

1. Suppose you flip  $n$  fair coins. What is the probability of getting exactly  $i$  heads, for each  $i$ . What is the probability of getting at least  $i$  heads for each  $i$ ?

$$1.1 \quad P = \binom{n}{i} / 2^n$$

$$1.2 \quad P = \sum_{k=i}^n \binom{n}{k} / 2^n$$

2. What is the probability of an odd sum when you roll three dice.

$$P = P(\text{all three are odd}) + P(1 \text{ odd} + 2 \text{ even}) = (3 \cdot 3 \cdot 3 + 3 \cdot 3 \cdot 3 \cdot 3) / (6 \cdot 6 \cdot 6) = 0.5$$

3. Suppose that each of 9 people are dealt 4 cards. What is the probability that one of the people has 2 or more kings. (I recently lost a poker hand where the only way I could have lost was if someone had 2 or more kings. I had been pretty sure I was going to win).

$$P = 1 - P(\text{every one has 0 or 1 King})$$

$= 1 - P(\text{no king is selected for everyone}) + P(1 \text{ king is selected for everyone}) + P(2 \text{ kings are selected, but only 1 max for everyone}) + P(3 \text{ kings are selected, but only 1 max for everyone}) + P(4 \text{ kings are selected, but only 1 max for everyone})$

$$= 1 - \frac{\binom{48}{36} \times \binom{4}{0}}{\binom{52}{36}} - \frac{\binom{48}{35} \times \binom{4}{1}}{\binom{52}{36}} - \frac{\binom{48}{34} \times \binom{4}{2}}{\binom{52}{36}} \times \frac{\binom{9}{2} \times \binom{4}{1} \times \binom{4}{1}}{\binom{36}{2}} - \frac{\binom{48}{33} \times \binom{4}{3}}{\binom{52}{36}} \times \frac{\binom{9}{2} \times \binom{4}{1} \times \binom{4}{1} \times \binom{4}{1}}{\binom{36}{3}} - \frac{\binom{48}{32} \times \binom{4}{4}}{\binom{52}{36}} \times \frac{\binom{9}{2} \times \binom{4}{1} \times \binom{4}{1} \times \binom{4}{1} \times \binom{4}{1}}{\binom{36}{4}}$$

$$= 0.4933$$

4. Which event is more likely:

(a) drawing an ace and a king, when you draw 2 cards from a 52 card deck.

(b) drawing an ace and a king, when you draw 2 cards from a 13 card deck consisting of only hearts.

Please explain with calculations.

52 card has 4 aces and 4 kings, but 13 card of only hearts has 1 ace and 1 king. So that:

$$P(a) = (\text{draw ace then draw king} + \text{draw king then draw ace}) / (\text{total numbers of draw 2 cards from deck})$$

$$= (4 \cdot 4 + 4 \cdot 4) / (52 \cdot 51) = 0.0060$$

$$P(b) = (\text{draw ace then draw king} + \text{draw king then draw ace}) / (\text{total numbers of draw 2 cards from deck})$$

$$= (1 \cdot 1 + 1 \cdot 1) / (13 \cdot 12) = 0.0128$$