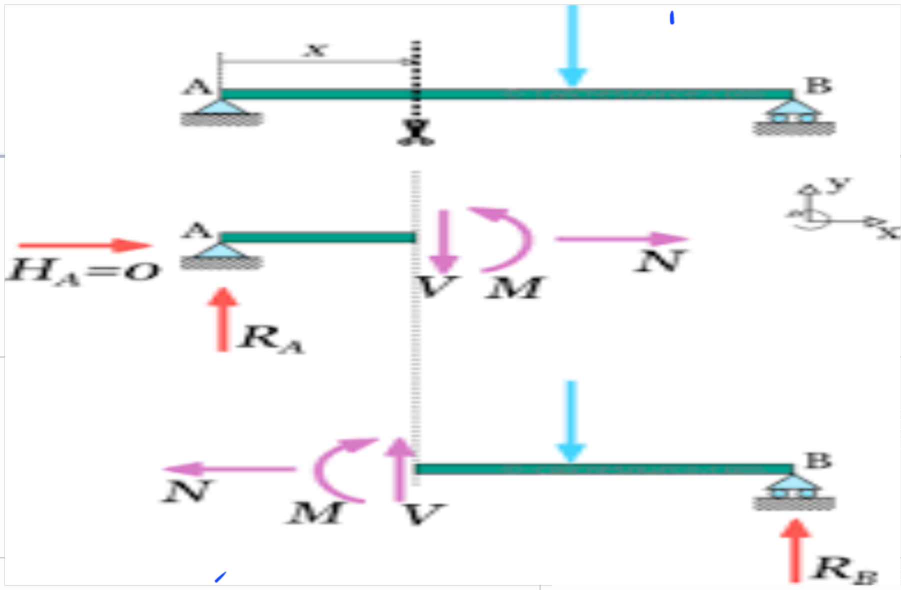
- If the resultant forces which are on the left side of the imaginary cutting plane is pointing upwards or if the resultant of forces which are on the right side of the imaginary cutting plane is pointing downwards ,then the shear force is considered positive.

+ imaginery cut

- If the resultant forces which are on the left side of the imaginary cutting plane is pointing downwards or if the resultant of forces which are on the right side of the imaginary cutting plane is pointing upward,then the shear force is considered negative.

- imaginery cut

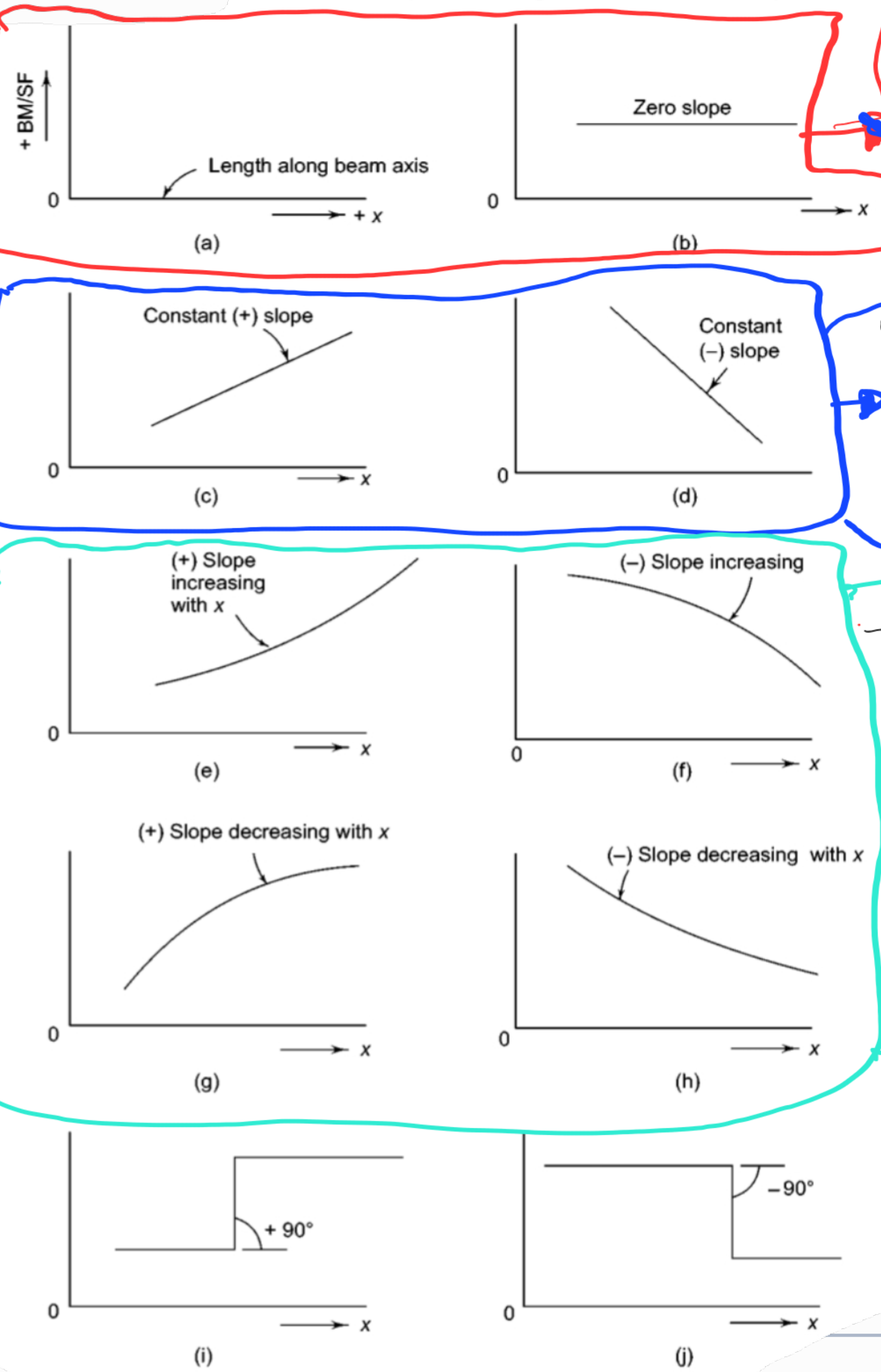
For section method



3

→ Important Point while making S.F & B.M diagram.

Different shapes of diagrams and their slopes



For Constant Shear Force and Bending moment eqn

For Linear Shear Force and Bending moment eqn

$y = mx + b$

For Parbolic and cubic Shear Force and Bending moment eqn

$y = x^2$ – Parabolic

or

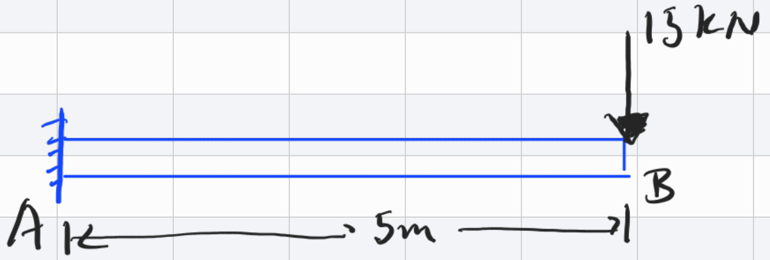
$y = x^3$ – Cubic

We try to draw the s.f and b.m diagram for cantiliver beam on following 5 different loading cases;

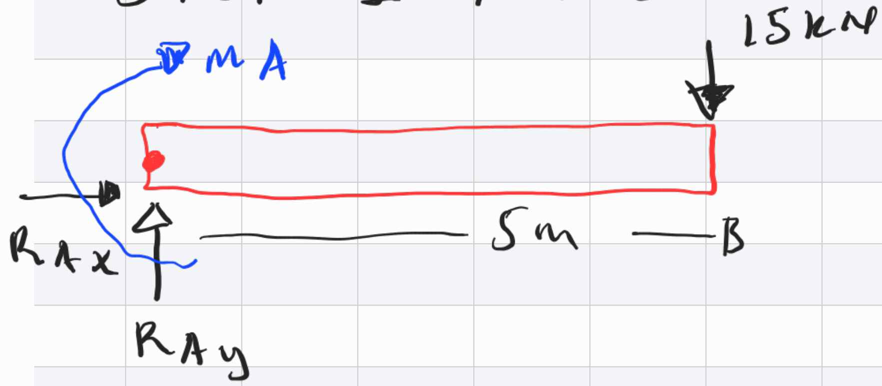
Case 1. With single point load
Case 2. With multiple point load
Case 3. With entirly uniformily distributed load
Case 4. With entirely uniformly varying load
Case 5. With combination of different loadings

Case 1.With single point load

- For the beam shown below draw the Shear Force and Bending moment diagram



→ Step 1 F.B.D



Step 2

- Find the Reaction Force

$\Sigma F_x = 0$, $R_{Ax} = 0$

$\Sigma F_y = 0 \; (\uparrow +) \; (- \downarrow)$

$R_{Ay} - 15 = 0$

$R_{Ay} = 15$

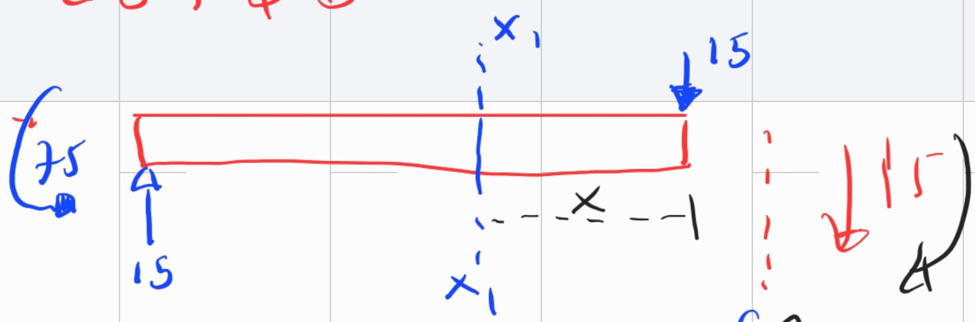
$\Sigma M_A = 0$

$M_A + 15(5) = 0$

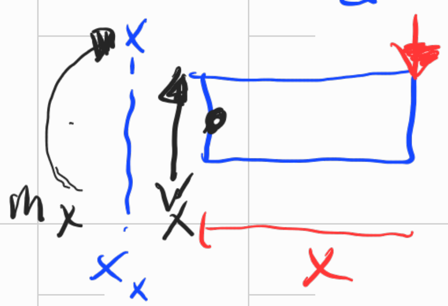
$M_A = -75 \text{ kN-m}$

$M_A = 75 \text{ CCW}$

→ Step 3 - Let us find internal Resistance Force (S.F & B.M)



→ Section method (R to L)



$\Sigma F_y = 0$

$V_x - 15 = 0$

$V_x = 15$

$+\Sigma m_{x\text{-}x} = 0$

$m_x + 15x = 0$

$m_x = -15x$

→ Step 4: Shear Force and Bending moment diagram

$\Sigma F_y = 0$

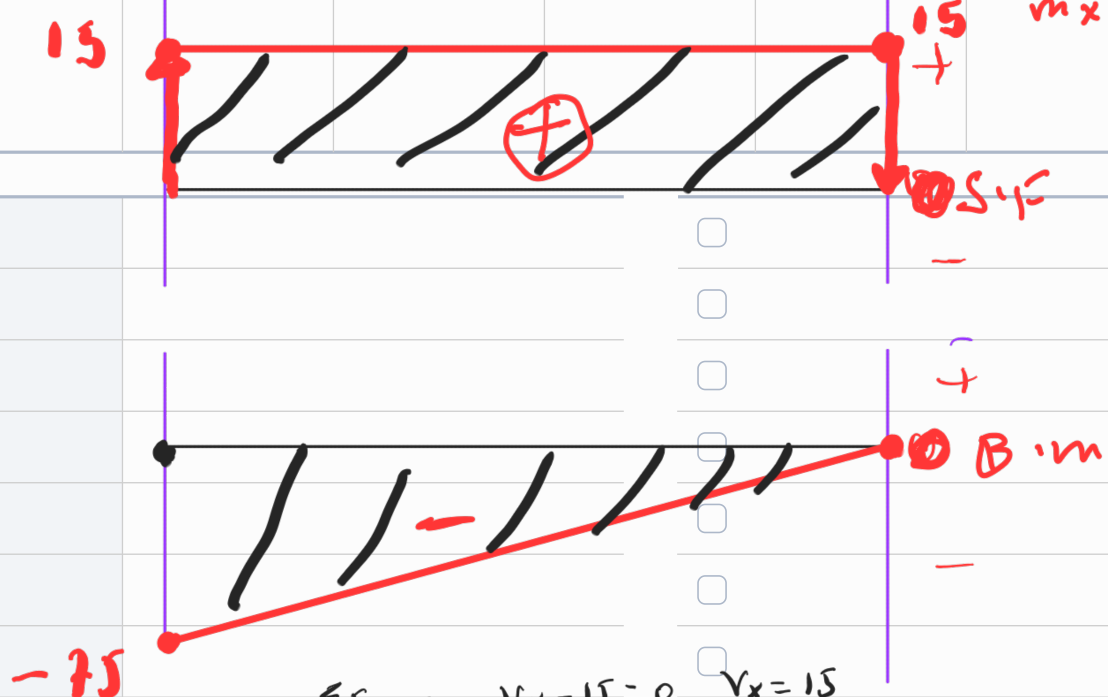
$V_x - 15 = 0$

$V_x = 15$

$+\Sigma m_{x\text{-}x} = 0$

$m_x + 15x = 0$

$m_x = -15x$

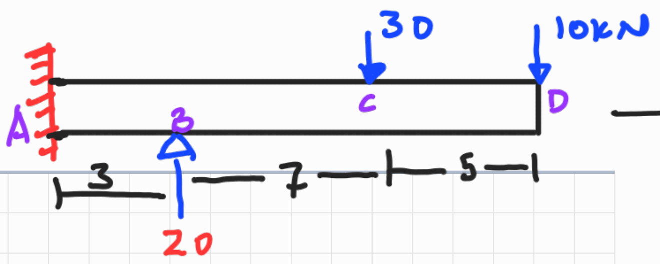


$\Sigma F_y = 0$ $\quad V_x - 15 = 0$ $\quad V_x = 15$

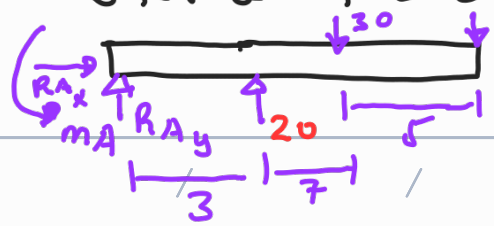
$+\Sigma m_{xx} = 0$ $\quad m_x + 15x = 0$

