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INVESTMENTS: DELINEATING AN EFFICIENT PORTFOLIO

Upasana Mitra and M. Kannadhasan wrote this case solely to provide material for class discussion. The authors do not intend to illustrate either effective or ineffective handling of a managerial situation. The authors may have disguised certain names and other identifying information to protect confidentiality.

> Hi Rahul!
>
> I want your advice in suggesting a portfolio of mutual fund for investment of my retirement fund. Last year, when I retired, I invested the full amount in a balanced fund. As it was a diversified fund, I thought that investment in one balanced fund would allow me to diversify my investment and I would get a decent return. Unfortunately, the fund has given negative return in spite of the fact that the stock index during the period has gone up by 5 per cent. Being a retired person, I cannot take much risk but would like to get maximum possible return. Can you make a list of best performing funds and propose an efficient portfolio.

Rahul Sharma, an MBA student in his final year at a premier business school, was to join an investment firm after completion of his courses. His uncle, who retired from government service in the previous year, had sought Sharma's advice on an efficient portfolio for his savings. His uncle had read a few articles about investments and was convinced that to reduce risk, he should diversify his investments. Accordingly, he had put forth the following constraints:

- He did not want to invest in individual stocks as he felt that doing so was too risky; instead, he preferred to invest in mutual funds, which had historically provided above-average returns.
- In the previous year, his investment in a single balanced fund had not generated a satisfactory return; hence, he decided to diversify his portfolio of funds.
- He also wanted the risk of the portfolio to not be more than 10 per cent per year. Nevertheless, with this limited risk, he wanted the portfolio to provide the best possible return.
- Additionally, he didn't want any short selling of securities.

Sharma had opted for an elective on Investments in his MBA course, and advising his uncle was his first opportunity to apply his academic knowledge to practice. As his uncle wanted to invest only in mutual funds, Sharma's job was easier. He looked out for a list of mutual funds that had generated good returns over the past few years.

SELECTION OF FUNDS

Fund performance reports were widely available, but Sharma found he liked the comprehensive mutual fund information provided at a popular investment review website, www.moneycontrol.com. The website provided the performance parameters of various types of mutual funds operating in India and ranked them based on predetermined performance criteria. It also provided quarterly return data for funds' previous five years.

Sharma shortlisted a few funds from each of the preferred categories based on their past five years of annualized returns, with the assumption that funds that had performed well in the past five years would also be expected to perform well in the future. Such assumptions of expecting future performance based on the past performance of assets were common in investment literature. According to Noble Laureate Sharpe:[1]

> Most performance measures are computed using historic data but justified on the basis of predicted relationships. Practical implementations use ex post results while theoretical discussions focus on ex ante values. Implicitly or explicitly, it is assumed that historic results have at least some predictive ability.

Sharma's shortlisted funds and their past performance are provided in Exhibit 1.

To make a detailed analysis of risk and return, Sharma required the historical net asset value (NAV) of the funds under consideration. Although the details were available elsewhere, www.moneycontrol.com also provided the quarterly returns of the funds for the past five years. Sharma wanted to make his task easier and hence decided to evaluate the performance of funds based on the previous five years' quarterly returns, as the required information was readily available. The time-series of quarterly returns of the selected funds over the previous five years were compiled and are provided in Exhibit 2.

CREATING A PORTFOLIO

Which portfolio would provide the optimal return? This question was asked by every equity investor. Markowitz[2] had formalized a measure of risk in his article "Portfolio Selection" and had developed a method to form an efficient portfolio based on the expected return and risk. For investments, diversifying the portfolio could reduce risk without compromising the expected return. Markowitz was the first to point out that variance of portfolio returns could be reduced by proper diversification. He suggested that assets could be selected on the basis of their overall risk–reward characteristics. However, the benefits of diversification depended on how returns of the individual assets correlated to each other.

As mutual funds returns are "uncertain" or "random," it is impossible to accurately predict the expected rate of return. Therefore, some form of historical averages was usually taken as the base from which to estimate the expected return.

The return of a portfolio over a time period *t* can be measured as follows:

$$r_t = \frac{V_t}{V_{t-1}} - 1$$

[1] *William F. Sharpe, The Sharpe Ratio, www.stanford.edu/~wfsharpe/art/sr/sr.htm. accessed January 15, 2014*
[2] *Harry M. Markowitz, "Portfolio Selection," Journal of Finance, 1952, 7 (1), pp. 77—91.*

where V_t is the value of a portfolio at the time t, and r_t is simply the percentage change of value from one period to another. Sharma decided to take the average of the past five years' return as the basis. The quarterly return was converted to the annual return using quarterly compounding as follows.

$$r_a = (1 + r_q)^4 - 1$$

where, r_a = annualized return and r_q = quarterly return.

