Homework #7

Yihan Feng

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Problem 1

In a pitch study, we are interested in the relationship between pitch and politeness.There are two levels politeness (a formal register: pol, and an informal register:inf). On top of that, we also have an additional fixed effect, gender. Each subject was tested on several scenarios (e.g., asking a peer for a favor (informal condition) or asking a professor for a favor (formal condition)). The pitch measurements are typically correlated for the same subject and in the same scenario.

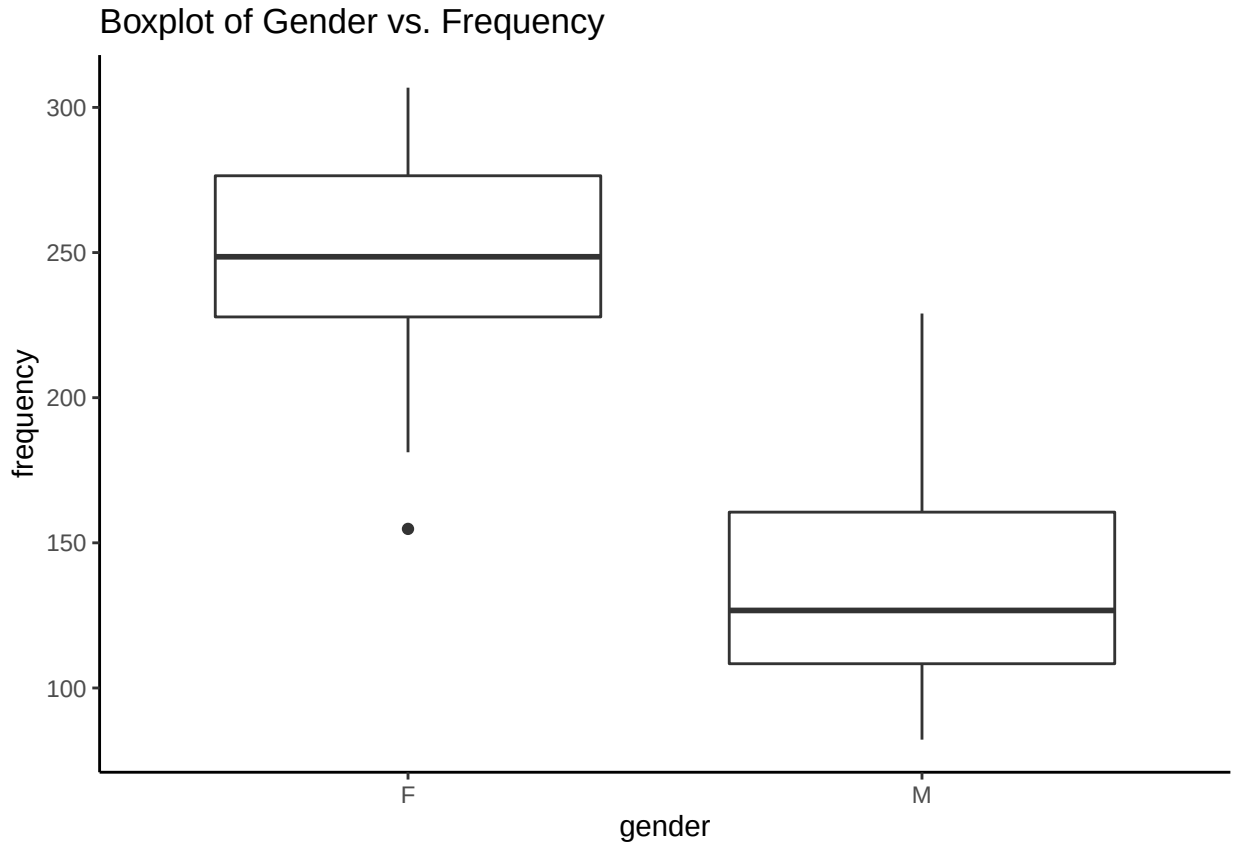
```
setwd("C:/Users/irene/OneDrive - cumc.columbia.edu/2021 M1 Spring/Biostatistical Methods 2/HW/HW7")

polite.df = read.csv("./politeness_data.csv") %>%
  janitor::clean_names()
head(polite.df)
```