$m_x = -15x$

②



Step 1 · F.B.D



Step 2 - Reaction Force & moment

$\Sigma F_y = 0, \quad R_{Ay} + 20 - 30 - 10 = 0$

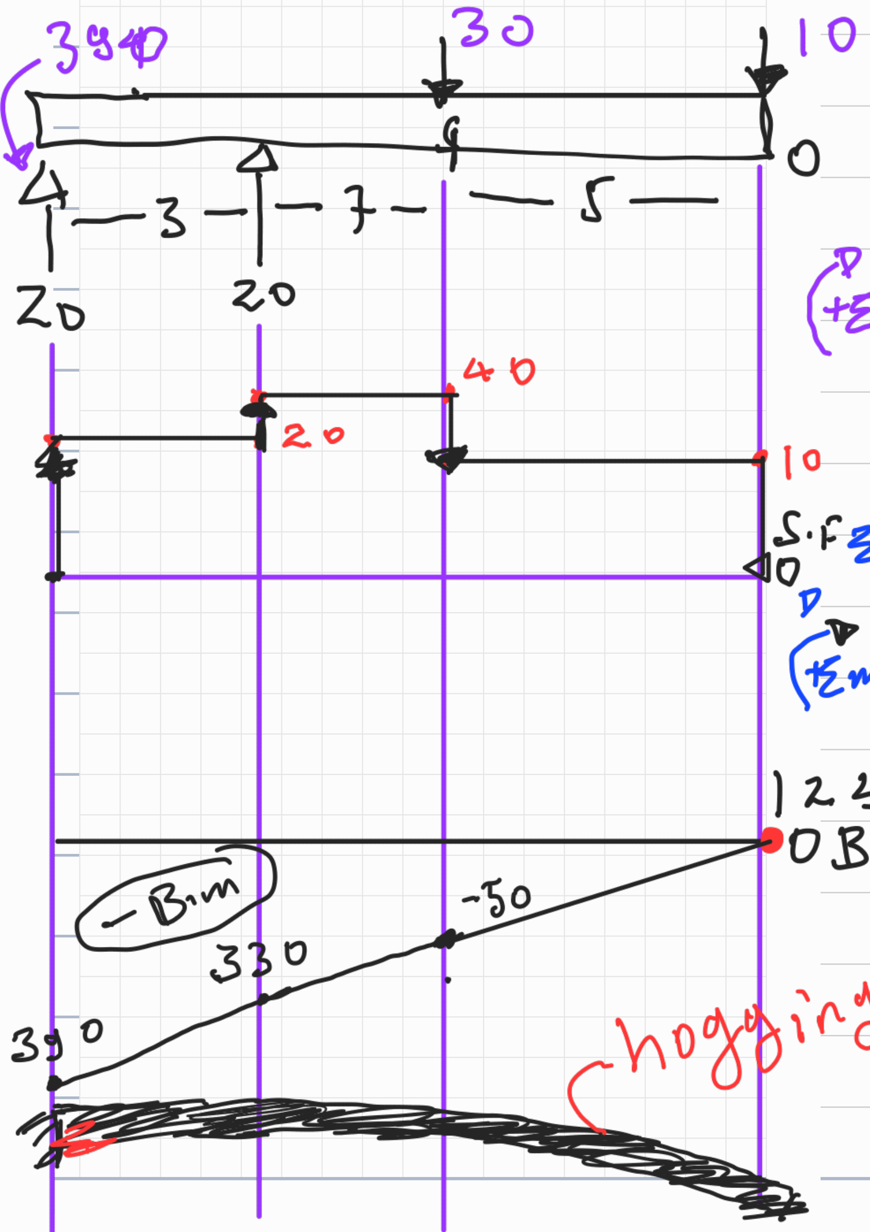
$R_{Ay} = 20$

$\Sigma M_A = 0 \quad M_A + 20(3) - 30(10) - 10(15) = 0$

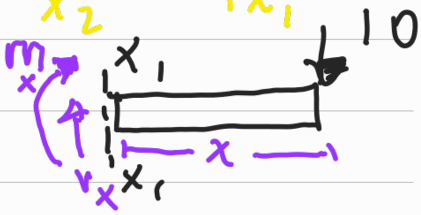
$M_A = 390 \, (CW)$

$\Sigma F_x = 0, \quad R_{Ax} = 0$

Step - S.F & B.m



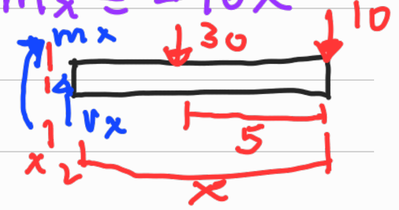
$0 \le x \le 5$



$\Sigma F_y = 0, \quad V_x = 10$

$\Sigma M_{xx_1} = 0 \quad M_x = -10x$

$5 \le x \le 12$

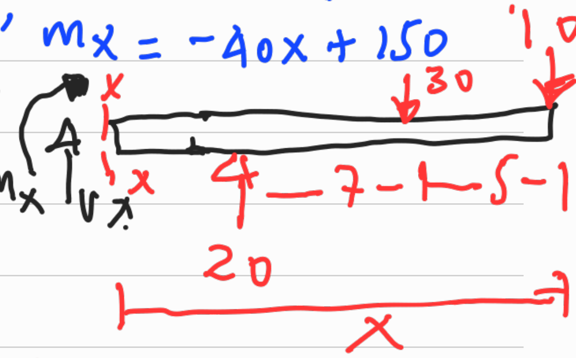


$\Sigma F_y = 0, \quad V_x - 30 - 10 = 0, \quad V_x = 40$

$\Sigma M_{xx} = 0, \quad M_x + 30(x-5) + 10x = 0$

$M_x = -40x + 150$

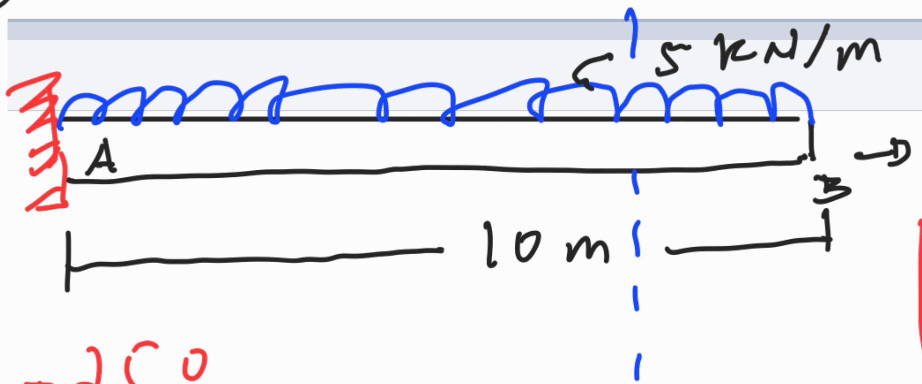
$12 \le x \le 15$



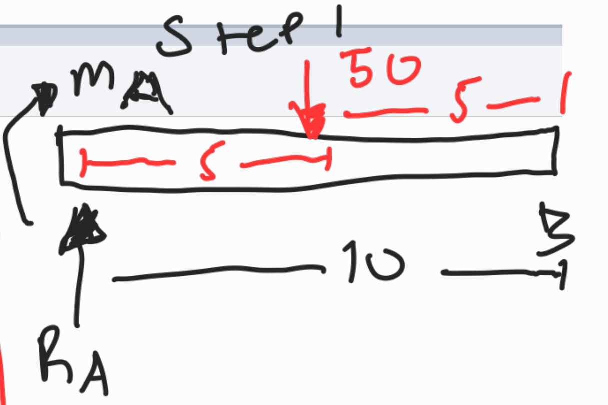
$\Sigma F_y = 0, \quad V_x = 20$

$\Sigma M_{xx} = 0, \quad M_x = -90 - 20x$

③ Case 2- Distributed load



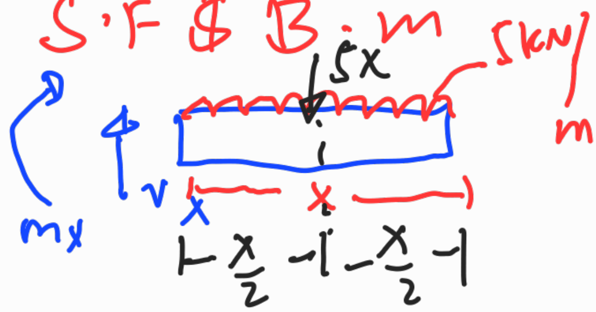
Step 1



$\Sigma F_y = 0, \quad R_A = 50$

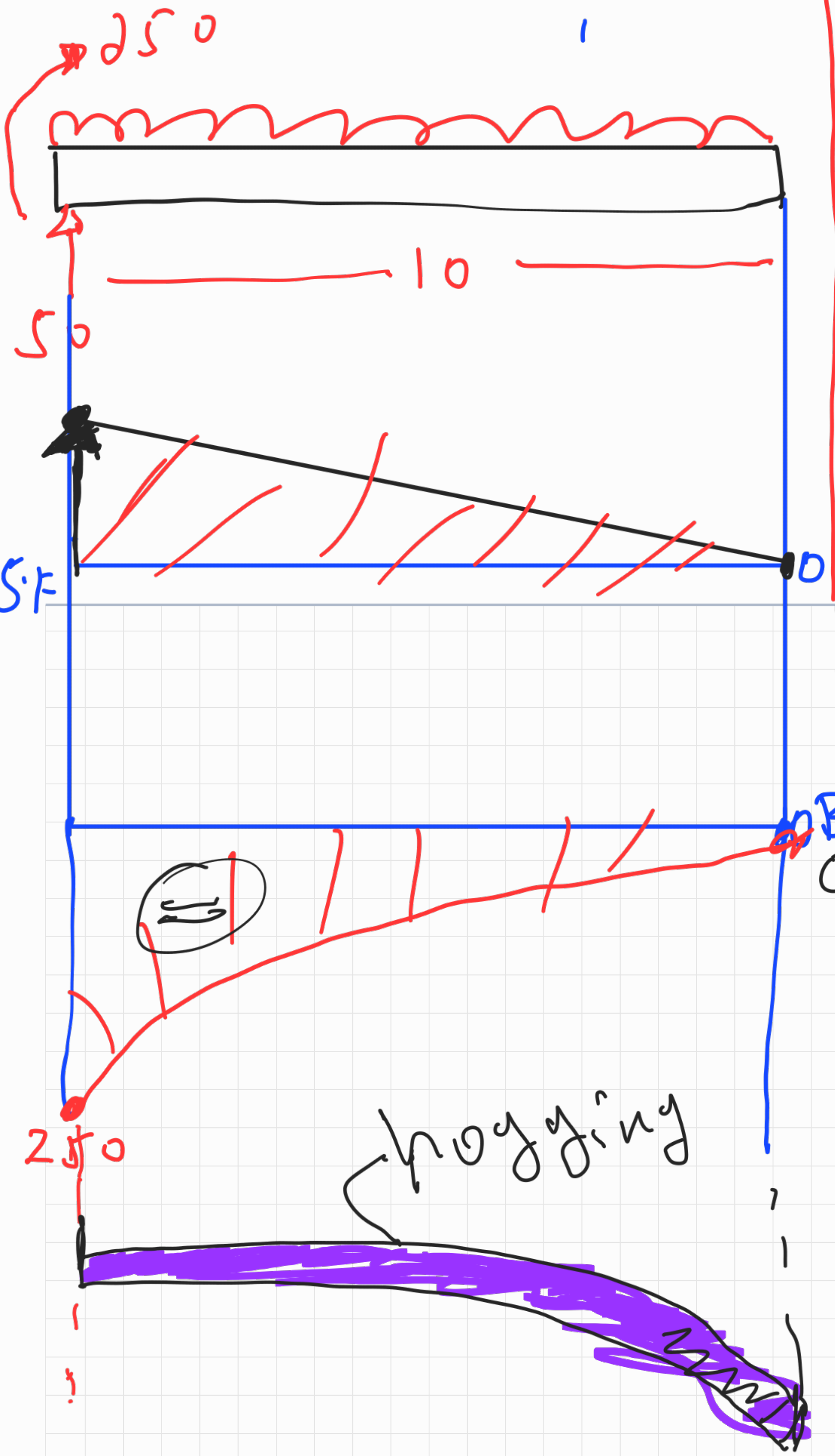
$\Sigma m_A = 0, \quad m_A = 250$ ccw

S.F & B.m

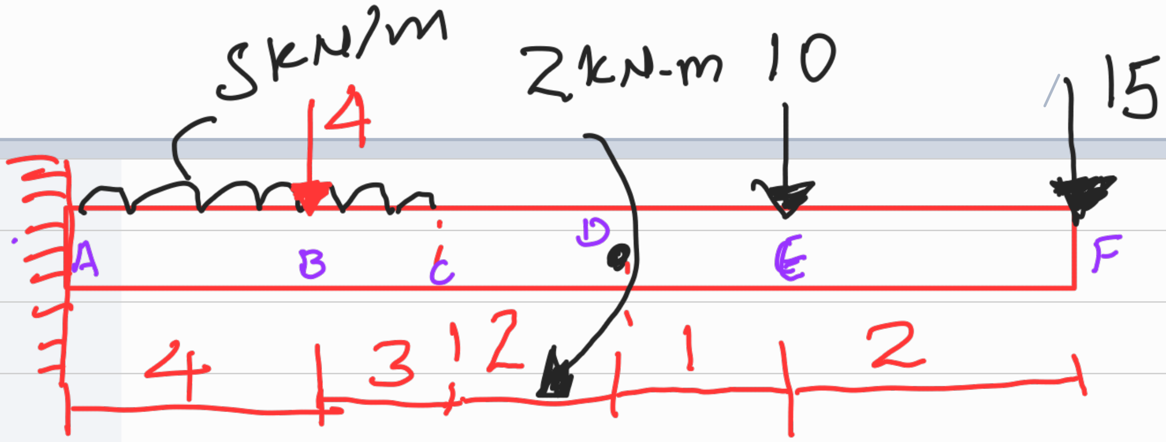


$\Sigma F_y = 0, \quad V_x = 5x$

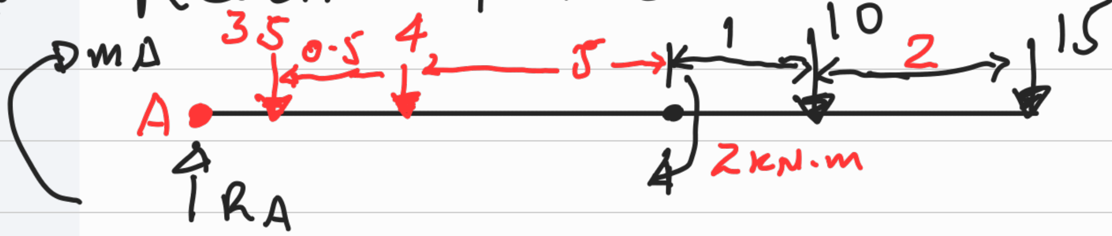
$\Sigma m_x = 0, \quad m_x = -\frac{5x^2}{2}$



④



Step 1 Reaction Force