Variance and standard deviation of returns are the popular measure of risk and are measured as follows.

$$V_q = \frac{1}{N}\sum_{i=1}^{N}\left(r_q - \overline{r_q}\right)^2$$

where V_q is variance of quarterly return, estimated from returns of past N quarters and $\overline{r_q}$ is mean of quarterly returns. For estimating population variance, the sum of squared deviations from their mean was divided by N. When sample variance is measured, the sum of differences is divided by $(N - 1)$ instead of N. Annualized variance (V_a) can be obtained from the quarterly variance by multiplying by 4.

$$V_a = V_q \times 4$$

The standard deviation of the return is the square root of variance.

$$\sigma_a = \sqrt{V_a}$$

The above procedure of estimating returns and risk apply to a single asset. For a portfolio consisting of multiple risky assets, the expected return (r_p) and variance (V_p) of the portfolio are as follows.

$$r_p = \sum_{i=1}^{n} \omega_i \times r_i$$

$$V_p = \sigma_p^2 = \sum_{i=1}^{N}\sum_{j=1}^{N} \sigma_{i,j}\omega_i\omega_j = \sum_{i=1}^{N}\sum_{j=1}^{N} \rho_{i,j}\sigma_i\sigma_j\omega_i\omega_j$$

where $\sigma_{i,j}$ is the covariance of asset returns between asset i and asset j, $\omega_i \geq 0$, $i = 1, 2, 3, .. N$ and $\sum_{i=1}^{N} \omega_i = 1$.

Covariance $\sigma_{i,j}$ is correlation $\rho_{i,j}$ of returns between two assets multiplied by standard deviations of respective assets as follows:

$$\sigma_{i,j} = \rho_{i,j}\sigma_i\sigma_j$$

In case of a two asset portfolio,

$$r_p = \omega_1 \times r_1 + \omega_2 \times r_2$$

and

$$\sigma_p = \sqrt{\omega_1^2\sigma_1^2 + \omega_2^2\sigma_2^2 + 2\rho_{1,2}\omega_1\omega_2\sigma_1\sigma_2}$$

Correlation $\rho_{1,2}$ is a measure of the tendency of two variables moving together. It measures the degree of association in the scale –1 to +1. The value of σ_p is minimum when the value of $\rho_{1,2}$ is –1. When returns are perfectly correlated, that is $\rho_{1,2}$ = +1, there is no reduction of risk. If two stocks are not perfectly correlated, combining stocks into a portfolio would reduce the risk compared to the risk inherent in individual assets.

DIVERSIFICATION

Sharma's task was to plot the risk and return characteristics of various portfolios that could be created by mixing selected mutual funds in different proportions. A graphical representation of the risk–return profiles of the portfolios consisting of multiple risky assets is provided in Exhibit 3.

Although various possible combinations of assets could be used to form a portfolio and plotted in a risk–return space in a graph, best performing portfolios may be chosen as follows:

- From the portfolios offering the same return, the investor would favour the portfolio with the lowest risk, and
- From the portfolios having the same risk level, an investor would favour the portfolio that offered the highest rate of return.

The line joining the extreme points of the upper edge of plotted points is known as the *efficient frontier*. Any point on the efficient frontier provides the maximum expected return for the respective risk profile. Obtaining returns higher than the returns provided by portfolios on the efficient frontier is not possible by using any combination of risky assets. However, the best choice of a portfolio among the portfolios on the efficient frontier was not obvious. An investor needed to make a tradeoff between the expected return and risk to choose a portfolio based on the efficient frontier. The portfolios on the lower borer line were not as efficient as alternative portfolios could be made using the same constituent assets that offered higher return for a given risk level.

James Tobin[3] extended the work of Markowitz by adding a risk-free asset to the efficient portfolio. Several portfolios could be made by mixing a risk-free asset and an efficient portfolio lying on the efficient frontier. Sharpe[4] developed the capital asset pricing model (CAPM) and presented the concept of market portfolio. The best risk-adjusted return can be made by using the Sharpe Ratio, which is a measure that provides additional return over a risk-free rate of return of a portfolio compared with the risk involved. The Sharpe Ratio is computed as follows.

$$SR_p = \frac{\bar{r_p} - r_f}{\sigma_p}$$

[3] *James Tobin, "Liquidity Preference as Behavior Towards Risk," The Review of Economic Studies, 1958, 25, pp. 65—86.*
[4] *William F. Sharpe, Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk," Journal of Finance, 1964, 19 (3), pp. 425—442.*

where SR_p is the Sharpe Ratio of the portfolio, $\bar{r_p}$ is expected return from a portfolio, σ_p is standard deviation of portfolio return and r_f is the risk-free rate. A portfolio that provides highest Sharpe Ratio is the market portfolio. It can be located on the efficient frontier by joining a tangent line from the risk-free rate point on the Y-axis to the efficiency frontier curve (see Exhibit 4). This tangent line has the highest possible slope among all lines that can be drawn joining the risk-free point and any other point on the efficient frontier. The line joining the risk-free rate and market portfolio offers the optimal investment opportunity for an investor and is also known as the Capital Market Line, or simply CML. A point on the CML beyond the market portfolio point implies borrowing at risk-free rate. A higher return beyond the return offered by market portfolio is possible when the investor borrows at the risk-free rate and invests in the market portfolio.