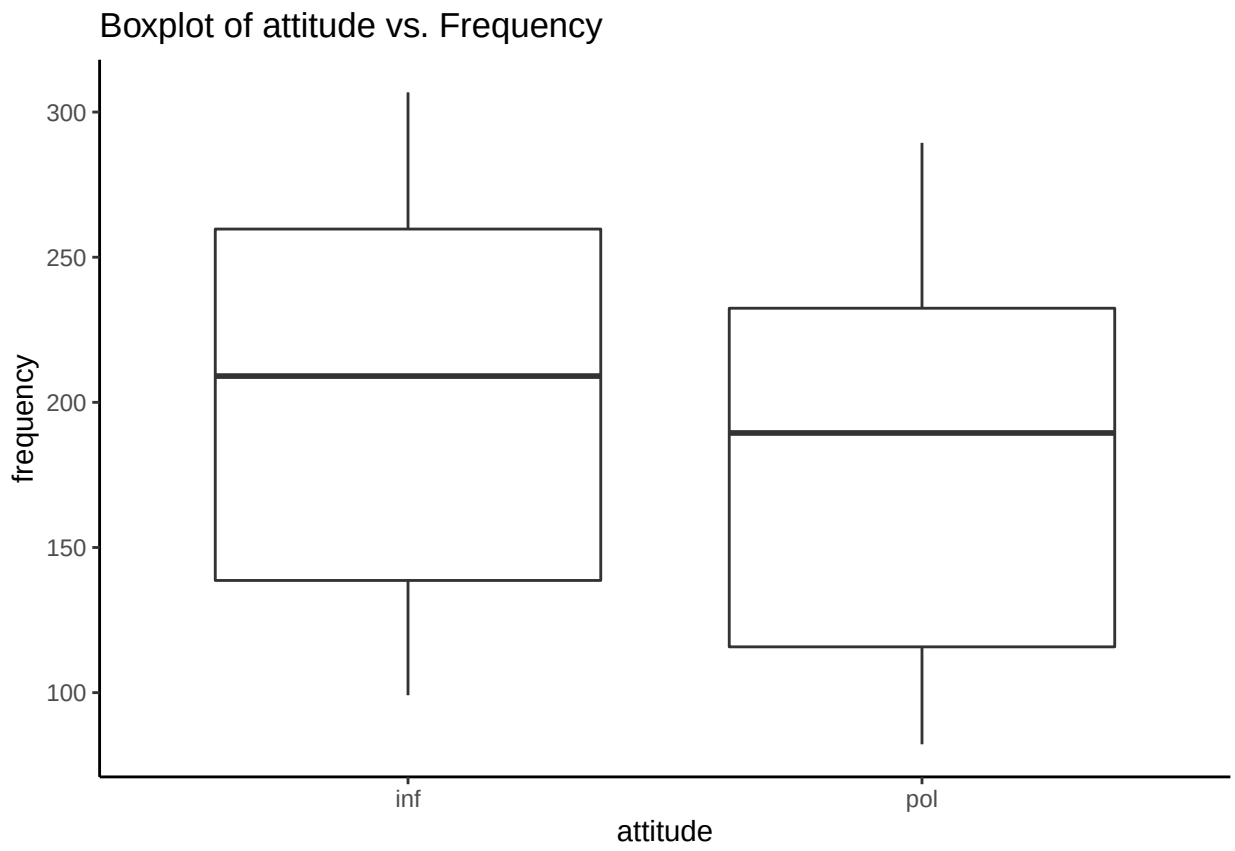
```
##   subject gender scenario attitude frequency
## 1      F1      F        1      pol     213.3
## 2      F1      F        1      inf     204.5
## 3      F1      F        2      pol     285.1
## 4      F1      F        2      inf     259.7
## 5      F1      F        3      pol     203.9
## 6      F1      F        3      inf     286.9
```

(a) Exploratory analysis: provide boxplots to show the relation between gender/attitude and pitch (ignoring different scenarios).

```
gender.plot = polite.df %>%
  ggplot(aes(x = gender, y = frequency)) +
  geom_boxplot() +
  labs(title = "Boxplot of Gender vs. Frequency") +
  theme_classic()
gender.plot
```




```
attitude.plot = polite.df %>%
  ggplot(aes(x = attitude, y = frequency)) +
  geom_boxplot() +
  labs(title = "Boxplot of attitude vs. Frequency") +
  theme_classic()
attitude.plot
```