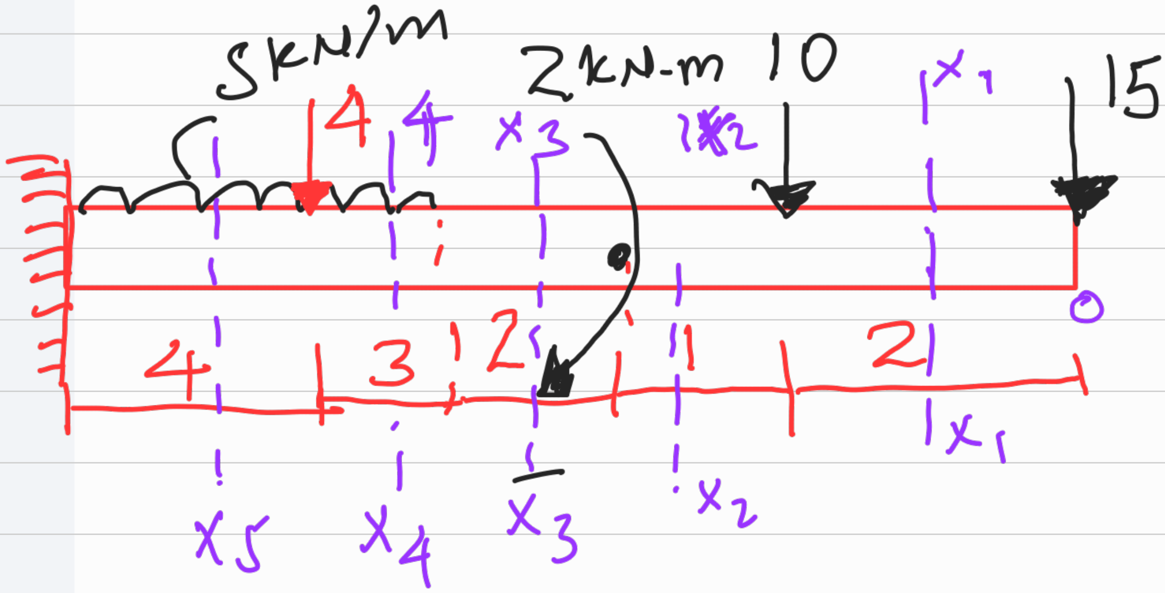


$R_A = 64$

$+\sum M_A = 0$ $\quad M_A + 35(3.5) + 4(4) + 2 + 10(10) + 15(12) = 0$

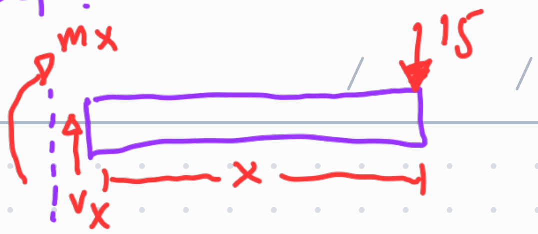
$M_A = -420.5 = 420.5 \text{ ccw}$

- Step - Shear Force and Bending moment at d/t Section



- From Right to left.

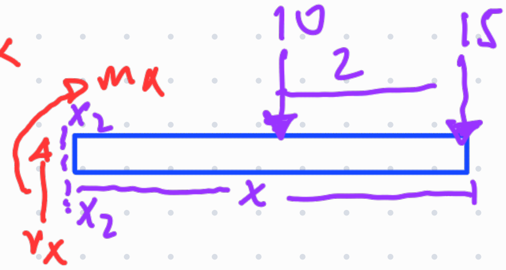
Section $x-x_1$, $0 \le x \le 2$



$V_x = 15$

$M_x = -15x$

Section $x-x_2$, $2 \le x \le 3$

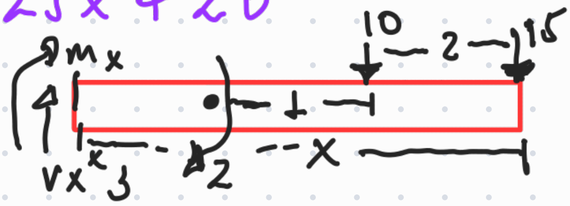


$V_x = 25$

$\circlearrowleft + \sum M_{xx_2} = 0, \quad M_x + 10(x-2) + 15x = 0$

$M_x = -25x + 20$

Section $x-x_3$, $3 \le x \le 5$

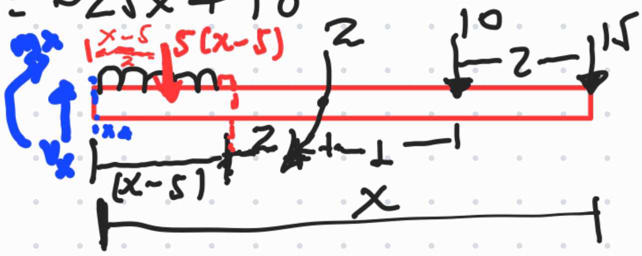


$V_x = 25$

$\circlearrowleft + \sum M_{x-x_3} = 0, \quad M_x + 2 + 10(x-2) + 15x = 0$

$M_x = -25x + 18$

Section $x-x_4$, $5 \le x \le 8$

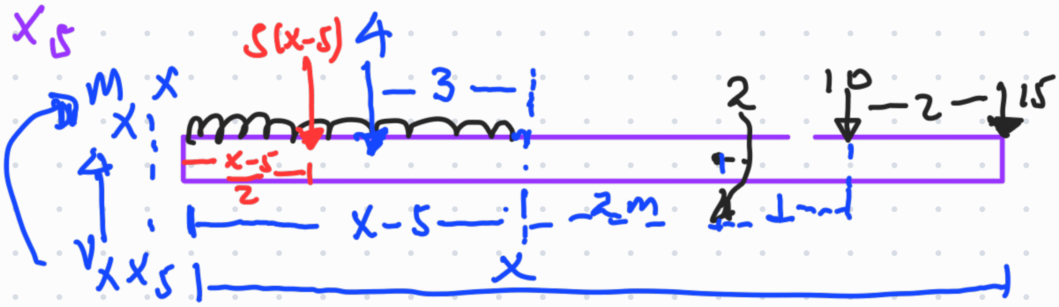


$\sum F_y = 0, \quad V_x - 5(x-5) - 10 - 15 = 0, \quad V_x = 5x$

$\circlearrowleft + \sum M_{x-x_4} = 0, \quad M_x + 5(x-5)\frac{(x-5)}{2} + 2 + 10(x-2) + 15x = 0$

$M_x = -2.5x^2 - 44.5$

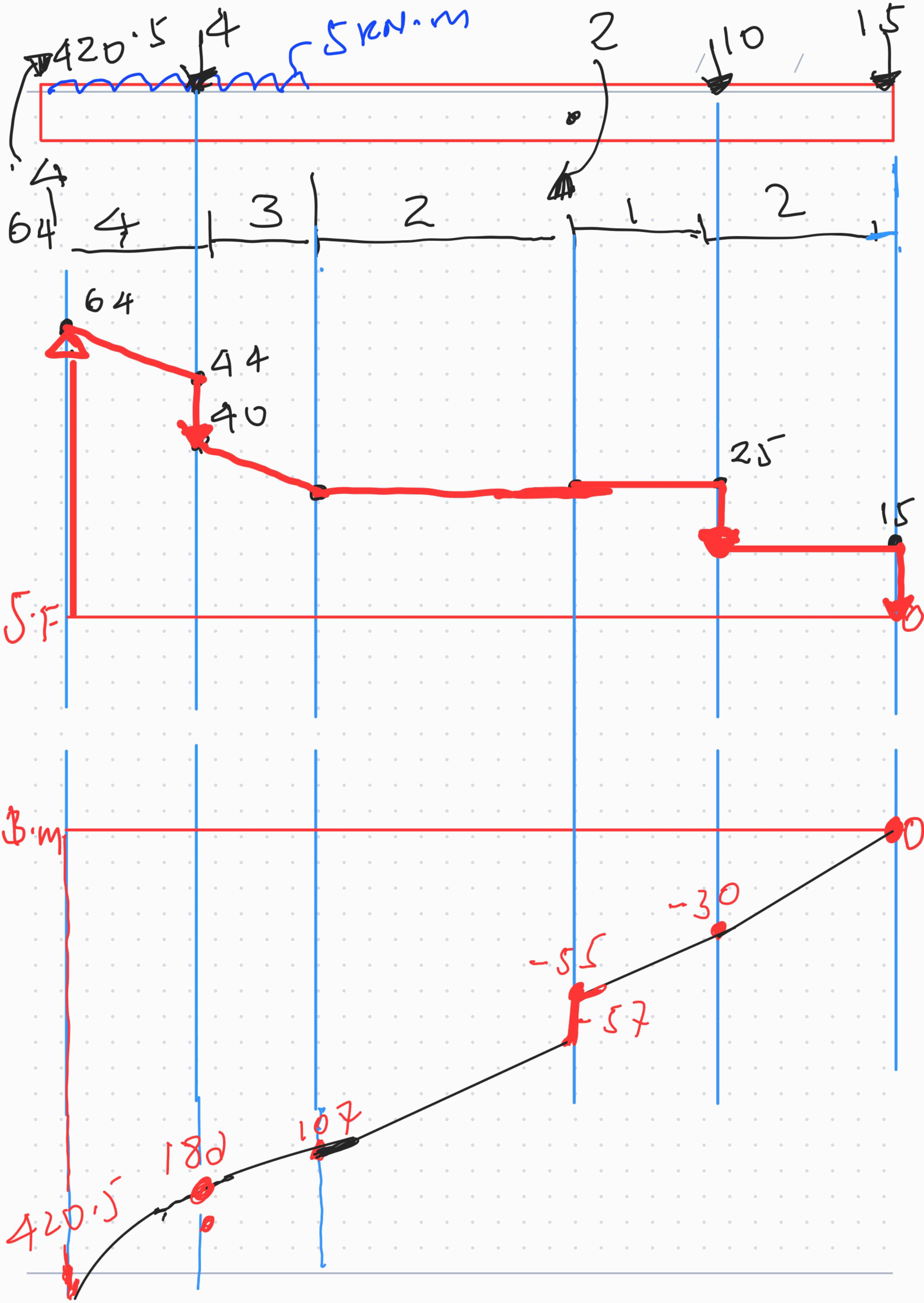
Section $x-x_5$



$\sum F_y = 0, \quad V_x - 5(x-5) - 4 - 10 - 15 = 0, \quad V_x = 5x + 4$

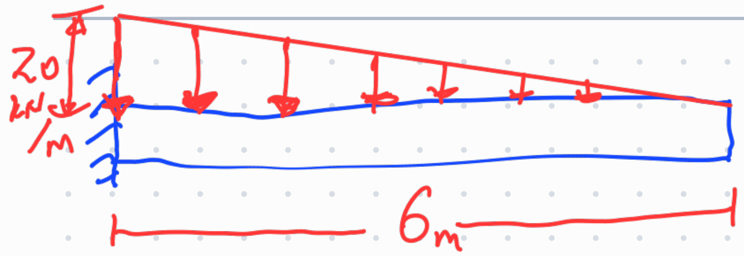
$\circlearrowleft + \sum M_{x-x_5} = 0, \quad M_x + 5(x-5)\frac{(x-5)}{2} + 4(x-8) + 2 + 10(x-2) + 15x = 0$