FINAL PORTFOLIO

The task before Sharma was to construct an *N-asset* portfolio of risky assets that could provide the best return for a given risk. Although the task appeared to be complex, Sharma remembered that he had done a similar exercise in his investment course, where he used Excel functions to calculate the portfolio risk and return and then carried out the return maximization procedure using the "solver" add-on in Excel. He searched for the Excel files used for his assignment in the investment course and prepared a portfolio that offered the highest Sharpe Ratio.

According to the CAPM, the addition of a risk-free asset in a portfolio of mutual funds was likely to yield a better risk-adjusted return. Therefore, Sharma included in his uncle's portfolio a risk-free security that would yield 8 per cent per annum. He also prepared a comprehensive write-up to explain the fundamental concepts behind the optimal portfolio and to explain how the addition of a risk-free asset in a portfolio of risky assets was useful for generating a better risk-adjusted return.

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EXHIBIT 1: SHARMA'S SELECTED MUTUAL FUNDS

Fund #	Mutual Fund	Fund Type	Quarterly Return (Per Cent)
1	ICICI Prudential Focused Bluechip Equity (G)	Large Cap	5.19
2	Quantum Long-Term Equity (G)	Large Cap	6.59
3	DSP BlackRock Micro Cap Fund – Regular Plan (G)	Small and Medium Cap	5.44
4	SBI Emerging Businesses (G)	Small and Medium Cap	5.76
5	Reliance Equity Opportunities Fund – Retail Plan (G)	Diversified Equity	5.03
6	ICICI Prudential Exports & Other Services – Regular Plan (G)	Diversified Equity	4.74
7	HDFC Balanced Fund (G)	Balanced Fund	1.80
8	HDFC Prudence Fund (G)	Balanced Fund	2.08
9	Birla Sun Life Government Securities - Long Term Fund (G)	Gilt Long Term	2.21
10	R*Shares Gold Exchange Traded Fund	Gold ETF	3.08

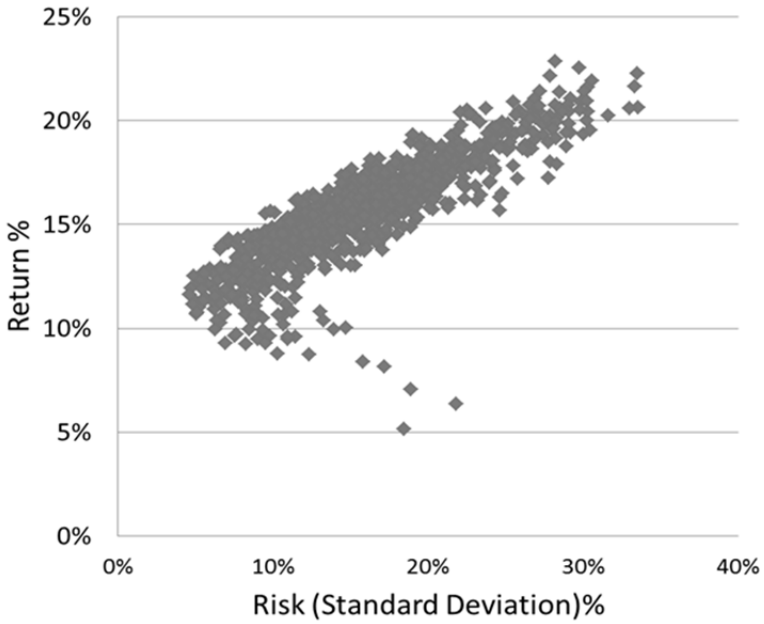
Note: (G) = growth. The quarterly returns represent the simple average of returns during the previous 20 quarters.
Source: www.moneycontrol.com, accessed on January 31, 2014.

