ISTA-Net: Interpretable Optimization-Inspired Deep Network for Image Compressive Sensing

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Image Restoration

Image restoration has received much attention over the last decade. There have been many applications such as magnetic resonance imaging (MRI), image denoising and compressive sensing. Generally, many image reconstruction problems can be formulated as a linear inverse problem.

$$y = Dx + \eta \tag{1}$$

where the vector $x \in \mathbb{R}^n$ denotes the image to be recoverd and has a total of n pixels, $y \in \mathbb{R}^m$ is the observed image or measurements, $D \in \mathbb{R}^{m \times n}$ is a degradation matrix and η denotes white Gaussian noise with variance σ^2.

Two dominant strategies for solving IR

Model-based optimization methods and discriminative learning methods have been the two dominant strategies for solving various inverse problems in low-level vision. Typically, those two kinds of methods have their respective merits and drawbacks, e.g., model-based optimization methods are flexible for handling different inverse problems but are usually time-consuming with sophisticated priors for the purpose of good performance; in the meanwhile, discriminative learning methods have fast testing speed but their application range is greatly restricted by the specialized task.

ISTA-Net for Image Compressive Sensing

Optimization-based CS reconstruction

$$\min_{x} \frac{1}{2}\|\Phi x - y\|_2^2 + \lambda\|\varphi x\|_1 \qquad (2)$$

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ISTA-iterration

$$\begin{cases} r_k = x_{k-1} - \rho\Phi^{\mathsf{T}}(\Phi x_{k-1} - y), \\ x_k = \arg\min_{z} \dfrac{1}{2}\|x - r_k\|_2^2 + \lambda\|\varphi x\|_1. \end{cases} \tag{3}$$

Proximity Operator

$$\mathrm{Prox}_{t\varphi}(a) := \arg\min \frac{1}{2}\|a - b\|_2^2 + t\varphi(b). \tag{4}$$

For example, if $\varphi(x) = \|Wx\|_1$, we have $\mathrm{Prox}_{\lambda\varphi}(r) = W^\mathsf{T}\mathrm{soft}(Wr, \lambda)$.

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In this paper, they replace traditional sparse regulariton with a nonlinear transform(learnable), then x_k subproblem is formulated as

$$x_k = \arg\min_{z} \frac{1}{2}\|x - r_k\|_2^2 + \lambda\|\mathcal{F}(x)\|_1. \tag{5}$$

Therefore x_k can be efficiently computed in closed form as

$$x_k = \tilde{\mathcal{F}}(\mathrm{soft}(\mathcal{F}(r_k), \theta)) \tag{6}$$

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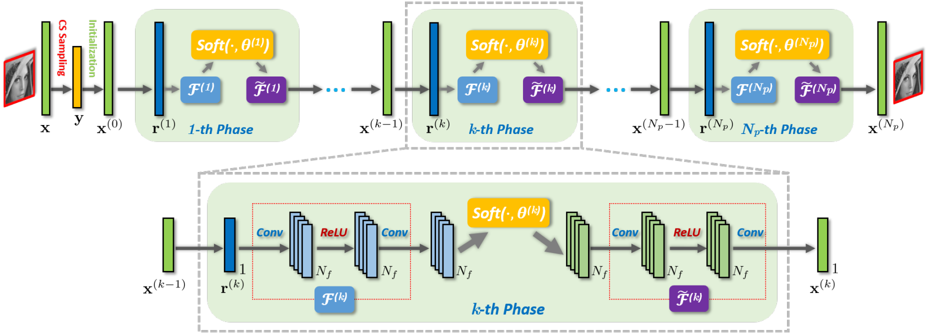
Loss function

$$\mathcal{L}_{\text{total}} = \mathcal{L}_{\text{discerpancy}} + \mathcal{L}_{\text{constraint}} \tag{7}$$


CS Sampling
Initialization
x
y
x(0)
r(1)
Soft(·, θ(1))
F(1)
F̃(1)
1-th Phase
x(k−1)
r(k)
Soft(·, θ(k))
F(k)
F̃(k)
k-th Phase
x(Np−1)
r(Np)
Soft(·, θ(Np))
F(Np)
F̃(Np)
Np-th Phase
x(Np)
Conv
ReLU
Conv
Nf
1
Conv
ReLU
Conv
x(k)

Future work

FISTA: A fast iterative shrinkage thresholding algorithm for linear inverse problems.