$M_x = -2.5x^2 - 4x - 12.5$

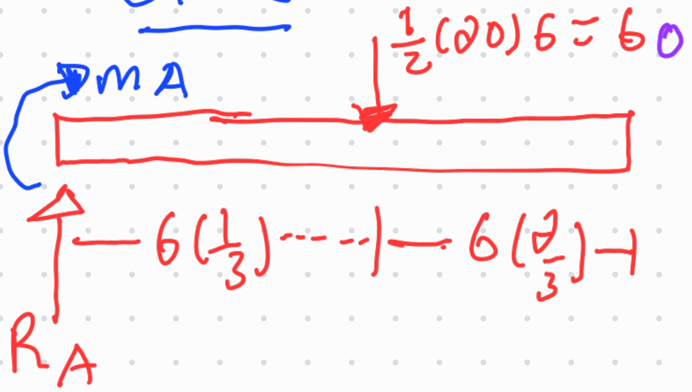


420·5
4
5KN·m
2
10
15
4
64
4
3
2
1
2
64
44
40
25
15
0
S·F
B·m
0
-30
-55
-57
107
180
420·5

(5) Case 3 - Cantilever beam with Uniformly Varying Load



Step 1 - F.B.D



- Reaction Force and moment

$R_A = 60$

$M_A = 120$ ccw

Step 2

- Shear and Bending
- taking Section R → L

$\frac{20}{6} = \frac{y}{x}$

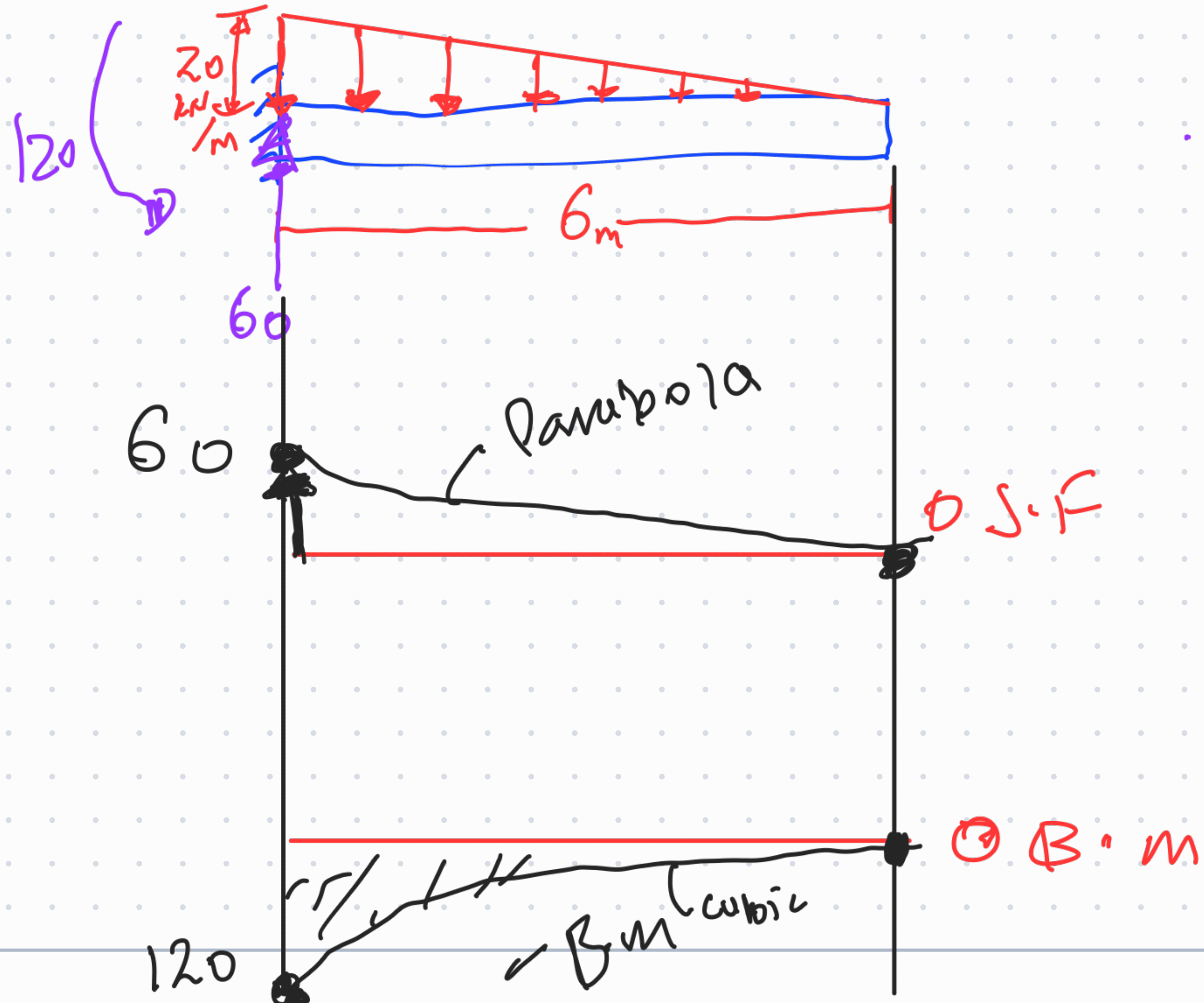
$y = \frac{20}{6}x$

$\sum F_y = 0, \quad V_x = \frac{20}{12}x^2$

$\circlearrowleft + \sum M_{xx} = 0 \quad M_x + \frac{20}{12}x^2\left(\frac{x}{3}\right) = 0$

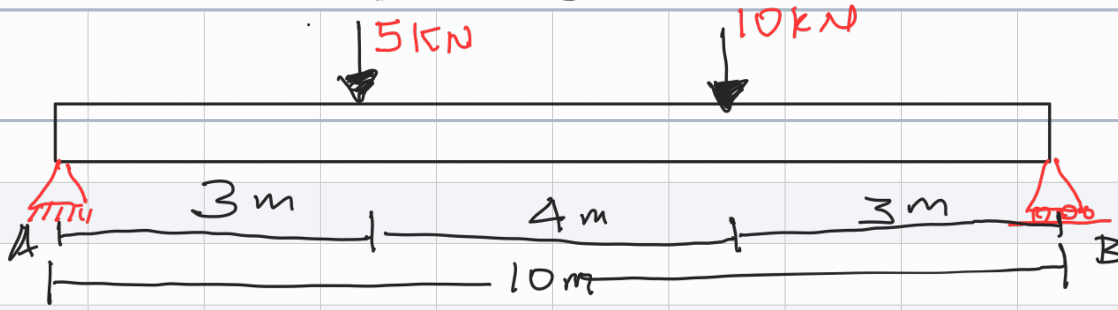
$M_x = \frac{-20}{36}x^3$

Step 3 S.F & B.M diagram.

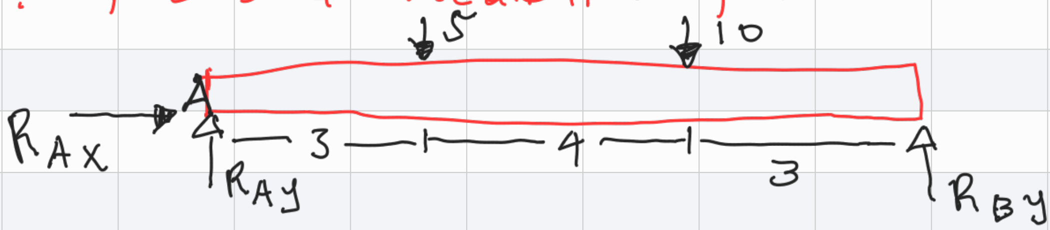


Draw the Shear force and Bending moment diagram for the following simply supported beams. also find the maximum bending moment

Problem 1



Step 1 : F.B.D & Reaction Force



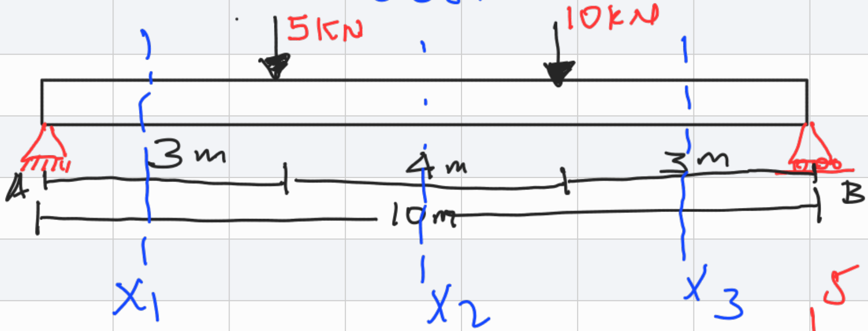
$\rightarrow \sum F_y = 0,\quad R_{Ay} + R_{By} = 15$ ---- eq (1)

$\rightarrow (+\sum M_A = 0 \qquad R_{By}(10) - 5(3) - 10(7) = 0$

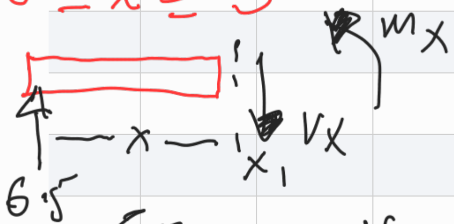
$R_{By} = \frac{85}{10} = 8.5\ kN$

i.e $R_{Ay} = 15 - R_{By} = 15 - 8.5 = 6.5\ kN$

Step B) :- internal Resistance (S.F & B.M)



$0 \le x \le 3$



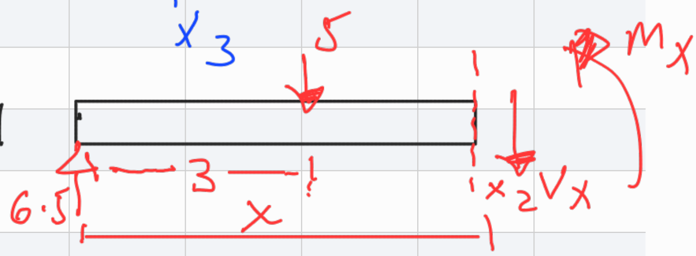
$\sum F_y = 0 \quad V_x = 6.5$

$(+\sum M_{xx_1} = 0,$

$M_x - 6.5x = 0$

$M_x = 6.5x$

$3 \le x \le 7$



$V_x = 1.5$

$(+\sum M_{xx_2} = 0, \quad M_x - 6.5x + 5(x-3) = 0$

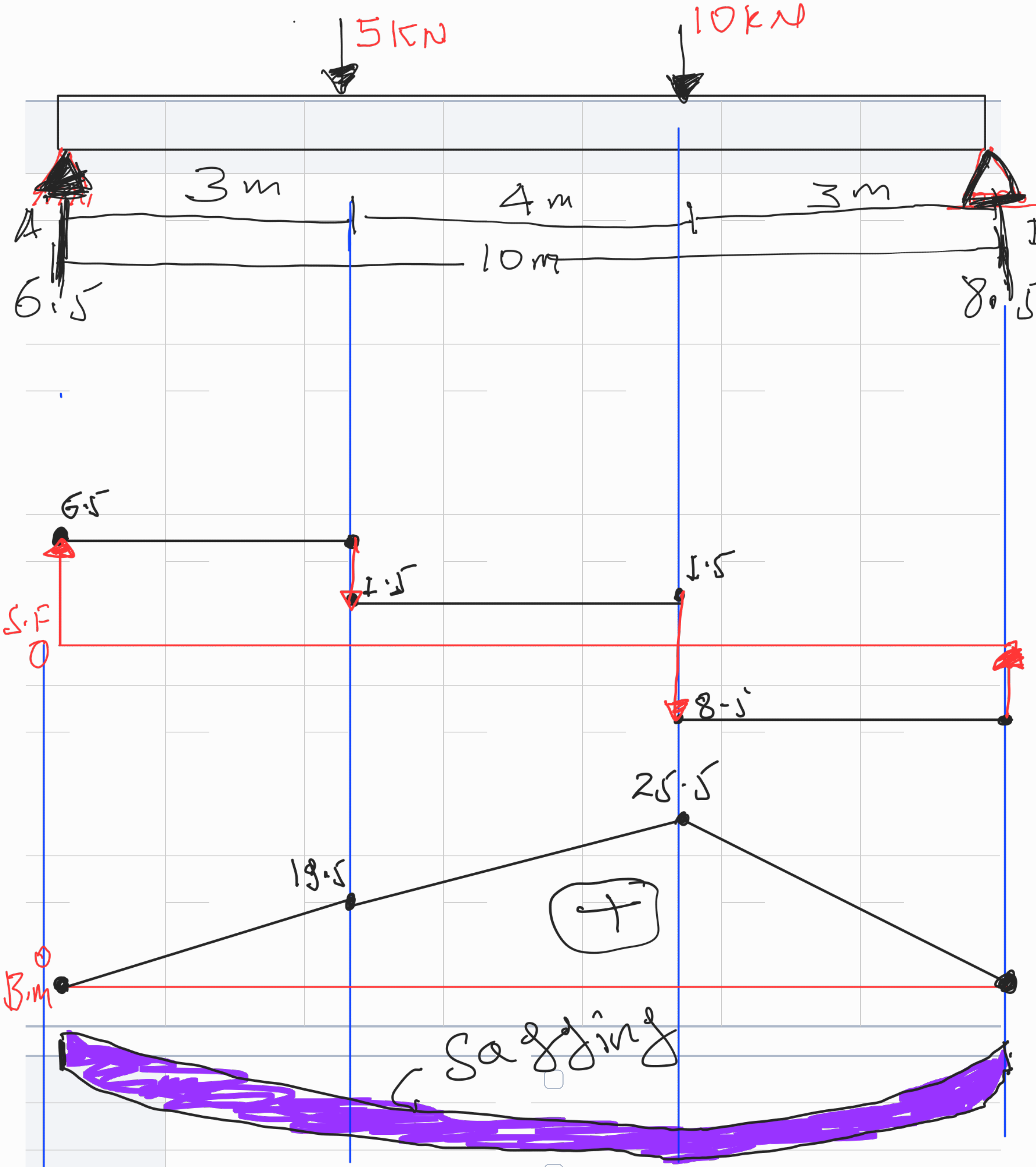
$M_x = 1.5x + 15$

$7 \le x \le 10$

$V_x = -8.5$

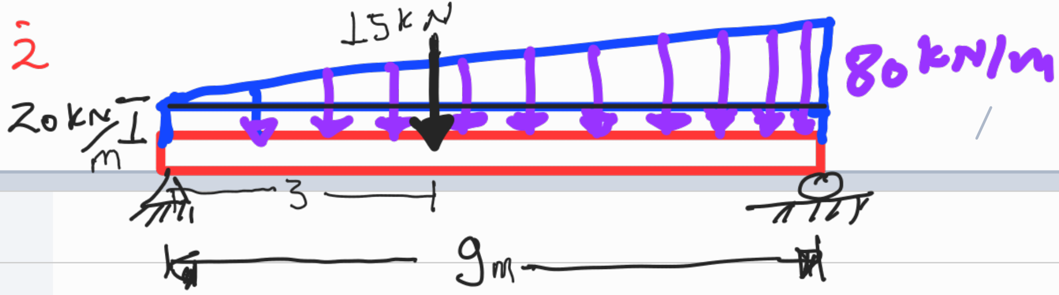
$(\sum M_{xx_4} = 0 \quad -6.5x + 5(x-3) + 10(x-7) + M_x = 0$