EXHIBIT 2: QUARTERLY RETURNS (BY PERCENTAGE) OF SHARMA'S SELECTED FUNDS, 2009 TO 2013

Fund #		MF1	MF2	MF3	MF4	MF5	MF6	MF7	MF8	MF9	MF10
Quarter		ICICI Prudential Focused Bluechip Equity - (G)	Quantum Long-Term Equity – (G)	DSP-BlackRock Micro Cap Fund – Regular Plan (G)	SBI Emerging Business (G)	Reliance Equity Opportunities Fund – Retail Plan (G)	ICICI Prudential Exports and Other Services-Regular Plan (G)	HDFC Balanced Fund (G)	HDFC Prudence Fund (G)	Birla Sun Life Government Securities - Long Term Fund (G)	R*Shares Gold Exchange Traded Fund
2009	Q1	2.90	0.10	-13.70	-14.70	-6.40	-11.40	-1.00	-4.20	7.40	10.40
2009	Q2	41.60	44.30	66.00	66.20	47.50	49.10	37.60	47.00	4.70	-4.10
2009	Q3	19.20	23.40	30.50	15.70	27.40	19.80	14.10	18.20	1.80	7.70
2009	Q4	-0.20	-0.20	14.10	-0.20	-0.20	5.60	8.50	-0.20	1.70	-0.20
2010	Q1	3.40	1.90	9.20	4.60	5.40	1.80	4.00	4.10	1.20	-2.80
2010	Q2	2.90	6.10	9.40	0.20	5.00	0.40	6.70	6.60	5.70	14.20
2010	Q3	15.40	16.20	14.20	19.10	18.30	14.30	9.90	12.50	1.40	1.50
2010	Q4	1.20	0.20	-4.40	0.10	-2.60	-1.90	0.60	-0.70	1.10	7.30
2011	Q1	-2.60	-5.80	-14.70	-9.00	-7.30	-4.90	-3.10	-3.20	1.20	1.30
2011	Q2	-1.00	-2.50	1.50	4.90	2.60	-2.70	4.60	1.60	1.10	4.90
2011	Q3	-9.40	-8.70	-5.20	-1.40	-8.90	-10.70	-5.10	-6.80	1.40	19.20
2011	Q4	-2.40	-4.00	-13.50	-8.10	-8.50	-7.30	-6.60	-7.50	3.20	2.00
2012	Q1	12.70	17.10	20.00	14.60	21.10	17.80	15.50	16.00	2.00	3.80
2012	Q2	-0.60	-1.90	0.40	3.10	1.40	-3.20	-0.80	-1.10	3.40	3.50
2012	Q3	8.10	8.60	9.60	10.20	10.70	9.30	6.20	6.50	2.00	5.20
2012	Q4	4.40	3.80	4.50	13.00	5.40	5.20	2.50	5.10	2.40	-2.30
2013	Q1	-4.90	-3.90	-15.80	-12.50	-7.90	1.90	-5.80	-8.60	2.60	-2.80
2013	Q2	1.80	-0.60	-0.70	-1.70	-3.10	2.10	-0.30	-0.40	5.20	-15.60
2013	Q3	1.30	-1.10	-4.50	-8.40	-4.30	17.00	-2.30	-7.30	-6.60	16.00
2013	Q4	10.00	12.30	24.90	13.10	19.50	16.30	15.40	17.10	1.30	-7.60

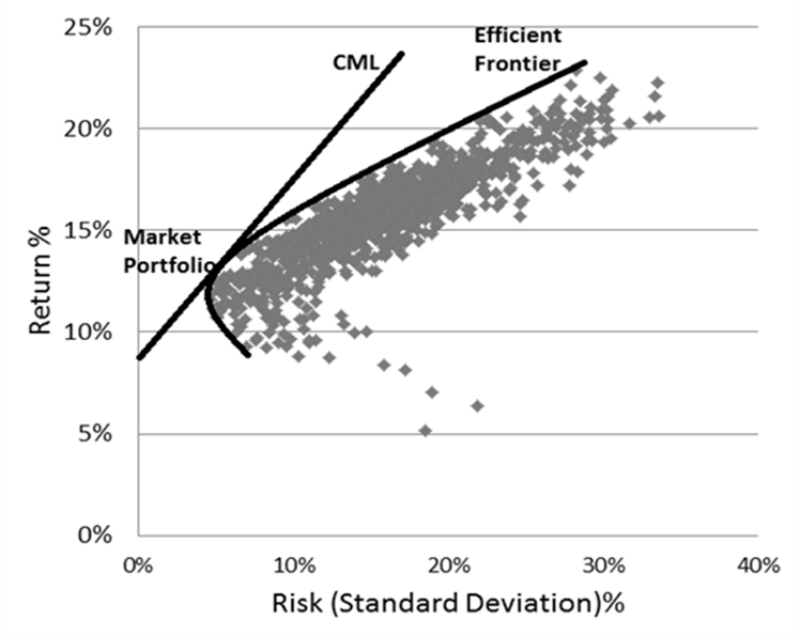
Source: www.moneycontrol.com, accessed on January 31, 2014.

EXHIBIT 3: RISK–RETURN PROFILE OF THE PORTFOLIOS CONSIDERED BY SHARMA



Source: Created by authors.

EXHIBIT 4: RISK–RETURN PROFILE OF THE PORTFOLIOS CONSIDERED BY SHARMA AND THE CAPITAL MARKET LINE



Note: CML = capital market line
Source: Created by authors.