Homework 4 of CS520 Theory of Programming Languages

Submit your solutions to the TAs by putting them in the homework submission box in the third floor of the E3-1 building by 2:00pm of 4 December 2018 (Tuesday). If you type up your solutions, you can email them to Mr Hyoungjin Lim (lmkmkr@kaist.ac.kr). The numbers in the questions refer to exercise questions in the textbook of the course, i.e. "Theories of Programming Languages" by John C. Reynolds.

Question 1

You need to do two things for this question. First, solve the parts (a), (b) and (c) of 10.1. Among the three expressions in 10.1, ignore the third and solve these parts with the first two expressions. Second, for these two expressions, write down the canonical forms that you would get if those expressions are evaluated according to the eager evaluation.

[Hint 1] Using the following abbreviations made it much easier for me to do the required calculations.

$$N \stackrel{\text{def}}{=} \lambda b. \, \lambda x. \, \lambda y. \, b \, y \, x, \qquad\qquad T \stackrel{\text{def}}{=} \lambda z. \, \lambda w. \, z,$$

$$D \stackrel{\text{def}}{=} \lambda d. \, d \, d, \qquad\qquad F \stackrel{\text{def}}{=} \lambda f. \, \lambda x. \, f \, (f \, x).$$

[Hint 2] N is the standard encoding of the negation of booleans in the lambda calculus, and T that of true. The expression F takes a function f and composes it with itself. Thus, the first expression composes the negation operation with itself, and then applies it to true. The second expression applies F to F. Intuitively, this should lead to the composition of the second F with itself because of the first F.

Question 2

Solve 10.12. You may assume the substitution lemma stated in 10.11. Using this assumption, you have to prove that

$$[\![(\lambda v. \, e) \, z]\!] \, \eta = [\![e/(v \to z)]\!] \, \eta$$

for all variables v, expressions e, canonical expressions or variables z, and environments η. Recall that an environment η is an element in $V^{\langle \text{var} \rangle}$ (i.e. function from the set of variables $\langle \text{var} \rangle$ to V), and that every canonical form is a lambda expression $\lambda w. \, e'$. As a result, if z is a canonical expression or a variable,

$$[\![z]\!] \, \eta \neq \bot.$$