$M_x = -8.5x + 85$

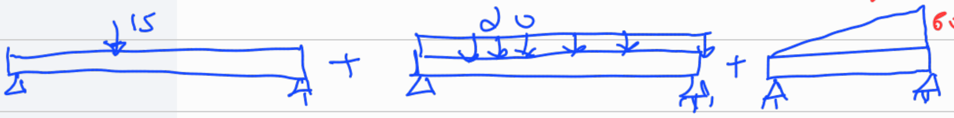


5KN
10KN
3m
4m
3m
A
10m
6.5
8.5
6.5
1.5
1.5
S.F
0
8.5
25.5
19.5
+
0
B.m
Sagging

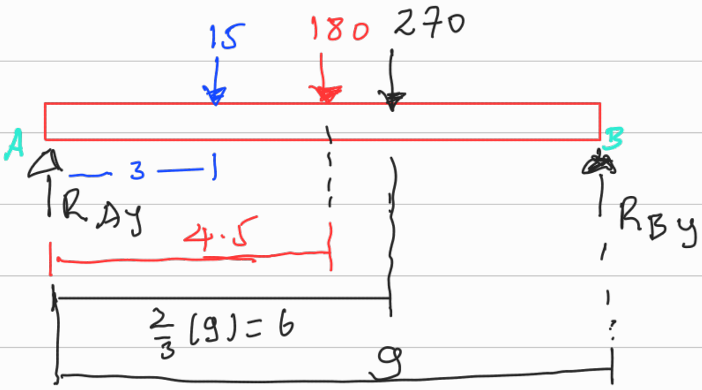
Problem 2



- Step 1 :- FBD & Reaction Force.



- the FBD of the three condition together →



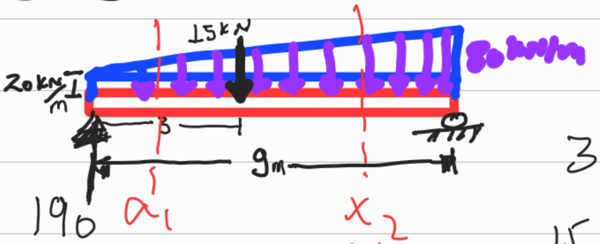
$\Sigma F_y = 0, \quad R_{Ay} + R_{By} = 465$

$+\Sigma M_A = 0 \quad R_{By}(9) - 270(6) - 180(4.5) - 15(3) = 0$

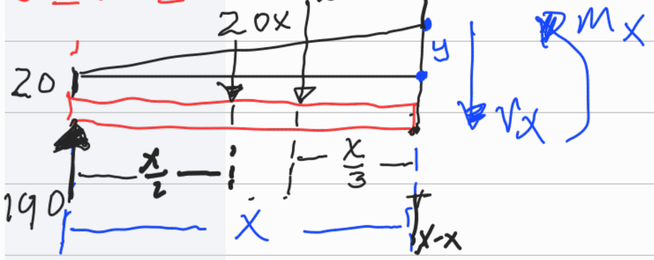
$$R_{By} = \frac{2475}{9} = 275 \text{ kN}$$

i.e $R_{Ay} = 465 - R_{By} = 190$ kN

- Step 2 :- the Shear Force and Bending moment



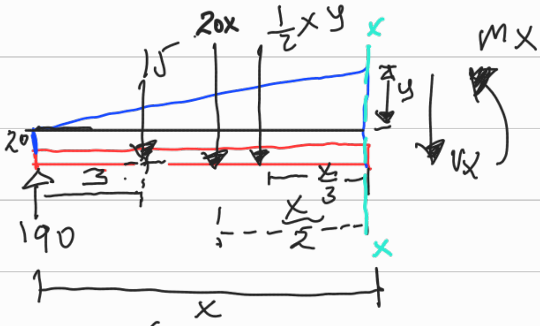
$0 \le x \le 3$



$$\frac{60}{9} = \frac{y}{x}$$

$$y = \frac{60}{9}x$$

$3 \le x \le 9$



$\Sigma F_y = 0, \quad V_x = -\frac{1}{2}\frac{60}{9}x^2 - 20x + 190$

$\Sigma M_{xx} = 0 \quad M_x + \frac{1}{2}\frac{60}{9}x^2\left(\frac{x}{3}\right) + 20x\left(\frac{x}{2}\right) - 190x = 0$

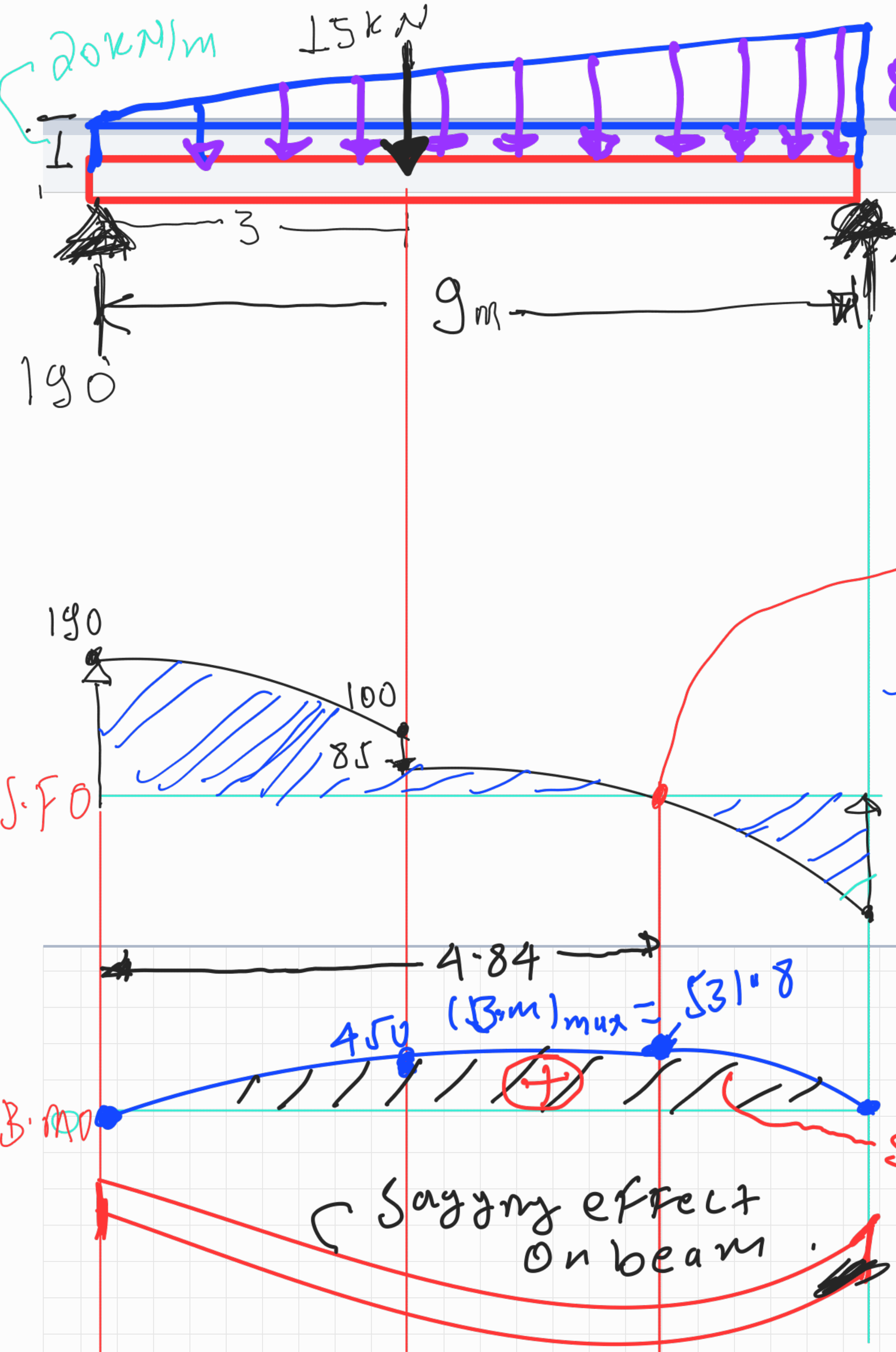
$$M_x = -\frac{10}{9}x^3 - 10x^2 + 190x$$

$$V_x = -\frac{60}{18}x^2 - 20x + 175$$

$+\Sigma M_{xx} = 0$

$M_x + \frac{60}{54}x^3 + 10x^2 + 15(x-3) - 190x = 0$

$$M_x = -\frac{10}{9}x^3 - 10x^2 + 175x + 45$$



B/n $3 \le x \le 9$

S.F = 0, S.O
B.m = max

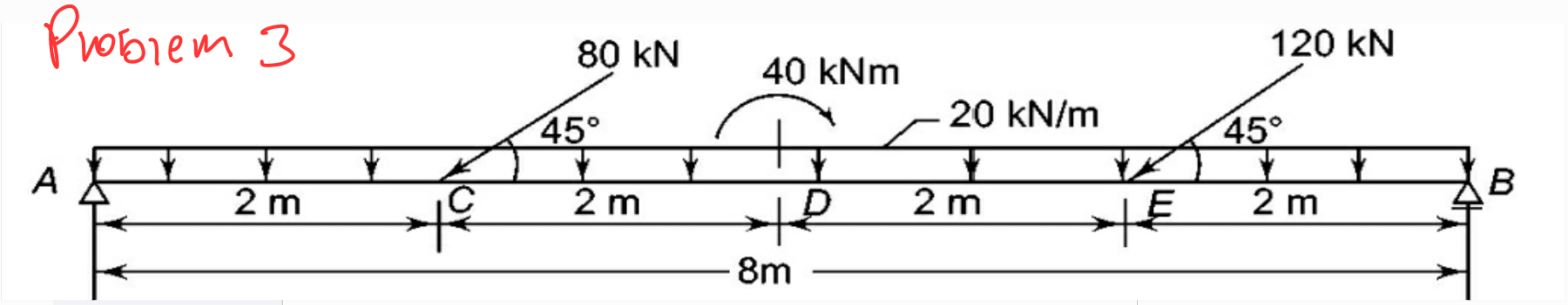
- to Find the d/c at w/n S.F become zero.