Boxplot of attitude vs. Frequency
300
250
200
150
100
frequency
inf
pol
attitude

(b) Fit a mixed effects model with random intercepts for different subjects (gender and attitude being the fixed effects). What is the covariance matrix for a subject Yi? What is the covariance matrix for the estimates of fixed effects (Hint: 3×3 matrix for intercept, gender and attitude)? What are the BLUPs for subject-specific intercepts? What are the residuals?

```
lmm = lme(frequency ~ gender + attitude,
          random = ~ 1|subject,
          data = polite.df,
          method = "REML")
summary(lmm)
```

```
## Linear mixed-effects model fit by REML
##   Data: polite.df
##        AIC      BIC    logLik
##   806.0805 818.0527 -398.0402
##
## Random effects:
##  Formula: ~1 | subject
##         (Intercept) Residual
## StdDev:    24.45803 29.11537
##
## Fixed effects:  frequency ~ gender + attitude
##                  Value Std.Error DF   t-value p-value
## (Intercept)  256.98690 15.154986 77 16.957251  0.0000
## genderM     -108.79762 20.956235  4 -5.191659  0.0066
## attitudepol  -20.00238  6.353495 77 -3.148248  0.0023
##  Correlation:
##             (Intr) gendrM
## genderM     -0.691
## attitudepol -0.210  0.000
##
## Standardized Within-Group Residuals:
##        Min         Q1        Med         Q3        Max
## -2.3564422 -0.5658319 -0.2011979  0.4617895  3.2997610
##
## Number of Observations: 84
## Number of Groups: 6
```

1. Covariance matrix for a subject Y_i:

$$\begin{pmatrix} \sigma_b^2+\sigma^2 & \sigma_b^2 & \dots & \sigma_b^2 \\ \sigma_b^2 & \sigma_b^2+\sigma^2 & \dots & \sigma_b^2 \\ \dots & \dots & \dots & \dots \\ \sigma_b^2 & \sigma_b^2 & \dots & \sigma_b^2+\sigma^2 \end{pmatrix}$$

with values in:

$$\begin{pmatrix} 1445.9 & 598.2 & \dots & 598.2 \\ 598.2 & 1445.9 & \dots & 598.2 \\ \dots & \dots & \dots & \dots \\ 598.2 & 598.2 & \dots & 1445.9 \end{pmatrix}$$

2. Covariance matrix for the estimates of fixed effects:

```
vcov(lmm, type = "fixed")
```

```
##             (Intercept)       genderM   attitudepol
## (Intercept)   229.67362 -2.195819e+02 -2.018345e+01
## genderM      -219.58189  4.391638e+02  6.451438e-15
## attitudepol   -20.18345  6.451438e-15  4.036690e+01
```

3. BLUPs for subject specific intercepts:

```
random.effects(lmm) %>%
  knitr::kable()
```

	(Intercept)
F1	-13.575831
F2	10.170522
F3	3.405309
M3	27.960288
M4	4.739325
M7	-32.699613

4. residuals:

```
residuals(lmm)
```

```
##          F1          F1          F1          F1          F1          F1
## -10.1086926 -38.9110735  61.6913074  16.2889265 -19.5086926  43.4889265
##          F1          F1          F1          F1          F1          F1
##  27.3913074  33.3889265   8.4913074   8.9889265 -42.2086926 -12.7110735
##          F1          F1          F3          F3          F3          F3
## -26.9110735 -68.6086926 -10.6898326 -23.0922136  -3.5898326  -9.3922136
##          F3          F3          F3          F3          F3          F3
##  26.6101674   5.6077864  35.0101674  46.4077864  -7.7898326  -7.8922136
##          F3          F3          F3          F3          M4          M4
## -13.8898326  18.4077864   4.0077864 -54.8898326 -22.2262298 -29.3286108
##          M4          M4          M4          M4          M4          M4
##  96.0737702 -38.0286108 -20.7262298  60.6713892  60.4737702   9.9713892
##          M4          M4          M4          M4          M4          M4
## -31.1262298 -26.0286108 -22.9262298 -16.7286108  -6.9286108  -6.4262298
##          M7          M7          M7          M7          M7          M7
##  -9.3872916 -16.3896725 -13.2872916 -11.1896725  -9.5872916  -5.2896725
##          M7          M7          M7          M7          M7          M7
##   1.6127084   4.5103275  -1.7872916 -12.5896725  13.3127084  -7.2896725
##          M7          M7          F2          F2          F2          F2
##   8.9103275  12.1127084 -14.4550462 -35.8574271  -0.8550462  -7.4574271
##          F2          F2          F2          F2          F2          F2
##  42.2449538  34.6425729  -3.9550462  29.0425729  30.5449538  27.0425729
##          F2          F2          F2          F2          M3          M3
## -39.1550462 -41.2574271  13.8425729 -19.9550462  -2.3471929  12.6504261
```

```
##          M3          M3          M3          M3          M3          M3
## -13.7471929  23.5504261   4.0528071   9.9504261  51.3528071  14.7504261
##          M3          M3          M3          M3          M3          M3
##   4.5528071 -19.6495739  -9.4471929 -18.1495739 -15.0495739  -2.8471929
## attr(,"label")
## [1] "Residuals"
```