$0 = -\frac{60}{18}x^2 - 20x + 175$

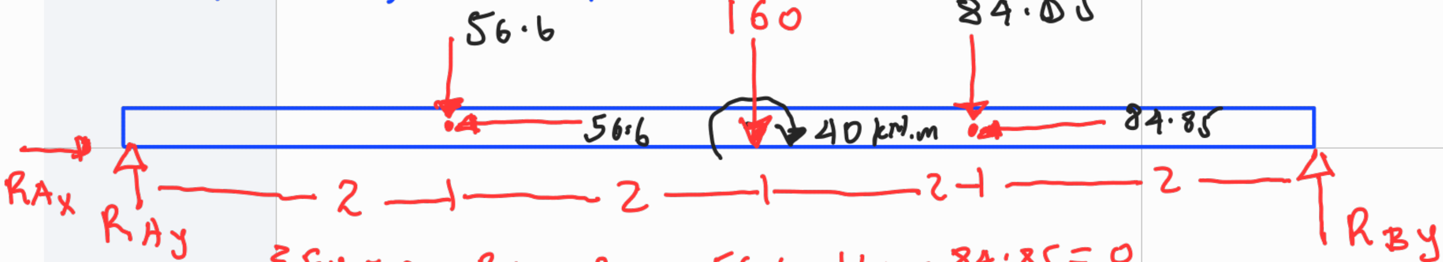
$X = 4.84m$

sagging

Problem 3



– Step 1 :- F.B.D & Reaction Forces



$\sum F_y = 0, \quad R_{Ay} + R_{By} - 56.6 - 160 - 84.85 = 0$

$R_{Ay} + R_{By} = 301.45$ .... eq 1

$\sum F_x = 0, \quad R_{Ax} - 56.6 - 84.85 = 0$

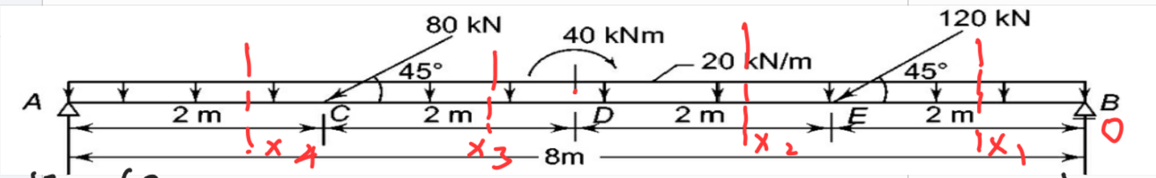
$R_{Ax} = 141.45$

$\sum M_A = 0, \quad R_{By}(8) - 84.85(6) - 160(4) - 56.6(2) - 40 = 0$

$R_{By} = 162.78$

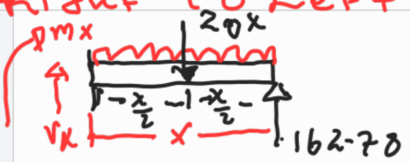
i.e $R_{Ay} = 301.45 - R_{By} = 138.67$ kN

– Step 2 :- Shear Force and Bending moment



– From Right to Left

$0 \le x \le 2$

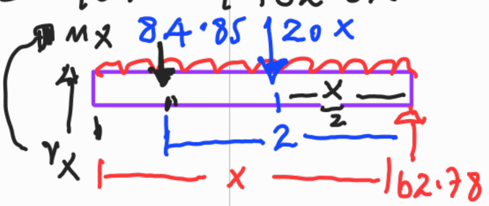


$V_x = 20x - 162.78$

$\sum M_{xx_1} = 0, \quad -m_x - 20x\left(\frac{x}{2}\right) + 162.8x = 0$

$m_x = -10x^2 + 162.8x$

$2 \le x \le 4$



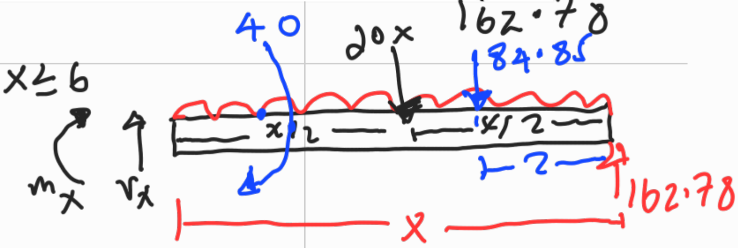
$V_x = 20x - 77.5$

$\sum M_{xx_2} = 0,$

$m_x + 84.85(x-2) + 20x\left(\frac{x}{2}\right) - 162.78x = 0$

$m_x = -10x^2 + 77.9x + 169.7$

$4 \le x \le 6$



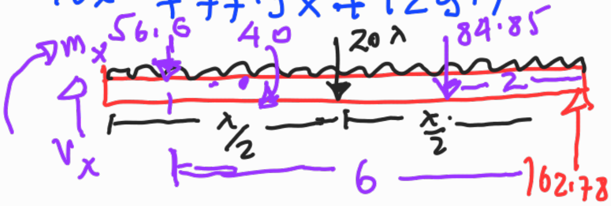
$V_x = 20x - 77.95$

$\sum M_{xx} = 0,$

$m_x = -10x^2 + 77.9x + 169.7 - 40$

$= -10x^2 + 77.9x + 129.7$

$6 \le x \le 8$

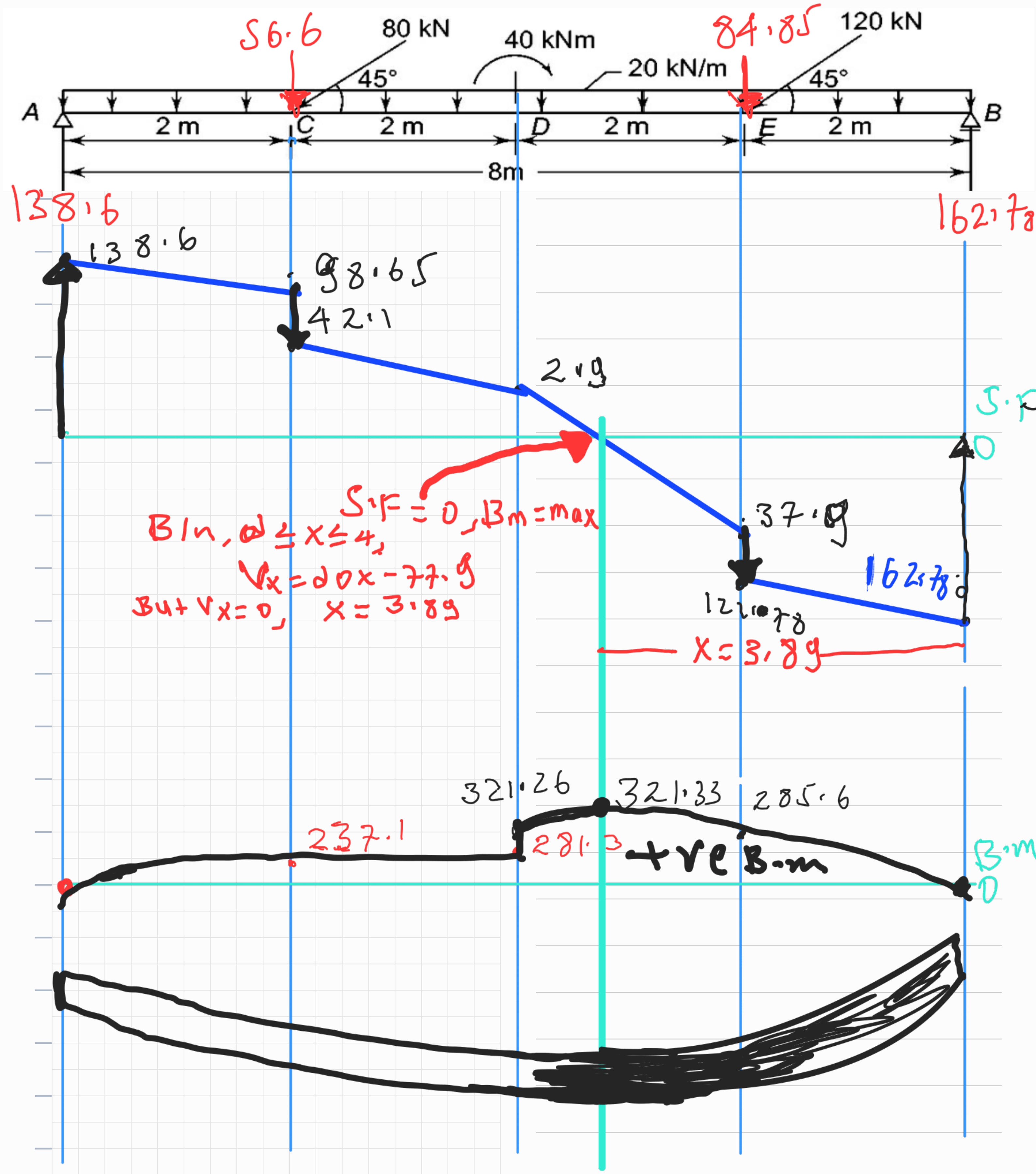


$V_x = 20x - 21.33$

$\sum M_{xx_4}$

$m_x + 56.6(x-6) + 40 + 10x^2 + 84.85(x-2) - 162.78x = 0$

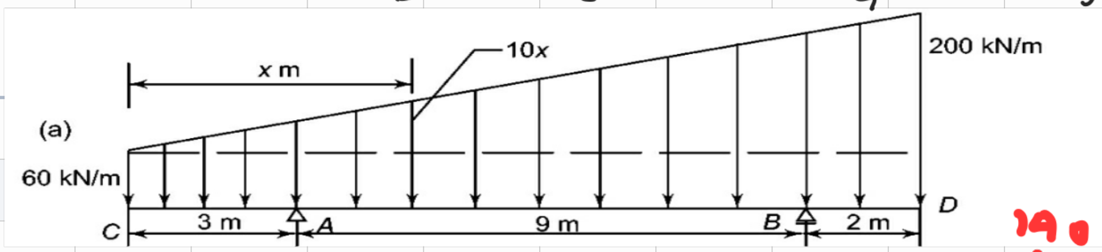
$m_x = -10x^2 + 21.35x + 469.3$



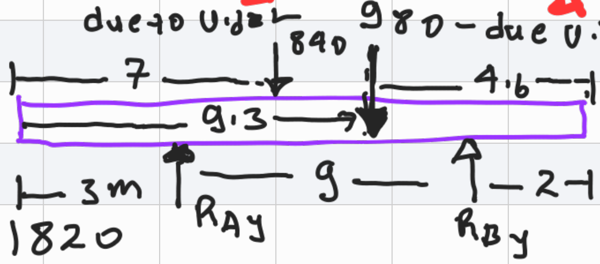
56.6
80 kN
40 kNm
84.85
120 kN
20 kN/m
45°
45°
A
B
C
D
E
2 m
2 m
2 m
2 m
8m
138.6
162.78
138.6
98.65
42.1
2.9
S.F
0
S.F = 0, Bm = max
B/n, 0 ≤ x ≤ 4,
Vx = 20x − 77.9
But Vx = 0, x = 3.89
37.9
162.78
121.78
x = 3.89
321.26
321.33
285.6
237.1
281.3
+ve B.m
B.m
0

Draw the Shear Force and Bending moment diagram for the following over hanging beams. also find the maximum bending moment & Counter flexure Point

Problem 1



-Step 1- F.B.D.



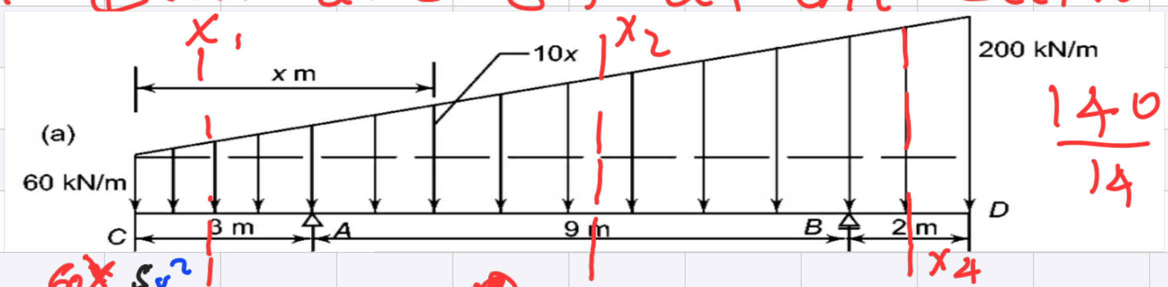
$\Sigma F_y = 0, \quad R_{Ay} + R_{By} = 1820$

$\Sigma M_A = 0 \quad R_{By}(9) - 980(6.3) - 840(4) = 0$

$R_{By} = 1059.3$

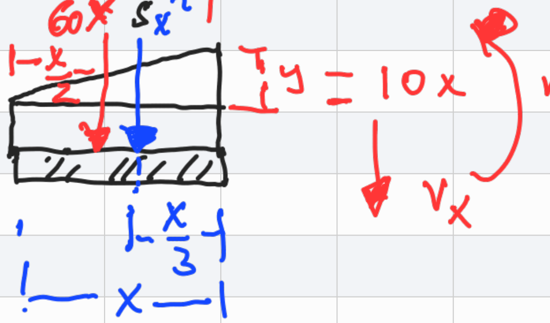
i.e $R_{Ay} = 1820 - R_{By} = 760.6$

Step 2: B.m and S.F at d/t Section.

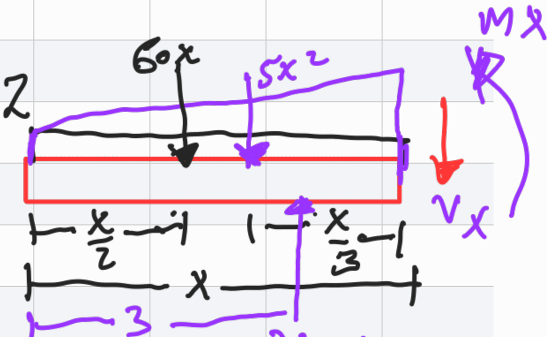


$\frac{140}{14} = \frac{y}{x}$

$0 \le x \le 3$



$3 \le x \le 12$



$\Sigma F_y = 0, \quad V_x + 60x + 5x^2 = 0$

$V_x = -5x^2 - 60x$

$V_x = -5x^2 - 60x + 760.6$

$\Sigma M_{xx} = 0$

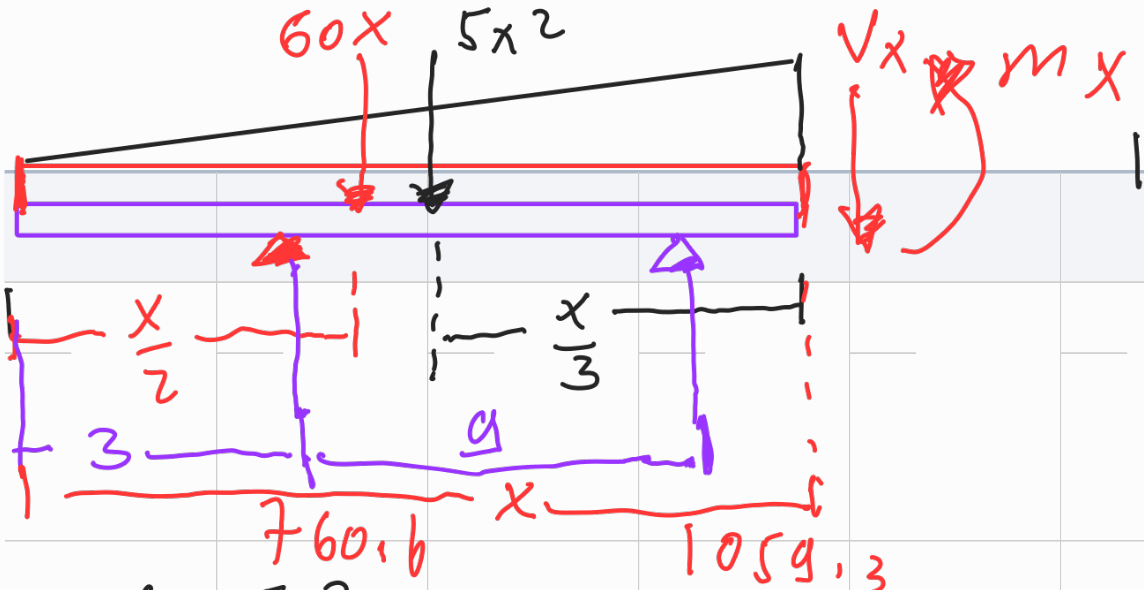
$m_x + 60x\left(\frac{x}{2}\right) + 5x^2\left(\frac{x}{3}\right) = 0$

$m_x = -30x^2 - \frac{5}{3}x^3$

$\Sigma M_{xx} = 0$

$m_x + \frac{5x^3}{3} + 30x^2 - 760.6(x-3) = 0$

$m_x = -\frac{5}{3}x^3 - 30x^2 + 760.6x - 2281.8$



$12 \le x \le 14$

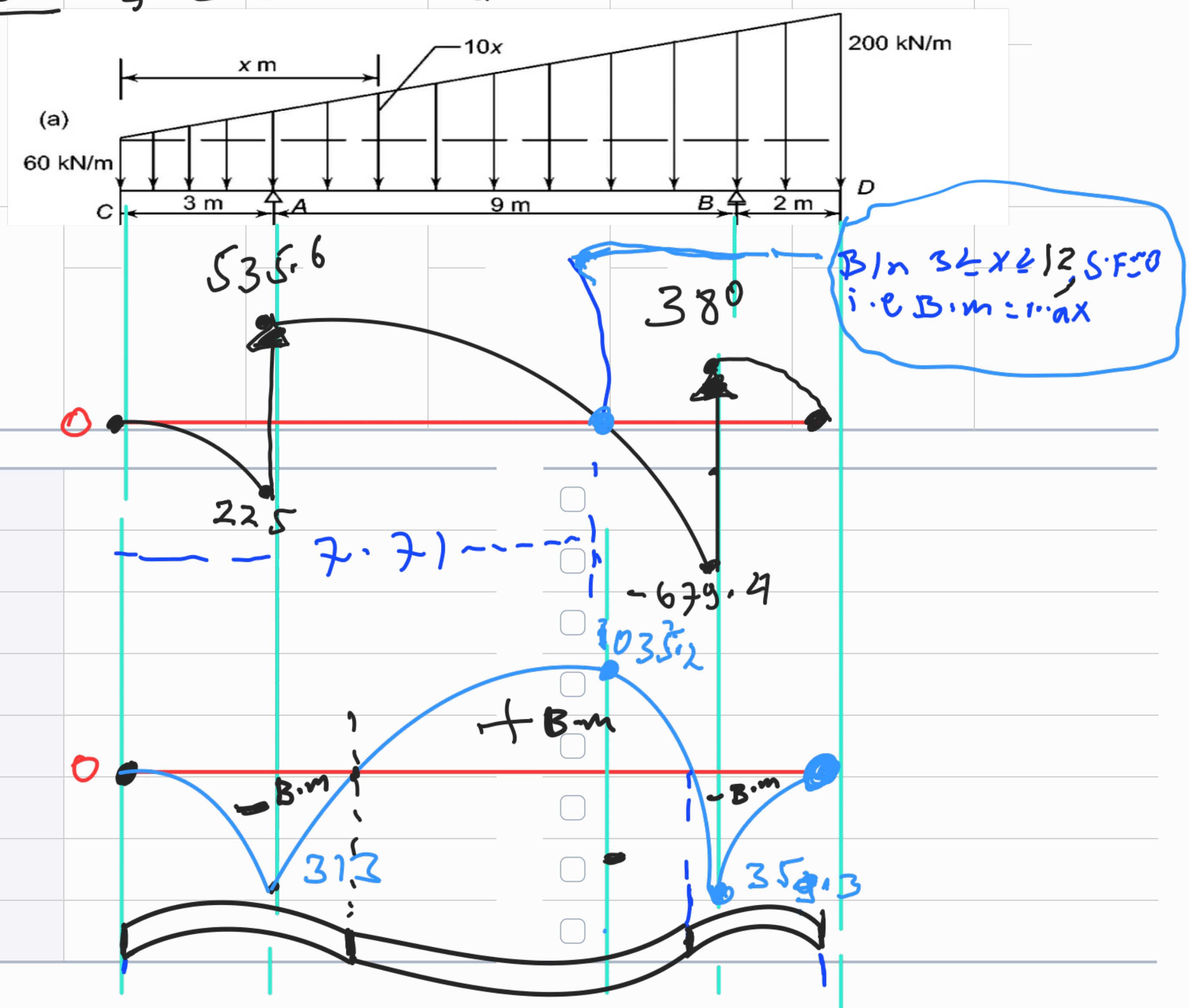
$Vx = -5x^2 - 60x + 1820$

$\Sigma m_{xx} = 0$

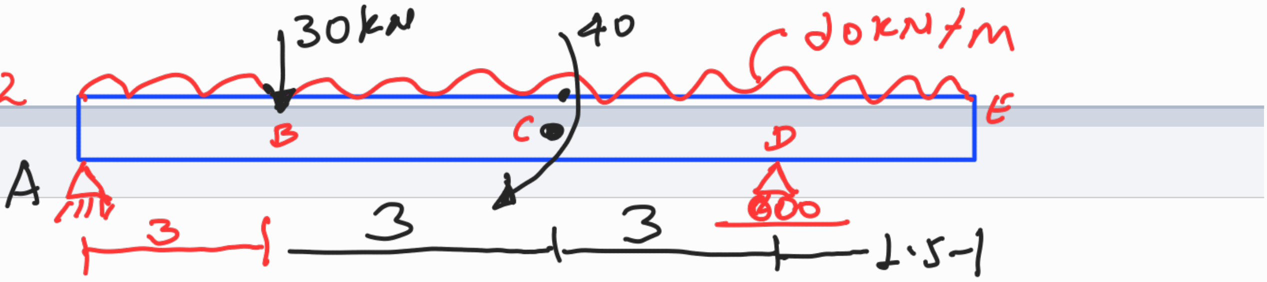
$mx - 1059.3(x-12) + \frac{5x^3}{3} + 30x^2 - 760.6(x-3) = 0$

$mx = -\frac{5x^3}{3} - 30x^2 + 1820x - 14993.4$

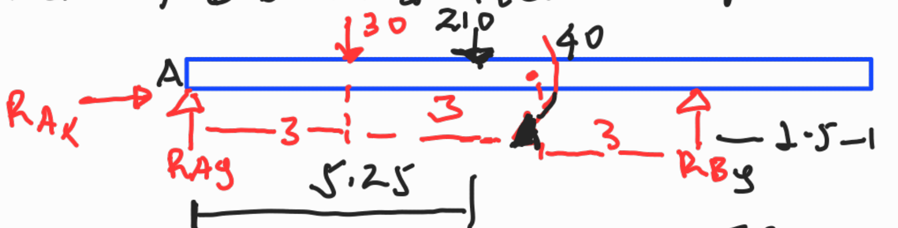
- Step 4 : S.F and B.m diagram



Problem 2



- Step 1 – F.B.D and Reaction Force



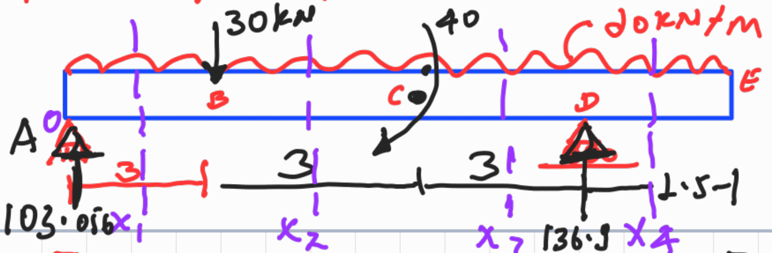
$\Sigma F_y = 0$, $R_{Ay} + R_{By} = 240$, $\Sigma F_x = 0$, $R_{Ax} = 0$

$\circlearrowleft + \Sigma M_A = 0$ $\quad -30(3) - 210(5.25) - 40 + R_{By}(9) = 0$

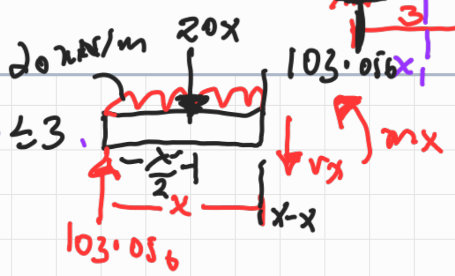
$R_{By} = \frac{1232.5}{9} = 136.944$ kN

i.e $R_{Ay} = 240 - R_{By} = 103.056$ kN

Step 2 :- Internal Resistance (B.M & S.F) at diff Section



$0 \le x \le 3$

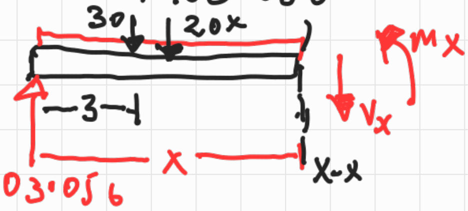


$V_x = -20x + 103.056$

$\circlearrowleft + \Sigma M_{x_x} = 0$ $\quad M_x + 20x\left(\frac{x}{2}\right) - 103.056x$

$M_x = -10x^2 + 103.056x$

$3 \le x \le 6$



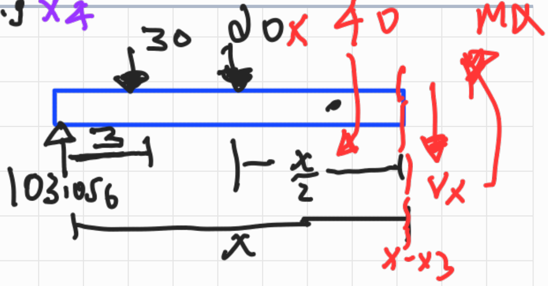
$\Sigma F_y = 0$ $\quad V_x = -20x + 73.056$