(c) Fit a mixed effects model with intercepts for different subjects (gender, attitude and their interaction being the fixed effects). Use likelihood ratio test to compare this model with the model in part (b) to determine whether the interaction term is significantly associated with pitch.

```
# subjects: gender, attitude, interaction
lmm1 = lme(frequency ~ gender + attitude + gender * attitude,
          random = ~ 1|subject,
          data = polite.df,
          method = "REML")
summary(lmm1)
```

```
## Linear mixed-effects model fit by REML
##   Data: polite.df
##        AIC     BIC    logLik
##   799.8018 814.094 -393.9009
##
## Random effects:
##  Formula: ~1 | subject
##         (Intercept) Residual
## StdDev:    24.46382 29.04716
##
## Fixed effects:  frequency ~ gender + attitude + gender * attitude
##                           Value Std.Error DF   t-value p-value
## (Intercept)          260.68571 15.481307 76 16.838740  0.0000
## genderM             -116.19524 21.893875  4 -5.307203  0.0061
## attitudepol          -27.40000  8.964149 76 -3.056620  0.0031
## genderM:attitudepol   14.79524 12.677221 76  1.167073  0.2468
##  Correlation:
##                      (Intr) gendrM atttdp
## genderM              -0.707
## attitudepol          -0.290  0.205
## genderM:attitudepol  0.205 -0.290 -0.707
##
## Standardized Within-Group Residuals:
##        Min         Q1        Med         Q3        Max
## -2.2344163 -0.5454437 -0.1646159  0.4697182  3.1800944
##
## Number of Observations: 84
## Number of Groups: 6
```

```
# likelihood ratio test
lmm.ml = lme(frequency ~ gender + attitude,
          random = ~ 1|subject,
          data = polite.df,
          method = "ML")

lmm1.ml = lme(frequency ~ gender + attitude + gender * attitude,
          random = ~ 1|subject,
          data = polite.df,
          method = "ML")

anova(lmm.ml, lmm1.ml)
```

```
##         Model df      AIC      BIC    logLik   Test  L.Ratio p-value
## lmm.ml      1  5 825.6363 837.7904 -407.8182
## lmm1.ml     2  6 826.2508 840.8357 -407.1254 1 vs 2 1.385523  0.2392
```

From the likelihood ratio test, the p value is $0.2392 > 0.05$. Therefore, we fail to reject the null hypothesis, and the smaller, without interaction model is better. Further, we can conclude that the interaction is not significantly associated with pitch.

(d) (Optional; required for PhD or DrPH) Write out the mixed effects model with random intercepts for both subjects and scenarios (gender and attitude being the fixed effects). Fit the model using lmer in the lme4 package. Write out the covariance matrix for a subject Yi. What is the interpretation of the coefficient for the fixed effect term attitude?

```
lmer = lmer(frequency ~ gender + attitude + (1 | subject) + (1 | scenario),
            data = polite.df)
summary(lmer)
```

```
## Linear mixed model fit by REML ['lmerMod']
## Formula: frequency ~ gender + attitude + (1 | subject) + (1 | scenario)
##    Data: polite.df
##
## REML criterion at convergence: 784.1
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -2.2690 -0.6331 -0.0878  0.5204  3.5326
##
## Random effects:
##  Groups   Name        Variance Std.Dev.
##  scenario (Intercept) 224.5    14.98
##  subject  (Intercept) 613.2    24.76
##  Residual             637.8    25.25
## Number of obs: 84, groups:  scenario, 7; subject, 6
##
## Fixed effects:
##             Estimate Std. Error t value
## (Intercept)  256.987     16.101  15.961
## genderM     -108.798     20.956  -5.192
## attitudepol  -20.002      5.511  -3.630
##
## Correlation of Fixed Effects:
##             (Intr) gendrM
## genderM     -0.651
## attitudepol -0.171  0.000
```

Covariance matrix for Y_i:

$$\begin{pmatrix} \sigma^2_{b_1} + \sigma^2_{b_2} + \sigma^2 & \sigma^2_{b_1} + \sigma^2_{b_2} & \dots & \sigma^2_{b_1} + \sigma^2_{b_2} \\ \sigma^2_{b_1} + \sigma^2_{b_2} & \sigma^2_{b_1} + \sigma^2_{b_2} + \sigma^2 & \dots & \sigma^2_{b_1} + \sigma^2_{b_2} \\ \dots & \dots & \dots & \dots \\ \sigma^2_{b_1} + \sigma^2_{b_2} & \sigma^2_{b_1} + \sigma^2_{b_2} & \dots & \sigma^2_{b_1} + \sigma^2_{b_2} + \sigma^2 \end{pmatrix}$$

with values in:

$$\begin{pmatrix} 1475.5 & 837.7 & \dots & 837.7 \\ 837.7 & 1475.5 & \dots & 837.7 \\ \dots & \dots & \dots & \dots \\ 837.7 & 837.7 & \dots & 1475.5 \end{pmatrix}$$