$\circlearrowleft + \Sigma M_{x_x} = 0$

$M_x + 20x\left(\frac{x}{2}\right) + 30(x-3) - 103.056x = 0$

$M_x = -10x^2 + 73.056x + 90$

$6 \le x \le 9$

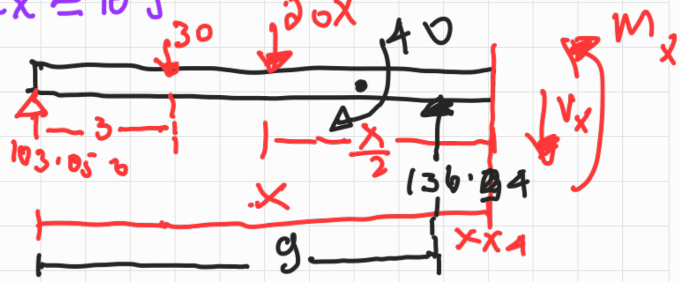


$\Sigma F_y = 0$, $V_x = -20x + 73.056$

$\circlearrowleft \Sigma M_{x_3} = 0$

$M_x = -10x^2 + 73.056x + 130$

$9 \le x \le 10.5$

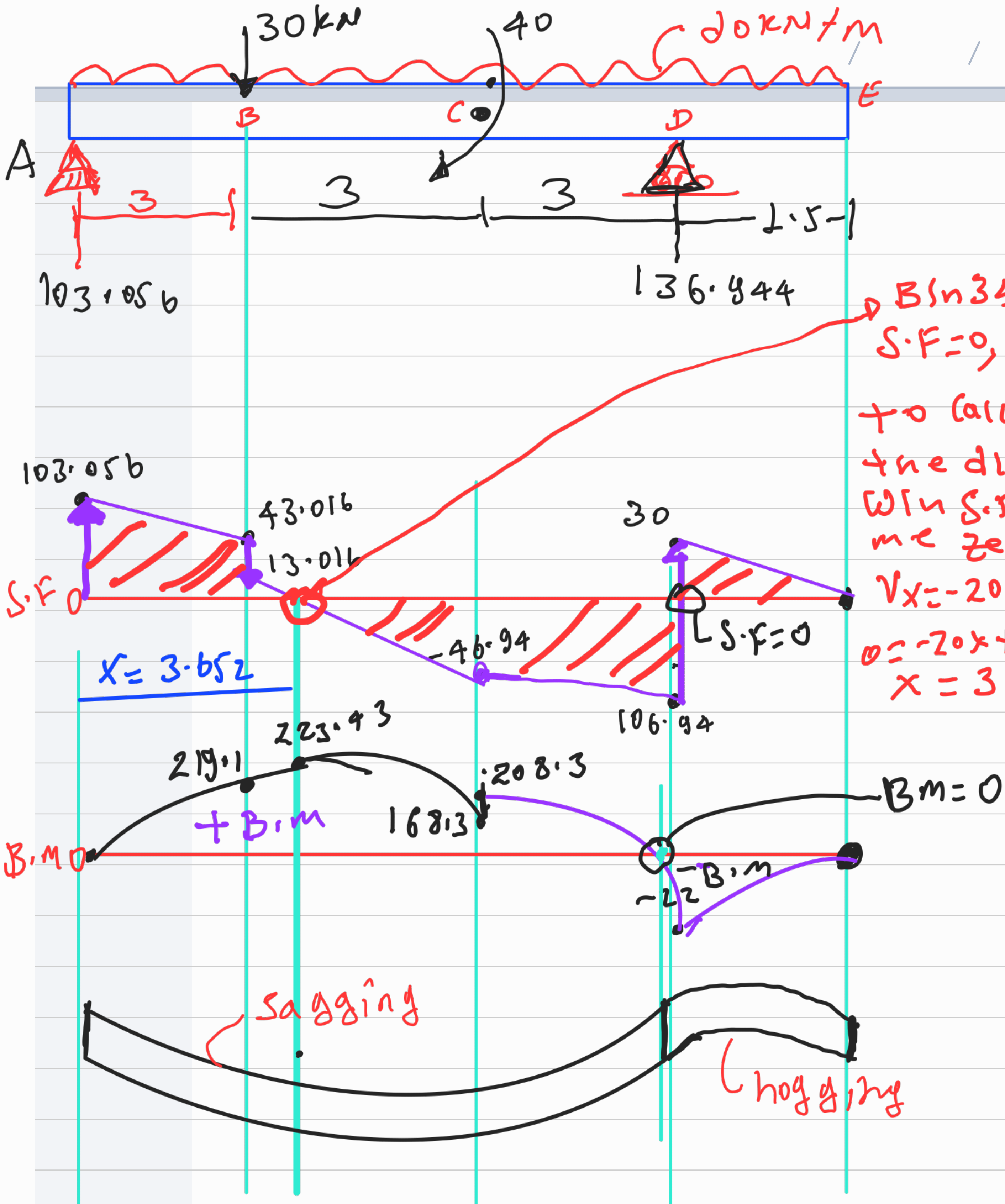


$V = -20x + 210$

$\circlearrowleft + \Sigma M_{x_4} = 0$

$M_x - 40 + 10x^2 + 30(x-3) - 103.056x - 136.94(x-9) = 0$

$M_x = -10x^2 + 210x - 1102.486$



B/n $3 \le x \le 6$
S.F = 0, BM = max

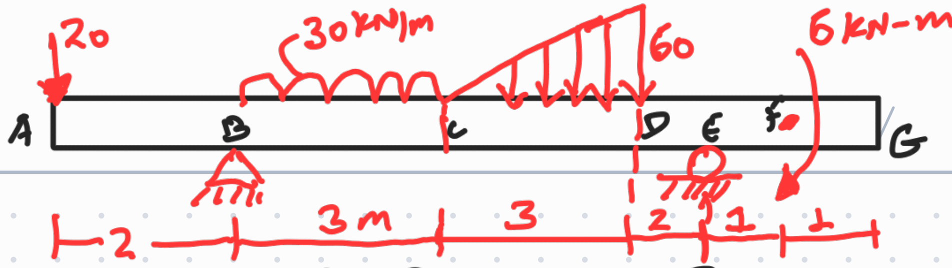
to calculate the dist at wich S.F become zero

$V_x = -20x + 73.056$

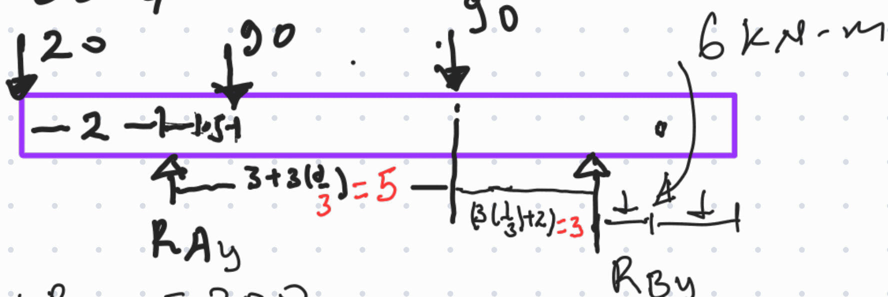
$0 = -20x + 73.056$

$x = 3.652$

Problem 3



Step 1 :- F.B.D & Reaction Force.

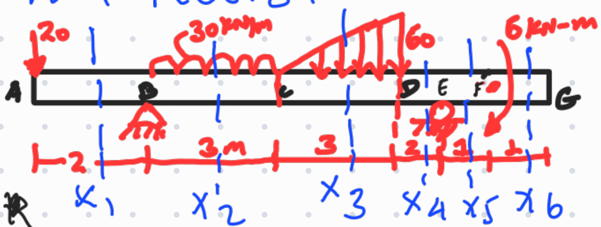


$\Sigma F_y = 0, \quad R_{Ay} + R_{By} = 200$

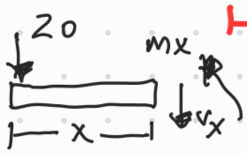
$\Sigma M_A = 0 \quad 20(2) - 90(1.5) - 90(5) + R_{By}(8) - 6 = 0$

$R_{By} = \frac{551}{8} = 68.875, \quad R_{Ay} = 200 - R_{By} = 131.125\,kN$

Step 2 :- Internal Resistance (S.F & B.M)



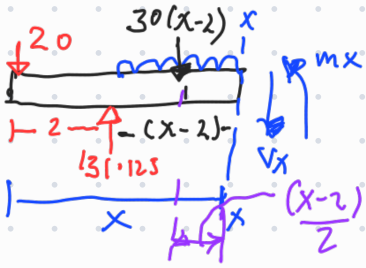
$0 \le x \le 2$



$V_x = -20$

$+\Sigma M_{x_x} = 0 \quad M_x = -20x$

$2 \le x \le 5$



$\Sigma F_y = 0 \quad -V_x - 20 + 131.125 - 30(x-2) = 0$

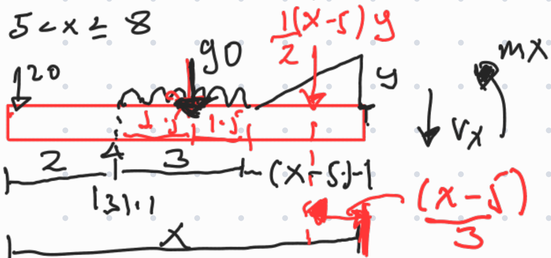
$V_x = -30x + 171.125$

$+\Sigma M_{x_x} = 0$

$m_x + 30(x-2)\left(\frac{x-2}{2}\right) + 20x - 131.125(x-2) = 0$

$M_x = -15x^2 + 171.12x - 322.2$

$5 < x \le 8$



$\frac{60}{3} = \frac{y}{(x-5)}$

$y = 20(x-5)$

$\Sigma F_y = 0, \quad V_x = -10(x-5)(x-5) + 131.1 - 110$

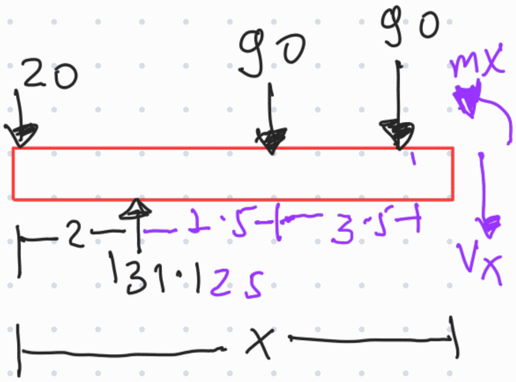
$= -10x^2 + 100x - 228.9$

$\Sigma M_{x_x} = 0$

$m_x + 10(x-5)(x-5)\left(\frac{x-5}{3}\right) + 90(x-3.5) + 20x - 131.1(x-2) = 0$

$M_x = -\frac{10}{3}x^3 + 50x^2 - 228.8x + 469.26$

$8 \le x \le 10$



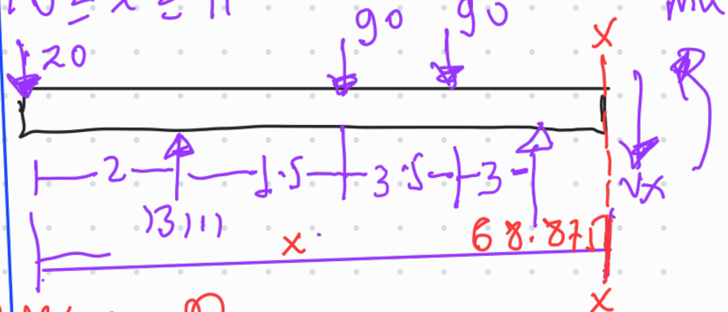
$\Sigma F_y = 0, \quad V_x = -68.9$

$\Sigma M_{x_x} = 0$

$m_x + 90(x-7) + 90(x-3.5) + 20x - 131.1(x-2) = 0$

$M_x = -68.9x + 682.8$

$10 \le x \le 11$



$V_x = 0$

$\Sigma M_{x_x} = 0$

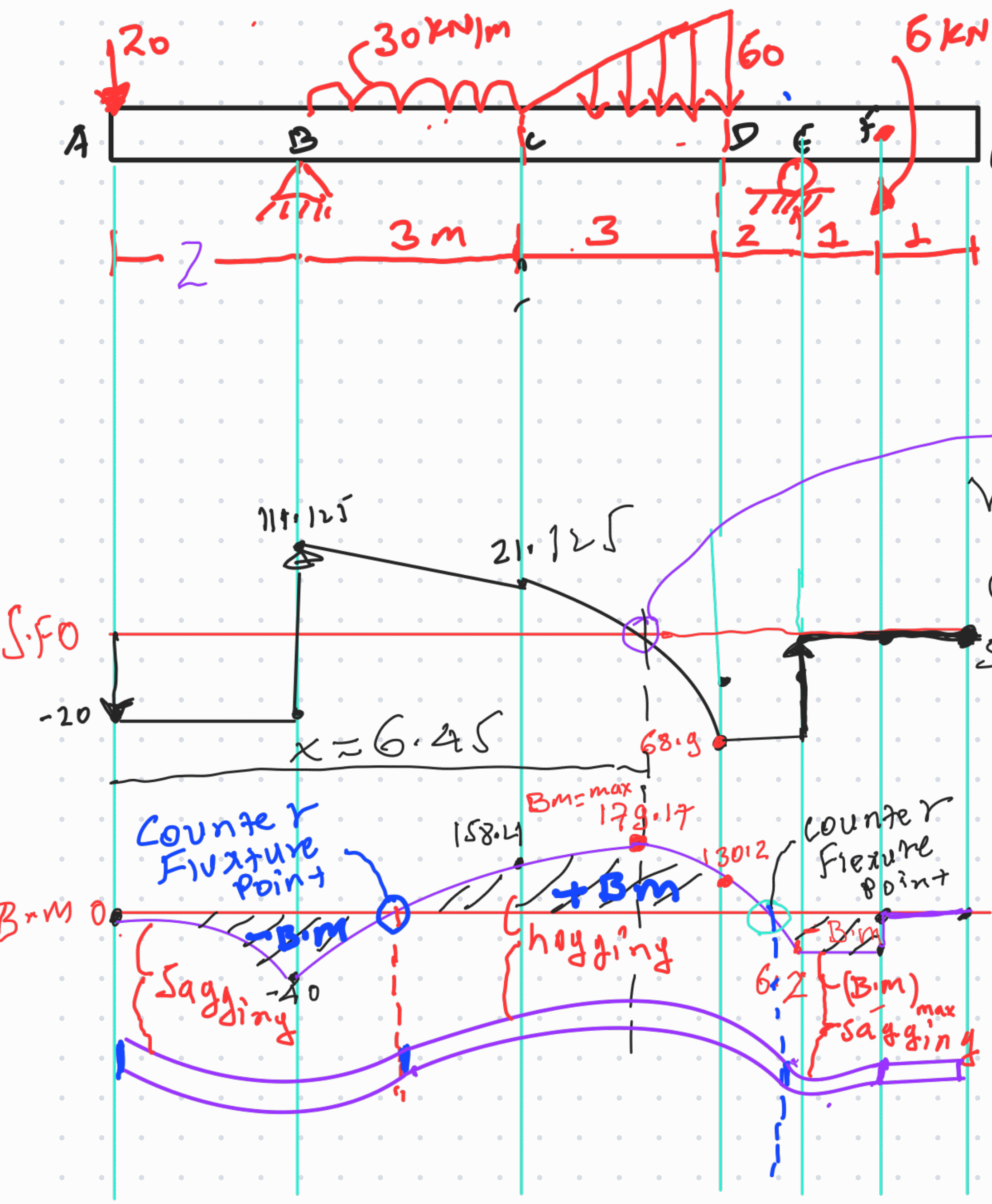
$M_x + 90(x-7) + 90(x-3.5) + 20x - 131.125(x-2) - 68.9(x-10) = 0$

$M_x = -6.2 \; kN\text{-}m$

$11 \leq x \leq 12$ $V_x = 0$

$M_x = -6.2 + 6 = 0$

→ Step 3 S.F & B.m diagram.



B In $5 \leq x \leq 8$

S.F = 0 i.e Bm = Max

$V_x = -10x^2 + 100x - 228.9$

$0 = 10x^2 - 100x + 228.9$

Solving for x, we have

$x_1 = 6.45$ or $x_2 = 3.54$

taking the root value b/n the interval

$x = 6.45$